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THE

DESIGN AND CONSTRUCTION

OF

METALLIC BRIDGES

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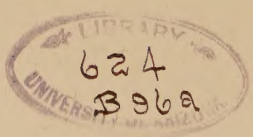
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PREFACE.

THIS book is based upon the ninth edition of "The Stresses in Bridge and Roof Trusses, Arched Ribs, and Suspension Bridges," by one of the Authors of this volume. That work, however, has been so thoroughly rewritten and such a large amount of new matter has been introduced that a new title has been adopted.

It has been the intention of the Authors to prepare a comprehensive book covering all types of trusses used to a material extent in American bridge practice, or at least all trusses forming the basis of that practice. In conformity with this plan, the most advanced methods of stress computations, as well as those that are older, have been employed.

The methods of influence lines and of deflections are used in the treatment of swing-bridges as well as the more common procedure with the theorem of three moments.

There will also be found a short chapter setting forth the main features of the application of the principle of least work to the determination of stresses in trusses.

As graphical methods of analysis have been completely set forth in the Authors' "The Graphic Method by Influence Lines for Bridge and Roof Computations," the graphic theory is but little used in this book. This work and that constitute a complete and comprehensive practi-

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cal treatment of metallic bridges, including the designs of structures, with the exceptions of suspension bridges and arched ribs. These two classes of bridge structures, together with certain other statically indeterminate forms, will be treated in a supplementary volume to be issued at an early day. This plan has been followed in order to keep the present volume within convenient proportions.

W. H. B.
M. S. F.

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THE DESIGN AND CONSTRUCTION OF METALLIC BRIDGES.

CHAPTER I.

A HISTORICAL SKETCH OF BRIDGE BUILDING.

Art. 1.—The Earliest Bridge Structures.

History of the earliest bridge building is lost in the obscurity of prehistoric times. Although the single log of timber and the long flat stone were probably the first bridges used by primitive man, the earliest evidences of genuine bridge construction are the ruins of the masonry arches of the ancient Assyrians, generally laid without cementing material in the joints. Perhaps the oldest arch of this kind yet discovered is one found at Nippur, belonging to a period about four thousand years B.C.

It is well established that the ancient Assyrians maintained a state of civilization involving the transportation of great quantities of agricultural products and other materials, and that their internal commerce was largely conducted through a perfect network of canals large enough to float boats of substantial tonnage. It is practically impossible that these conditions could have existed through many centuries without bridge structures to span

those canals. Again, the Assyrians were constructors of great buildings, some of them requiring roof-coverings of possibly 40 or 50 feet span or more. It is also known that they were skillful users of timber. Under these circumstances there is no reasonable doubt that they built timber-beam bridges of spans as great as the safe reach of a single log. The ruins of enough stone or brick arches have been found in recent years to demonstrate clearly their ability to build arches for small bridges at least.

The ruins of structures dating back probably to 2500 B.C. show that the Egyptians were also familiar with the construction of the arch. It is further demonstrated by their constructions requiring the transportation of large blocks of stone considerable distances that they were at least familiar with the use of timber beams as long as single tree-trunks. The maintenance of their industrial operations throughout many centuries amounts to conclusive evidence that they were familiar with the construction of both arches and beam bridges of at least small spans.

The same general observations may be applied to the early Greeks, and probably to other ancient nations who had made substantial progress in civilization prior to the Christian era.

The Chinese are reputed to have built ancient bridges both of the cantilever and arch types by using either sticks of timber or stonework, but there is no conclusive tangible evidence as to their ancient character.

Art. 2.—Early Roman Bridges.

The old Romans appear as the first systematic builders of bridges of magnitude among ancient peoples. They possessed the sturdy character, force, and structural intelligence necessary for the production of great engineering works, among which their early bridges, chiefly across the

Tiber at Rome, are the most prominent instances. They built both in timber and stone.

The oldest of these Tiber bridges was built in timber and is known as Pons Sublicius, which indicates that it was a pile bridge or, at least, probably with pile piers. It should be remarked in this connection, however, that Lanciani, an excellent authority on the works of the ancient Romans, believes that the abutments and piers were of

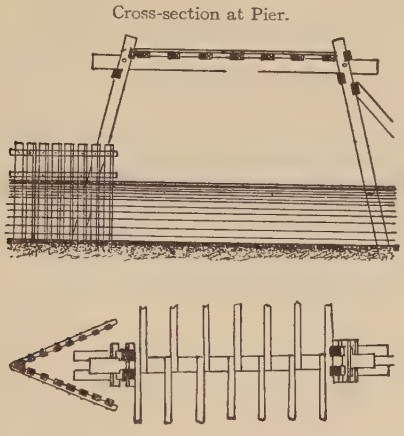


FIG. 1.—Bridge Thrown Across the Rhine by Julius Cæsar.

masonry. It is also supposed that this may have been a bridge of timber arches. As near as can be determined its date was probably about 625 B.C.

The military timber-pile bridge which Julius Cæsar built across the Rhine, reputedly in ten days, for the passage of his army, is also historical.

Art. 3.—Roman Arches.

A masonry arch bridge was built across the Tiber near Rome by the Censor Ælius Scaurus in the year 100 B.C., and as rebuilt and repaired from time to time it is now

known as the Ponte Molle. The spans of the arches vary from 51 to 79.8 feet, and the width of the structure is a little less than 29 feet.

In or about the year 104 A.D. the emperor Trajan constructed what is supposed to be a wooden arch bridge with masonry piers across the Danube just below the Rapids of the Iron Gate.

A bas-relief on the Trajan Column at Rome exhibits the timber arches, but fails to give the span lengths, which have been the subject of much controversy, some supposing them to have been as much as 170 feet.

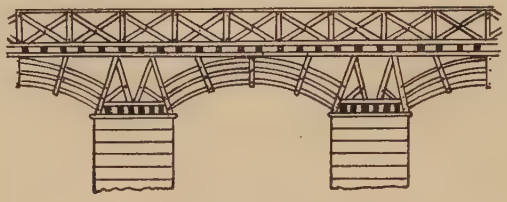


FIG. 2.—Trajan's Bridge.

The ancient Pons Fabricius, now known as Ponte Quattiro Capi, still exists, and it is the only one which remains intact after the expiration of nearly two thousand years. It has three arches, the fourth being concealed by the modern embankment at one end; a small arch pierces the pier between the other two arches. This structure is divided into two parts by the island of Æsculapius. It is known that a wooden bridge must have joined that island with the left bank of the Tiber as early as 192 B.C., and a similar bridge on the other side of the island is supposed to have completed the structure. While Lucius Fabricius was Commissioner of Roads in the year 62 B.C. he reconstructed the first-named portion into masonry arches. An engraved inscription below the parapets shows that the work was duly and satisfactorily completed, and further

that it was the custom to require the constructors or builders of bridges to guarantee their work for the period of forty years.

The Pons Cestius is a bridge since known as the Pons Gratianus and Ponte di S. Bartolomeo. Its first construction is supposed to have been completed in or about 46 B.C., and it was rebuilt for the first time in A.D. 365. A third restoration took place in the eleventh century. The modern reconstruction in 1886–1889 was so complete that only the middle arch remains as an ancient portion of the structure. The island divides the bridge into two parts the Ship of Æsculapius lying between the two, but it is not known when or by whom the island was turned into that form.

Another old Roman bridge, of which but a small portion is now standing, is Pons Æmilius, the piers of which were founded in 181 B.C., but the arches were added and the bridge completed only in 143 B.C. It was badly placed, so that the current of the river in times of high water exerted a heavy pressure upon the piers, and in consequence it was at least four times carried away by floods, the first time in the year A.D. 280.

The discovery of what appears to be a row of three or four ruins of piers nearly 340 feet up-stream from the Ponte Sisto seems to indicate that a bridge was once located at that point, although little or nothing is known of it as a bridge structure. Some suppose it to be the bridge of Agrippa.

The most historical of all the old Roman bridges is that which was called Pons Ælius, now known as Ponte S. Angelo, built by Hadrian A.D. 136. Before the reconstruction of the bridge in 1892, six masonry arches were visible, and the discovery of two more since that date makes a total of eight, of which it is supposed that only three were needed in a dry season The pavement of the

approach to this bridge as it existed in 1892 was the ancient roadway surface. Its condition at that time was an evidence of the substantial character of the old Roman pavement.

Below this bridge the remains of another can be seen at low water. It is supposed that this structure was the work of Nero, although its name is not known. The modern Ponte Sisto is a reconstruction of the old Pons Valentinianus, or bridge of Valentinian I. The latter was an old Roman bridge, and it was regarded as one of the most impressive of all the structures crossing the river. It was rebuilt in A.D. 366–367.

The most of these bridges were built of masonry and are of the usual substantial type characteristic of the early Romans. They were ornamented by masonry features in the main portions and by ornate balustrades along either side of the roadway and sidewalks.

Among the early Roman bridges should be mentioned that constructed at Alcantara in Spain, supposedly by Trajan, about A.D. 105. It is 670 feet long and its greatest height is 210 feet. One of its spans is partially destroyed.

In the old Roman military expeditions, the art of constructing temporary timber structures along lines of communication was well known and practiced with a high degree of ability. Just what system of construction was employed cannot be determined, but piles were constantly used. At least some of these timber military bridges, and possibly all, were constructed with comparatively short spans, the trusses being composed of such braces and beams as might be put in place between bents of piles. Some of the sticks of these bridges have been found in the beds of German rivers, and at other places, perfectly preserved after an immersion of about two thousand years.

The Romans developed the semicircular arch to a high degree of excellence, and used it most extensively in many

sewers, roads, and aqueducts. While the aqueduct spans were usually made with a length of about 18 or 20 feet, they built arches with span lengths as much as 120 feet or more, comparing favorably with modern arch-bridge work. They seldom used any other curve for their arches than the circle, and when they built bridges an odd number of spans was usually employed with the central opening the largest, possibly in obedience to the well-known esthetic law that an odd number of openings is more agreeable to the eye than an even number. Apparently they were apprehensive of the safety of the piers from which their arches sprang, and it was not an uncommon rule to make the thickness of the piers one third of the clear span. Nearly one fourth of the entire length of the structure would thus be occupied by the pier thicknesses. Although the use of mortar, both lime and cement, early came into use with the Romans, they usually laid up the ring-stones of their arches dry, i.e., without the interposition of mortar joints.

In addition to the Roman bridges to which reference has already been made the aqueducts supported upon masonry arches, built for the purpose of supplying water to the city of Rome, have for centuries been famous as great engineering works, as have also other and similar masonry viaduct arches built in France, Spain, Greece, and other points both in Europe, Asia Minor, and Africa.

Art. 4.—Bridges of the Middle Ages.

After the decadence of Rome the construction of bridges as well as of other large public works was nearly abandoned throughout the most of Europe, although occasionally a notable structure was completed. In 1146 a stone arch bridge was built over the Danube at Regensburg or Ratisbon, Bavaria. Forty-two years later, in

1188, the famous masonry arch bridge over the river Rhone at Avignon was finished. The arches of this bridge were of elliptical outline with the smallest radius of curvature at the crown. This structure was built by a religious brotherhood called "Fratres Pontis," which devoted itself to the building and repair of bridges during the middle ages.

The old London bridge, a series of stone arches over the river Thames, was begun by the priest and chaplain, Peter of Colechurch, in 1176, but he died four years prior to its completion in 1209. This bridge therefore required thirty-three years for its construction. It succeeded a timber bridge, which was the first bridge structure built across the Thames at London at some earlier date not now determinable.

The old masonry London bridge built by Colechurch has become famous from the series of timber houses three stories high built along it from end to end, the roadway being carried through the centre of the first floors of the houses in a covered arched passageway. These houses, said to be beautiful structures, were fitted on the roofs as promenades. They were burned in 1666, but were rebuilt more handsomely than before. The thickness of the piers ranged from 25 to 34 feet, two thirds of the entire waterway, about 337 feet in width, being occupied by them. The new London bridge now standing, which took the place of the old bridge, was designed by George Rennie and built by his brother Sir John Rennie, the eminent engineer. The centre arch has a span of 152 feet and a rise of 29.5 feet above Trinity high-water mark. The other span lengths are 140 and 130 feet.

The Trinity arch bridge at Florence, which some have considered the handsomest arch bridge in existence, was built in 1566. The longest span is 96 feet in the clear, while the adjoining spans are 83 feet and 86 feet in the

clear. The famous bridge of the Rialto at Venice was begun in 1587 and finished in 1591. It is a segmental arch 91 feet in span, having a width of footway of 72 feet. Its rise is about 24 feet.

A remarkable triangular arch bridge of three pointed arch openings with a common crown was built at Croyland in England at an early date, possibly as early as 943, although some think it was built in the early part of the fourteenth century.

Other masonry arch bridges were built at various points in Europe prior to 1700, possessing generally less interest than those named. Some were of unusual design with the apex of the roadway at the crown of the arch, approached on either side by a steep grade. Masonry arches continued to be built throughout the eighteenth century, but they possessed few characteristics different from those which had previously been constructed.

Art. 5.—Early Timber Bridges.

Foreign Bridges.

In the history of bridge building the middle and latter half of the eighteenth century are notable as witnessing the construction of the first timber bridges, resembling to some extent the truss type, and the construction of the first iron bridges, which were cast-iron arches. Probably the most remarkable of all timber structures was the bridge at Wittingen, built in 1758 by the Grubenmann Brothers. Its design was a mixture of arch and truss which would certainly defy analysis by stress computations. It possessed what may be termed three horizontal chords, one at the top of the structure, one about midway of its depth, and the other at the bottom. It had some characteristics of an arch, although there was no true arch member running throughout its length. It consisted

more nearly of a superposition of a number of queen-post trusses with some timbers disposed throughout its length in such a way as to act somewhat like an arch. The span of this bridge was 390 feet. It was burned about 1799. The Schaffhausen bridge built by the same engineers at nearly the same time as the Wittingen bridge was of essentially the same type, but it had two spans, one of 172 feet and the other of 193 feet.

American Bridges.

In 1660 what was called the "Great Bridge," a bridge on piles, was built across the Charles River near Boston. Other similar structures followed, but the first long-span timber bridge, where genuine bridge trussing or framing was used, appears to have been completed in 1792, when Colonel William P. Riddle constructed the Amoskeag Bridge across the Merrimac River at Manchester, N. H., in six spans of a little over 92 feet from center to center of piers. From that time timber bridges, mostly on the combined arch and truss principle, were built, many of them examples of remarkably excellent engineering structures for their day. Among these the most prominent were the Bellows Falls Bridge, in two spans of 184 feet each from center to center of piers, over the Connecticut River, built in 1785–92 by Colonel Enoch Hale; the Essex-Merrimac Bridge over the Merrimac River, three miles above Newburyport, Mass., built by Timothy Palmer in 1792. consisting of two bridges with Deer Island between them, the principal feature of each being a kind of arched truss of 160 feet span on one side of the island and 113 feet span on the other; the Piscataqua Bridge, seven miles above Portsmouth, N. H., in which a "stupendous arch of 244 feet chord is allowed to be a masterly piece of architecture, planned and built by the ingenious Timothy Palmer of Newburyport, Mass.," in 1794; the so-called

"Permanent Bridge" over the Schuylkill River at Philadelphia, built in 1804–06, in two arches of 150 feet and of 195 feet, all in the clear, after the design of Timothy Palmer; the Waterford Bridge over the Hudson River, built in 1804 by Theodore Burr, in four combined arch and truss spans, one of 154 feet, one of 161 feet, one of 176 feet, and the fourth of 180 feet, all in the clear; the Trenton Bridge, built in 1804–06 over the Delaware River at Trenton, N. J., by Theodore Burr, in five arch spans of the bowstring type, ranging from 161 feet to 203 feet in the clear; a remarkable kind of wooden suspension bridge (Fig. 3) built by Theodore Burr in 1808 across the Mohawk River at Schenectady, N. Y., in spans ranging in length from 157 feet to 190 feet; the Susquehanna Bridge at Harrisburg, Pa., built by Theodore Burr in 1812–16 in twelve spans of about 210 feet each; the so-called Colossus Bridge (Fig. 4), built in 1812 by Lewis Wernwag over the Schuylkill River at Fairmount, Pa., with a clear span of 340 feet $3\frac{3}{4}$ inches; the New Hope Bridge, built in 1814 over the Delaware River, in six 175 foot combined arch and truss spans, and a considerable number of others built by the same engineer.

Some of these wooden bridges, like those at Easton, Pa., and at Waterford, N. Y., remained in use for over ninety years with only ordinary repairs and with nearly all of the timber in good condition. In such cases the arches and trusses have been housed and covered with boards, so as to make what has been commonly called a covered bridge. The curious timber suspension bridge built by Theodore Burr at Schenectady was used twenty years as originally built, but its excessive deflection under loads made it necessary to build up a pier under the middle of each span so as to support the bridge structure at those points. These bridges were all constructed to carry highway traffic, but timber bridges to carry railroad traffic were

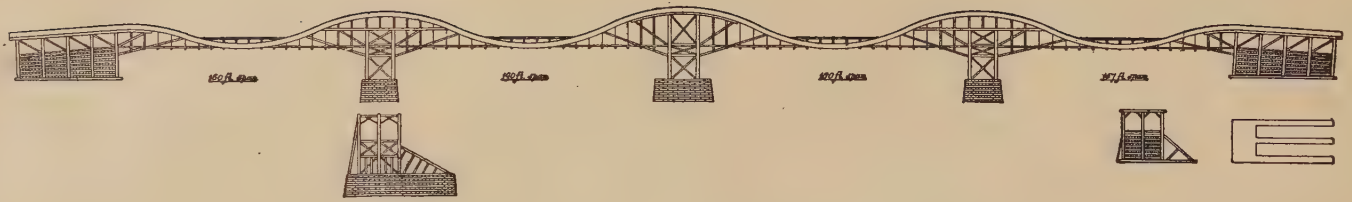


FIG. 3.—Mohawk Bridge at Schenectady, N. Y. Built by Theodore Burr.

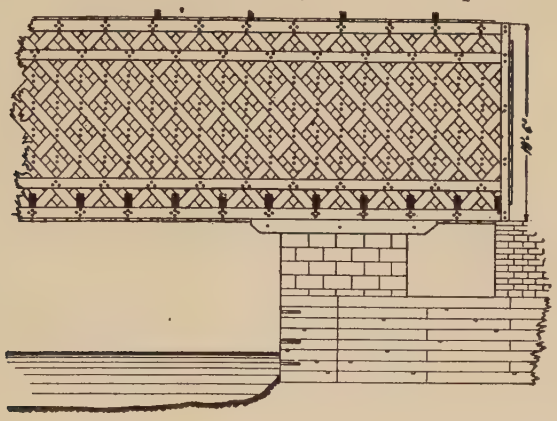


Single Arch 340 feet 3¾ inches.

FIG. 4.—Patent Bridge "Colossus," Across the River Schuylkill at Philadelphia. Built by Lewis Wernwag.

subsequently built on similar plans, except that Burr's plan of a wooden suspension bridge at Schenectady was never repeated.

A later type of timber bridge which was most extensively used in this country was invented by Ithiel Town in January, 1820, which was known as the Town lattice bridge. This timber bridge was among those used for railroad structures. As shown in Fig. 5 it was composed of a close



178 feet clear span.

FIG. 5.—Town Lattice Truss.

timber lattice, heavy planking being used as the lattice members, and they were all joined by wooden pins at their intersections. This type of timber structure was comparatively common not longer ago than twenty-five years, and probably some structures of its kind are still in use. The close latticework with its many pinned intersections made a safe and strong framework, and it enjoyed deserved popularity. It was the forerunner in timber of the modern all-riveted iron and steel lattice truss. It is worthy of statement, in connection with the Town lattice, that its inventor claimed that his trusses could be made of wrought or cast iron as well as timber. In many cases timber arches were combined with them.

The next distinct advance made in the development of bridge construction in the United States was made by Brevet Lieutenant-Colonel Long of the Corps of Engineers, U. S. A., in 1830–39, and by William Howe, who patented the bridge known as the Howe truss (Fig. 6), although the structure more lately known under that name is a modification of Howe's original truss. Long's truss was entirely of timber, including the keys, pins, or treenails required, and it was frequently built in combination with the wooden

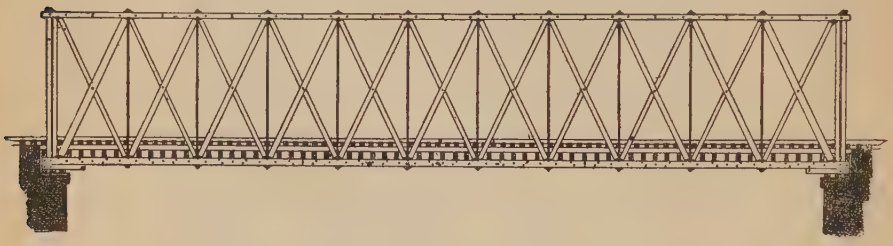


FIG. 6.—A Howe Truss Bridge of about 120 Feet Span.

arch. It was considerably used, but it was not sufficiently popular to remain in use.

The Howe truss was not an all-wooden bridge. The top and bottom horizontal members, known as "chords," the inclined braces between them and the vertical end braces, all connecting the two chords, were of timber and they were bolted at all intersections; but the vertical braces were of round iron with screw ends. These rods extended through both chords and received nuts at both ends bearing on cast-iron washers. These wrought-iron round rods were in groups at each panel-point, numbering as many as existing stresses required. The ends of the timber braces abutted against cast-iron joint-boxes. The railroad floor was carried on heavy timber ties running entirely across the bridge and resting upon the lower-chord members. It was a structure simple in character, easily framed, and of materials easily secured. It was also easily erected

and could quickly be constructed for any reasonable length of span. It possessed so many merits that it became widely adopted and is used in modified form at the present day, particularly on lines where the first cost of construction must be kept as low as possible. The large amount of timber in it and the simple character of its wrought-iron or steel members greatly reduces its first cost.

In 1844 the two Pratts, Thomas W. and Caleb, patented the truss, largely of timber, which has been perpetuated in form by probably the largest number of iron and steel spans ever constructed of a single type. The original Pratt trusses had timber upper and lower chords, but the vertical braces were also made of timber instead of iron, while the inclined braces were of wrought iron with screw ends, the reverse of the web arrangement in the Howe type. This truss had the great advantage of making the longest braces (of iron) resist tension only, while the shorter vertical braces resist compression. As a partial timber bridge it could not compete with the Howe truss, because it contained materially more iron and consequently was more costly. This structure practically closed the period of development of timber bridges.

Art. 6.—Early Iron Bridges.

Probably the first application of iron to the construction of a bridge of any considerable length of span was the cast-iron arch bridge built at Coalbrookdale, England, in 1779. The span of this bridge was 100 feet. It is supposed that an attempt was made to build a cast-iron bridge at Lyons in 1755, but that the attempt was ultimately abandoned.

The Wearmouth bridge in England is also a cast-iron arch of open panels which act as voussoirs. This bridge has a span of 236 feet with a rise of 34 feet. It is said by

Professor Fleeming Jenkin to contain 214 tons of cast iron and 46 tons of wrought iron. It is a remarkable structure for those early days of bridge building and was designed by a Mr. Rowland Burdon. In both these early structures of cast iron it was clearly in the mind of the designer simply to construct in a manner best adapted to the material open voussoirs which when put in place in the arch would act precisely as do the stone voussoirs of a masonry arch. The idea of a braced arch rib of modern type was obviously not considered by the builders of these early iron bridges.

It is a curious historical fact that Thomas Paine endeavored to introduce long cast-iron arches with a small rise of about one twentieth of the span in 1786. He constructed and tested at his own expense a model 90 feet in length which he intended as a basis for a 400-foot span. The French Academy of Sciences at Paris made a favorable report on his design, but the matter never reached a successful issue.

Soon after the construction of the Wearmouth bridge, however, Thomas Telford built what is probably the first cast-iron arch bridge embodying the advanced idea of braced members forming what would now be called an arched rib with braced spandrels. This bridge was built at Craigellachie in the beginning of the nineteenth century. From that time on braced metallic arched ribs have been matters of common construction, first in Europe and then in America.

The first bridge in America, completely of iron, was built in 1840 at Frankfort, N. Y., over the Erie Canal, by Mr. Earl Trumbull.

Art. 7.—First Iron Railway Bridges.

The first iron railway bridge appears to be a small cast-iron structure in four spans of 12½ feet each, built in England for the Stockton and Darlington Railway over the river Gaundless in 1823. As shown in Fig. 7, each span was lenticular in its outline, the curved top and bottom members being wrought iron and the vertical members

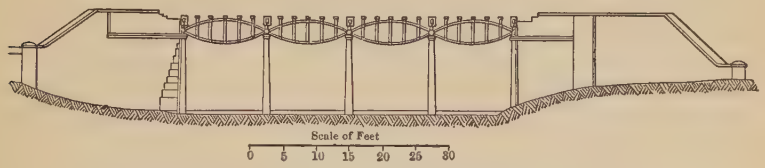


FIG. 7.—Diagram Elevation of Iron Railway Bridge Built in 1823, for the Stockton & Darlington Ry., England.

cast iron. The ends of these four spans were carried on cast-iron bents with cross-bracing. It will be noticed that the ends of the end spans did not rest on the masonry, but on the two cast-iron bents immediately adjacent to it. This bridge was in full use until 1842, but for a number of years subsequent to that date its duty was largely reduced and eventually it was put out of service entirely.

Art. 8.—Early Railway Bridges in the United States.

The beginning of railroad-bridge building in the United States is not a matter of clear record in all its details. The first wooden railway bridge is said to have been built by Wernwag for the B. & O. R. R. at Monaguay in 1830. It seems to be beyond much doubt, however, that the earliest of the timber trestles which have been used so much for railroad purposes in this country were built on what is now known as the Catawissa branch of the Philadelphia and Reading road in or about the year

1840. These trestles varied from 53 to 129 feet in height and were as much as 1000 feet long.

The first iron truss railroad bridge constructed in the United States was built in the year 1845 for the same railroad. This bridge was a combined wrought- and cast-iron structure designed on the Howe type by Mr. Richard B. Osborne, then chief engineer of the Reading Company. It crossed a small stream near Manayunk and was in use up to 1901, although for many years it had been supported by timber framework placed underneath it. The clear span was a little over 34 feet, and the depth of truss about 3 feet 6 inches. The compression members were of cast iron and the tension members of wrought iron.

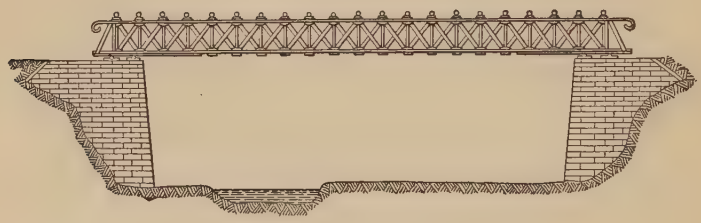


FIG. 8.—Elevation of Manayunk Bridge.

It was a three-truss double-track bridge. The accompanying figure shows the general character of the structure. It was constructed entirely by hand, as there were no facilities available at that time for power-forging and the use of tools other than by hand. Three other bridges of the same type were also constructed and erected in 1845 on the same railroad.

Art. 9.—Wrought-iron Tubular Bridges.

One of the most remarkable advances in wrought-iron bridge construction in the first half of the nineteenth century was the Britannia tubular bridge (1845) built across the Menai Straits to carry the Chester and Holyhead

Railway, each span consisting of two independent parallel tubes. Its design was conceived by Robert Stevenson, under whose direction it was constructed. The tubes are rectangular in outline, 15 feet wide, and vary in depth from 23 feet at the ends to 30 feet at the centre. Each tube is built of wrought-iron plates and angles, and has eight cells each 1 foot 9 inches square for the top chord and six cells each 2 feet 4 inches wide by 1 foot 9 inches high for the bottom chord. The spans of the Menai bridge, four in number, are about 459 feet, and 230 feet in the clear. The construction of this tubular bridge was followed by the building of that across the St. Lawrence River at Montreal, Canada.

Art. 10.—Early Truss Structures.

Probably the first true truss of magnitude in England is found in the Newark Dyke bridge built in 1851–53. The length of these trusses, composed of equilateral triangles forming a true Warren girder, is 259 feet and the depth from center to center of chords is 16 feet, the clear span being 240 feet 6 inches. The tension members are of wrought iron and the compression members of cast iron, in such proportion as to make a little less than six tenths of the total truss weight of cast iron and a little over four tenths of wrought iron. This bridge carries the Great Northern Railway over a branch of the river Trent near Newark. It was designed by Mr. Charles Wild and built by Mr. Joseph Cubitt in 1851–53.

Art. 11.—Whipple's Early Work. Howe, Pratt, Bollman, and Fink Trusses.

The evolution of the modern railroad truss-bridge began in the United States soon after the publication of Squire Whipple's historic work on Stresses in Bridge-trusses in

1840. Prior to Whipple's publication the design of bridges was essentially a matter of experience and judgment. No rational or philosophic conception of stresses in truss members had been acquired up to that time. The Howe and Pratt trusses were rational frameworks, and lent themselves readily to the system of analysis set forth by Whipple. They were therefore rapidly developed into bridge structures of exceptional excellence, and they form the foundation of the greater part of American truss railroad bridges.

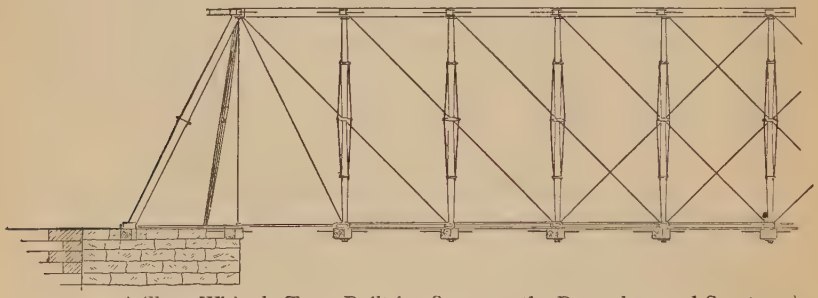


FIG. 9.—A Skew-Whipple Truss Built in 1852–3 on the Rensselaer and Saratoga R.R., 7 Miles North of Troy, N. Y. Panel length = 11 ft.

The Whipple truss as adapted to railway bridge construction consisted of cast-iron upper chords, cast-iron vertical interior and inclined end posts and wrought-iron tension members It was the initial truss of a large part of subsequent American bridge work.

Between about 1840 and 1852 there were developed two other types of bridge structures of considerable merit, the Bollman and the Fink. Strictly speaking, both these trusses are formed by superposing a number of inverted king-post trusses. In the Bollman bridge (Fig. 10) these superposed elementary trusses are of uniform depth and span, while in the Fink truss (Fig. 11) they are of varying span and may be of varying depth. Both trusses were used for through- and deck-bridges. In neither case, as is evident from the illustrations, is there a true lower or tension chord.

Wendell Bollman brought out his invention and applied it extensively on the Baltimore and Ohio Railroad between 1840 and 1850.

The Fink truss was brought out by Mr. Albert Fink about 1851, the first structure of prominence which he

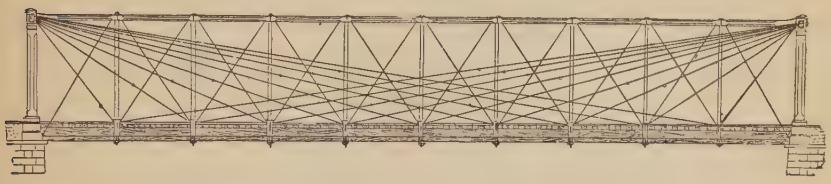


FIG. 10.—The Bollman Patent Iron Suspension Truss Bridge.

built being a bridge (Fig. 11) over the Monongahela River at Fairmont, West Virginia, in which there were three spans of 205 feet each. It was completed in 1852. Although both the Bollman and the Fink trusses possessed marked simplicity, the Fink proved to be the structure of greater merit, and soon displaced the Bollman truss. A large number of

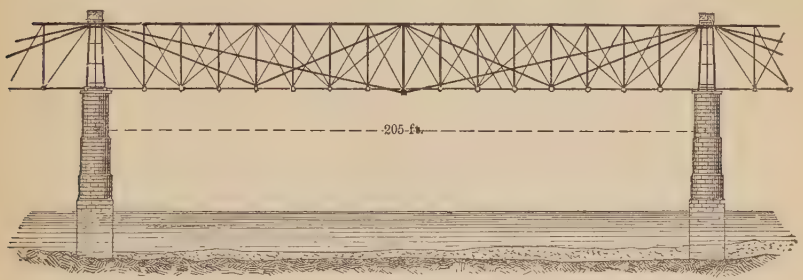


FIG. 11.—Fink Truss over the Monongahela River on the B. & O. R.R. Constructed under B. H. Latrobe, Chief Engineer, 1851–2.

Fink bridges were built during the fifteen or twenty years subsequent to their invention. The single-track iron railroad bridge across the Ohio River at Louisville, Ky., built between 1867 and 1870 was the largest of the Fink structures. The decadence of this type of bridge began about

that time, and soon after the completion of the Louisville bridge it was entirely displaced by other trusses.

The first plate-girder bridge was built in 1860 by Mr. E. S. Philbrick for the Boston and Albany Railroad.

Art. 12.—Pin Connections for Trusses.

Concurrently with the evolution of the American railroad truss, the pin connection at panel-points began to be used in American practice and has become a distinct feature in the greater part of bridge construction in the United States. This mode of connecting truss members at the panel-points in both upper and lower chords was a feature of the earliest bowstring and parallel-chord trusses of Whipple, and of the Howe and Pratt trusses when built in iron. Fink and Bollman also used the detail. A pin connection possesses the merit of simplicity and enables a true concurrence of stresses in all the members coupled at a panel-point to be attained without difficulty. It also conduces to economy and rapidity of erection. The pin detail was naturally crude in its earlier use, but it has been developed to a great degree of perfection in modern long-span bridges.

Art. 13.—Riveted Connections for Trusses.

The method of connecting the main members of bridge-trusses at panel points by suitably riveted details has also been used in American bridge practice almost from the earliest adaptation of iron and steel to this field of engineering work. It makes a more rigid connection for short-span bridges and for light trusses. Its peculiar excellence has always been recognized, and with the advance in shop facilities it has found a wide application in American bridge practice, until at the present time the consensus of engineering opinion gives it the preference over all types

of railroad-bridge structures for spans ranging from 125 to about 200 feet. Indeed it has been used with advantage for much longer spans and in work of the heaviest character.

Art. 14.—The Work of Whipple, Murphy, and Linville.

Among the names associated with the actual construction of early American railroad bridges are those of Squire Whipple, who built a single-track railroad bridge containing a span of 146 feet in 1832 near Troy, N. Y.; John W. Murphy, who in 1863 at Mauch Chunk, for the Lehigh Valley R.R., was probably the first engineer to use wrought iron for all the compression members

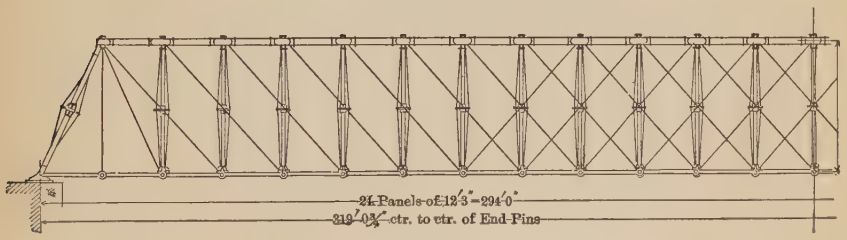


FIG. 12.—Linville Truss Built in 1863–4 at Steubenville.

in his trusses in connection with cast-iron joint-blocks and pedestals; and J. H. Linville (Fig. 12), who introduced, in 1861, the use of forged eye-bars in connection with wrought-iron posts for web members. In his first truss-bridges the latter used cast-iron upper chords, but subsequently adopted wrought iron for those members. Whipple appears to be the first American engineer to use a double-intersection web system. Murphy subsequently improved this type of truss so that for many years double-intersection trusses with parallel chords were frequently called Murphy-Whipple trusses.

It was only after the Ashtabula bridge disaster in 1876 that cast iron became fully discredited for main bridge mem-

bers. The metal was used for many years subsequent to that date for short blocks subject to compression only, until it was displaced by steel castings.

Art. 15.—The Post Truss.

American pin-bridge practice has trended constantly towards the employment of vertical compression web members, those in tension only being inclined. In 1865, however, an American engineer, Mr. S. S. Post, patented what is known as the Post truss (Fig. 13), and which was widely used for perhaps fifteen or eighteen years subsequently to that date. In this truss the compression web members

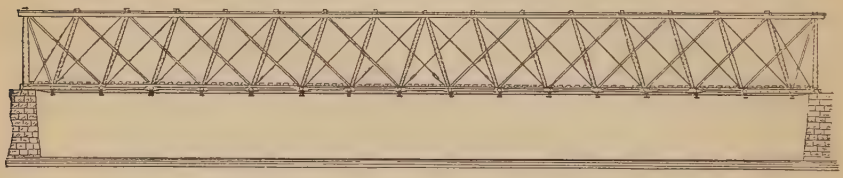


FIG. 13.—A Post Bridge of 200 Feet Span.

were inclined at such an angle as was supposed to give the greatest economy of material to the web system. Not only single systems of bracing of this type were employed, but double systems also. The pin connection was used in the Post truss. Although much was made of this system for a number of years, it was not able to maintain its place and for the past twenty years it has scarcely been known, except as a matter of history.

The first bridge entirely of steel was the Glasgow bridge over the Missouri River, built in 1879; but steel had been used in the main parts of the St. Louis (Eads's) bridge, completed in 1874.

Art. 16.—Present Types of Trusses.

During the past ten to fifteen years American railroad bridges of spans under 600 feet have in the main developed into plate girders up to 100 feet in span or more, trusses with riveted connections from approximately 100 to 200 feet in length, and pin-connected spans from 175 or 200 feet to the upper limit of length for simple truss-bridges. This classification is necessarily largely approximate, but it gives a general idea of sufficient definition for the purpose.

In the early years of iron and steel bridge building the sizes of individual members were limited by the shop capacity for handling and manufacturing and by the relatively small dimensions of bars of various shapes and of plates which could be produced by rolling mills. As both mill and shop processes have advanced and their capacities increased, corresponding progress has been made in bridge design. Civil engineers have availed themselves of these advances, so that at the present time single-system trusses with panel lengths of almost 40 feet and with depths as great as 85 feet or more and with spans of 550 feet are not considered especially remarkable. In order to construct trusses of such lengths in former times, double- and triple-intersection trusses, or trusses with subdivided panels, were required; the latter system is still deservedly employed, for it combines the theoretic advantage of the long panel with the advantages of a floor system with short and therefore light stringers.

The three longest simple spans in this country to-day include the two over the Ohio River: one, the Louisville and Jeffersonville bridge, built in 1894, with a length of 546 feet 6 inches, the other the Cincinnati and Covington bridge, built in 1889 with a length of 542 feet 6 inches, and the bridge over the Delaware River at Philadelphia, erected in 1896, with a span length of 533 feet.

SUSPENSION BRIDGES.

Art. 17.—Early Suspension Bridges.

Suspension bridges consisting simply of roadways suspended from unstiffened cables have been used probably from a remote period. It is supposed that such structures were used by the early Chinese, as well as by other peoples. The first authentic account of a genuine suspension bridge built entirely of iron of crude type is that of a 70-foot chain suspension bridge built over the river Tees in England in 1741. It was but 2 feet wide and intended for passenger traffic only.

Art. 18.—Suspension Bridges of James Finley.

Mr. James Finley, an American engineer of Fayette County, Pa., was the earliest builder in the United States to approximate to a rational design for a suspension bridge. He built his first structure in 1796, a chain suspension bridge, across Jacob's Creek, between Uniontown and Greenburg, Pa. He used masonry towers, and in order to make the reactions vertical at the tops of the towers, he gave the same angle of inclination to the cables on the two sides of the tower. The links of his chains were long enough to make a single panel in the bridge. The timber floor platform, so formed as to afford some stiffness, was suspended from the cables by vertical suspenders. He made the sag, or deflection of the cable, one seventh of the span. His bridges had no stiffening trusses, the stiffness of the floor platform constituting the whole stiffness of the structure. Finley patented his system of design in 1801. It is stated that he built forty such suspension bridges between 1800 and 1808.

The next span to which he adapted his bridge was 306 feet. This bridge crossed the Schuylkill River near Philadelphia. He built bridges, among other localities, across the Potomac River at Washington with a span of 130 feet, across Brandywine Creek near Wilmington, Del., with a span of 145 feet, and two bridges of 120 feet span each at Brownsville, Pa. The width of roadway employed by him was as much as 30 feet for the Brandywine Bridge.

Another suspension-bridge structure was that built across the Merrimac River, near Newburyport, Mass., by John Templeman on Finley's design. This bridge was built with a span of 244 feet with two roadways, each 15 feet wide. The cables of this bridge consisted of ten chains, each 576 feet long; four were placed in the middle of the structure and three on either side. The suspension bridge built across the Lehigh River in 1811 near Northampton, Pa., is supposed to be the first suspension bridge of more than one span. The structure consisted of two whole and two half spans, making a total length of 475 feet. It has two roadways and two 6-foot sidewalks or footways.

Art. 19.—First Wire Suspension Bridge.

The first wire suspension bridge ever built is supposed to be that which was constructed across the Schuylkill River near Philadelphia in 1816. It took the place of a chain suspension bridge destroyed in January of the same year by a heavy load of snow and ice. A firm of wire-manufacturers, White and Hazard, had a wire-mill near the site of the bridge, and they used their manufactured product in rebuilding the bridge, which had a span of 408 feet. The cables carried but a single foot-walk 18 inches wide, and the structure was considered safe for only eight people at a time. It has been stated that the entire cost of the bridge was $125. It is believed that the first

wire suspension bridge in Europe was built at Fribourg, Switzerland, in 1834.

While cable suspension bridges were built at subsequent times and in various places in the United States during the first half of the nineteenth century, all were of more or less indifferent design. None of these structures was a truly stiffened suspension bridge, nor were stiffened suspension bridges built in America prior to 1851, when Roebling's pioneer and historic work at Niagara began in the United States.

Art. 20.—First True Stiffened Suspension Bridge.

The Niagara wire-cable suspension bridge, built by John A. Roebling between 1851 and 1855, was a true stiffened suspension bridge. The stiffening trusses were 18 feet deep and designed to carry a single-track railway on the top of the stiffened structure 24 feet wide, and a highway on the lower floor with a span of 821 feet 4 inches between centers of towers. This structure was 245 feet high above the water. The railway floor was suspended from two cables, and the highway floors from two other cables of 10 feet greater sag. These cables were each composed of 3640 charcoal-iron wires divided into seven strands of 520 wires each. The wire was of such size as to require sixty to make 1 square inch of section, the total area of each cable being 60.4 square inches. The towers were originally of masonry, 60 feet high, 15 feet square at the base, and 8 feet square on top. These masonry towers were displaced with others of steel in 1886. This historic suspension bridge was displaced by a double-track spandrel braced arch in 1897. The Niagara suspension bridge was the first bridge of that type to carry railroad traffic.

Art. 21.—Other Stiffened Suspension Bridges over Niagara River.

Two other suspension bridges have spanned the Niagara River as highway structures, one built in 1851 at Queenston, and the other in 1889 near the Falls; the latter was replaced in 1898 by a steel arch. The former was partially wrecked in a wind-storm in 1864 and was never repaired, remaining a wreck until 1898, when it was taken down and replaced by a new suspension bridge, 1040 feet between towers, and used to carry both highway traffic and a surface trolley line.

Art. 22.—Cincinnati and Covington Suspension Bridge.

Another prominent suspension-bridge structure was built across the Ohio River between Covington and Cincinnati by the elder Roebling in 1867. The span of this bridge is 1057 feet between centers of towers. This structure carries street traffic only, and has but little stiffening in the shallow trusses carrying the road floor.

Art. 23.—East River Stiffened Suspension Bridges.

The other prominent stiffened suspension bridges in the United States are the two now (1905) spanning the East River between New York and Brooklyn. The older, or Brooklyn Bridge, has a span 1595½ feet between centers of towers. The main span and approaches make a total length of structure of 5989 feet. Although this structure is a true stiffened bridge, its trusses do not possess sufficient stiffening capacity to meet completely the requirements of a modern bridge of that type. There are four cables, each 15¾ inches in diameter, containing 5296 steel wires. The structure is 85 feet wide, and has two outside roadways for

highway traffic and trolley cars, two inner tracks for electric trains, and a middle deck, 12 feet above the other platforms, for foot-passengers.

The East River suspension (the Williamsburg Bridge) has a span of 1600 feet, and is a completely stiffened suspension bridge. This structure is suspended from four cables, each about 19 inches in diameter and containing 37 strands of 208, No. 6, steel wires. The elevation of the center of the cable, where it rests in the saddle, is 332.7 feet above mean high water, and the elevation of the cable at its lowest point in the main span is 161 feet. The center of the structure, like that of the old Brooklyn Bridge, has a clear height of 135 feet above the water beneath it.

Art. 24.—Projected North River Suspension Bridges.

Stiffened suspension bridges with spans of nearly 3000 feet in length have been designed to cross the North River at New York, but they have never been built.

Art. 25.—Position of Stiffening Truss.

Although in the great majority of American stiffened suspension bridges the stiffening truss has been placed along the roadway, it may be placed along the cables, and indeed be made a part of the cable system. A number of such designs have actually been constructed. A bridge of this character at Pittsburg and another crossing a number of railroad tracks in the city of St. Louis, in which the stiffened truss is located along the cable, have been designed and constructed by Mr. G. Lindenthal. He followed the same general plan in his design for a stiffened suspension bridge across the North River between Hoboken and New York, the span being over 2800 feet.

Again, a stiffened suspension bridge with the stiffening

truss made a part of the eye-bar cable system was designed and constructed across the Ohio River at Pittsburg by Hemberle, and has been known as the Point Bridge. In this case each half of the span is practically a bowstring truss with the cables acting as the tension chords, one end of each bowstring truss being carried at the top of a tower, and the other two ends coupled together with a pin at the center of the span. Another stiffened suspension bridge across the East River, known as the Manhattan Bridge, is now being constructed with a span of practically 1600 feet, like that of the Williamsburg and Brooklyn suspension bridges.

ARCHED RIBS.

Art. 26.—Nature of Arched Ribs.

If a stiffened suspension bridge should be reversed in position so that the tension members, suitably designed, would have to take compression, there would result the arched rib. In other words, the general principles of the theory of stiffened arched ribs and of stiffened suspension bridges are identical.

The stiffened arch-rib, or, as it is called, the arched rib, is a development from the metallic arch or the ordinary arch of blocks. If the arch-ring of the latter be so made as to enable it to resist bending as well as compression, the arched rib would result. These ribs may have their ends fixed or hinged, and there may or may not be a hinge at the crown.

Art. 27.—Captain Eads's Bridge at St. Louis.

The arched rib in crude form began gradually to be introduced soon after the construction of metallic arches, but it was not until the Eads arched-rib bridge structure

across the Mississippi River at St. Louis was built that this type of structure took a prominent position in American bridge building. This bridge was designed to carry a double-track railroad, and is still used for that purpose. There are three spans: a central span of 520 feet, with a rise of 47 feet $\frac{1}{2}$ inch, and two flanking spans of 502 feet each, the rise of the latter arches being 43 feet 9 inches. The chords of the stiffened arch are composed of four steel tubes, each 18 inches outside diameter. These hollow tubes are each composed of six lines of steel staves, varying from $1\frac{3}{16}$ to $2\frac{1}{8}$ inches thick, carefully fitted to form a tube encased in $\frac{1}{4}$-inch steel plates bent to make a close fit to the tubes. Each vertical pair of the latter are 12 feet apart between centers.

Art. 28.—Recent Arched Ribs.

Since the completion of the Eads Bridge many arched ribs have been constructed for various purposes, prominent among which are the large railroad-station train-sheds of the three-hinged type like those of the Pennsylvania Railroad at Jersey City, with a span of 252 feet 8 inches, built in 1892, and the Philadelphia and Reading Railroad at Philadelphia, with a span of 259 feet, built in 1893. The latest developments of the arched rib are the numerous structures built of reinforced or armored concrete, known generally as concrete-steel arches. These structures are thoroughly characteristic arched ribs, although some of them have hinge-joints either at the springing-lines or at the crown, or both. Concrete-steel arched ribs are adapted to all spans from the shortest to those from 300 to 400 feet in length, or more.

In the following tables are given the lengths of some of the largest steel arches constructed.

STEEL ARCHES WITHOUT HINGES.

	When Built.	Length.
Cornhouse Bridge, Berne, Switzerland	1898	376.6 ft.
Paderno Viaduct, Italy	....	492 "
Eads Bridge, St. Louis	....	520 "
Kaiser Wilhelm Bridge, Wupper River, Germany	1897	526 "
WITH TWO HINGES.		
Rio Grande Bridge, Costa Rica	1902	448 ft. 8 ins.
Washington Bridge, Harlem River, New York	1898	510 "
Gruenenthal, over the North Baltic Ship-canal	....	513 "
Garabit Viaduct, France	....	541 " 4 "
Spandrel Arch, Niagara Falls	1897	550 "
Düsseldorf Bridge, Germany	....	594 " 6 "
Bonn Bridge, Germany	....	614 " 2 "
Braced Arch, Niagara Falls	1898	840 "
WITH THREE HINGES.		
Pennsylvania Railroad, at Philadelphia	1894	300 ft. 8 ins.
Alexander III. Bridge, Paris, France (cast steel)	1898	353 "
Machinery Hall, Paris Exposition	1889	362 " 9 "
Manufactures and Liberal Arts Building, Chicago	1893	368 "
Lake St. Bridge, Minneapolis (wrought iron)	....	456 "
Bellows Falls, Vermont	1905	540 "
Viaur Viaduct, France (Spandrel)	....	721 " 6 "

Among the largest stone arches are the Cabin John arch, near Washington, with a span of 220 feet, and the Luxemburg arch, of 277.65 feet.

CANTILEVERS.

Art. 29.—The First Cantilevers in America.

A cantilever structure is essentially a continuous truss or beam with the points of contraflexure fixed in position; that portion of the structure between those points being the suspended span. The latter, i.e., the suspended span, has usually been designed with a length about one third to one half of the total cantilever opening, although the suspended span of the great Forth Bridge is scarcely more than one fifth of the total opening.

The cantilever bridge is generally considered to belong to a very old type, but it has been developed into a modern bridge structure only within the past twenty-five years. The first real cantilever in this country was designed and built by Mr. C. Shaler Smith in 1876 across the Kentucky River, on the line of the old Cincinnati Southern Railway. This was a true cantilever bridge, although the uniform depth of trusses obscures the ordinary division of such a structure into the two cantilever arms and the suspended span. In this structure the three openings between the piers and abutments were each equal to 375 feet. The suspended span is a standard feature of the modern cantilever, but it may be omitted, as in the Blackwell's Island bridge now being built (1905) across the East River.

Art. 30.—Prominent Cantilever Structures.

The first cantilever bridge built in the United States with a varying depth of cantilever arms, characteristic of the latest structures of that type, was the Niagara double-track bridge, designed and built by Mr. C. C. Schneider in 1883. The completion of that structure was soon followed by many others from short spans up to 1700 feet or more. The most prominent of these bridges is the great cantilever across the Firth of Forth in Scotland, completed in 1889, which carries a double-track line of railway. Its two longest spans are each 1710 feet, and each has a suspended span 350 feet in length. The total length of this structure is over one and one-half miles. The height of towers is 361 feet above high water, and the centers of the main spans have 152 feet clear headway above high water. The more prominent cantilever structures in the United States are those over the Colorado River at the Needles in California (660 feet), the Memphis Cantilever (790 feet 5 inches),

the Wabash Railroad Co.'s bridges at Pittsburg (812 feet) and at Mingo Junction (700 feet), the bridge at Thebes, Ill. (671 feet), the highway bridge (650 feet) at Marietta, O., and the Blackwell's Island structure, now (1905) in process of construction. The channel spans of the latter are 846 feet each.

One advantage of the cantilever structure is the entire and easy practicability of erecting it without false works in the main or cantilever span, each half being built out as a cantilever bracket until they meet at the center of the opening. This type of structure is economically adapted to spans up to 1800 or 2000 feet in length, or possibly more. Beyond that limit the stiffened suspension bridge exhibits economic adaptability.

A cantilever bridge is now (1905) being built across the St. Lawrence River near Quebec with a main span of 1800 feet. Suspension-bridge spans have not yet exceeded about 1600 feet in length as actually constructed, the Williamsburg Bridge between the Boroughs of New York and Brooklyn having that span length, and the Brooklyn suspension-bridge span being but little less.

MOVABLE BRIDGES.

Art. 31.—Types and Examples of Movable Bridges.

Movable bridges are also among old types of structures, especially in their use at the entrance of mediæval castles and fortresses in the shape of the trunnion bascule. There are three principal types of movable bridges in modern use:

1. The ordinary swing-bridge, turning in a horizontal plane about a vertical axis. This bridge usually has two equal arms, continuous over the pivot-pier on which it turns.

2. The trunnion bascule with either one or two leaves. This form has been used successfully in some of the latest examples of movable bridges.

3. The rolling lift or bascule bridge with either one or two leaves, which roll into a vertical position, the land end of each leaf being suitably shaped and designed to roll on a smooth track. The Scherzer rolling lift-bridge is the most prominent example of this type.

There are other special forms of movable bridges, like the Halsted St. lift-bridge over the Chicago River, 130 feet in length, designed and built by Mr. J. A. L. Waddell, and advancing and retreating or traversing movable bridges, but such special types are seldom employed.

The ordinary swing-bridge has been most extensively used for railroad structures across navigable streams since iron or steel bridges were first built in the United States. Indeed there have also been many timber swing-bridges. The largest of these iron or steel swing-bridges have reached lengths of 500 feet and more for both railroad and street traffic. The total weights turned have been as high as three thousand tons or more.

The most prominent example of the trunnion bascule structure of two leaves is the Tower Bridge at London with its opening of 240 feet, although a number of bascule bridges of this type were built even in the first half of the ninteenth century. The distance between the centers of trunnions of the Tower Bridge at London is 226 feet. Lately movable bridges of this type have been built in the United States, although none with as great length of leaves as the Tower Bridge.

The rolling lift bascule bridge has been developed chiefly within the past twenty years, the past ten years having seen the greatest extension of its use. It has been successfully applied with two leaves up to a clear opening of 255 feet, and with a length of 275 feet from center to center of the seats, the total weight of iron and steel in this case being 2250 tons. This bridge carries a double-track railroad at Chicago.

IRON RAILWAY VIADUCTS.

Art. 32.—American Development of Timber and Iron Viaducts.

The railway viaduct is peculiarly an American development. Its origin has already been indicated in the reference to the timber viaducts originally built on the railway lines now forming a portion of the Philadelphia and Reading system.

There is little doubt that the first iron trestles were built on the Baltimore and Ohio Railroad in 1852–3, as designed by Albert Fink, with heights not exceeding 58 feet. The compression members were of cast iron. In 1856–7 Mr. F. C. Lowthrop constructed spans of 100 and 111 feet, each supported on cast-iron trestle towers about 89 feet high, for the Catasauqua and Fogelsville Railroad, which formed a species of railroad viaduct, the compression members of which were of cast iron.

Art. 33.—Prominent Railway Viaducts.

The first iron viaduct of prominence of the strictly modern type was that built by Mr. George S. Morison at Portage on the Erie Railroad. The old Portage viaduct was a timber structure 850 feet long and 234 feet high above the bed of the river. It was designed by Mr. Silas Seymour and built in 1851–2. In 1875 it was entirely destroyed by fire. Mr. Morison rebuilt this viaduct immediately after the wooden structure was destroyed, on what is essentially a modern general plan. The towers were built of four columns suitably braced with a batter of 1 to 8 in a plane at right angles to the axis of the structure. The tower spans were 50 feet in length, and the open spans, i.e., the spans between the towers, included one span 118 feet and two spans each 100 feet in length.

Since the reconstruction of the Portage viaduct in iron,

the iron or steel viaduct has been one of the most common of American railway structures. In the earlier viaducts the open and tower spans were frequently made respectively 40 and 30 feet each. In the later development the spans are almost invariably plate girders, but of increased length, 60 and 40 feet for the open and tower spans being frequently found.

The common type of tower shows it to be composed of four main legs or columns braced in planes both longitudinal and transverse to the axis of the structure. The batter or inclination of these legs is not infrequently one horizontal to six vertical in vertical transverse planes. Provision has sometimes been made for expansion and contraction both longitudinally and transversely in each tower, although that is not common practice. The open spans between the towers are so designed as to afford provision for longitudinal expansion and contraction.

The Verrugas viaduct, a single-track structure, on the Lima and Oroya Railroad in Peru, 52 miles from Callao and 5836 feet above sea-level, was a remarkable structure at the time of its construction, 1871–72. The total length of the viaduct was 575 feet. There were three iron towers, each consisting of three transverse bents with four columns to a bent. The towers, built in stories of 25 feet, were fully braced with struts and ties; they were 146 feet, 252 feet, and 179 feet high. The spans between the towers and between the latter and the abutments were three of 100 feet each, and one of 125 feet. They were Fink trusses. The compression members in the towers were Phœnix columns with cast-iron connections. This structure was on a 3 per cent grade. It was designed by Mr. C. H. Latrobe. The erection of the viaduct, planned and executed by Mr. L. L. Buck, was completed in three and a half months. In 1889 a heavy freshet washed out the central pier and caused the collapse of the structure.

The most prominent railway viaduct structures in the United States are the Kinzua viaduct on the Erie Railroad at Bradford, Pa., and the Pecos viaduct on the Southern Pacific Railway over the Pecos gorge in Texas. The Kinzua viaduct is 2053 feet long, with a maximum height of tower of 285 feet and with open and tower spans respectively 61 feet and 38 feet 6 inches. It was originally built in 1881 with Phœnix-column compression members, but it was rebuilt as an all-riveted structure in 1900. In the reconstruction of this viaduct the inclined members of the tower bracing were largely omitted, reliance for stiffening the tower legs being placed upon heavy plate brackets between the legs and the horizontal struts. The Pecos viaduct has a total length of 2180.5 feet and a total height above the water surface of the creek in the bottom of the gorge of 321 feet. Its open and tower spans are mainly 65 feet and 35 feet respectively. The central span over the creek is of the cantilever type, 185 feet long in the clear. These viaducts were erected without false works.

The Gokteik viaduct, in Burmah, about 400 miles from Mandalay, built by the Pennsylvania Steel Co., is 2260 feet long and 320 feet high at the highest point,

The highest viaduct of the present day (1905) is probably the Fades viaduct, being erected over the Sioule River on the railway from Clermont to Tulle, about 100 miles from Paris, France. It consists of three continuous deck-truss spans, the central span, 478 feet between centers of piers, being flanked by two spans, each 378 feet between centers of piers. The two supports at the extremities of the 472-foot span are hollow masonry piers, each 303 feet high. The tops of rails of this central span are 435 feet above the bed of the Sioule River.

CHAPTER II.

GENERAL TYPES OF TRUSSES, LOADS AND SPECIFICATIONS.

Art. 34.—Definitions and Equations of Conditions.

A TRUSS may be defined to be a framework, or articulate structure, so composed of individual pieces connected at joints or panel-points that external forces called loads applied at the joints produce the stresses of tension and compression only in those pieces. If the loads are applied along the members of the truss, those members will generally be subjected to bending in addition to the direct stresses of tension and compression.

The simplest of all trusses is a triangle, and all trusses however complicated containing no superfluous members must contain the triangle as the elementary figure. The triangle is the truss element, for the reason that it is the only geometrical figure whose form may not be changed without varying the length of its sides.

The following laws or conditions of equilibrium deduced in the study of statics are employed in determining the stresses in trusses or framed structures:

$$\left.\begin{aligned} \Sigma X &= 0. \\ \Sigma Y &= 0. \\ \Sigma Z &= 0. \\ \Sigma M_x &= 0. \\ \Sigma M_y &= 0. \\ \Sigma M_z &= 0. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (1)$$

X, Y, and Z are respectively the components of the external forces and of the stresses in the members of the structure in the directions of the X, Y, and Z axes, and M_x, M_y, and M_z, respectively, their moments about the same axes. These six equations of condition are applicable to any structure whatever, but they reduce to the following three when applied to a coplanar framework; that is, a framework situated in one plane:

$$\left.\begin{aligned} \Sigma X &= 0. \\ \Sigma Y &= 0. \\ \Sigma M_z &= 0. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

These equations further reduce to two only, viz., ΣH and $\Sigma V = 0$, in the treatment of the equilibrium of concurrent coplanar forces.

Since the majority of framed structures are coplanar, eqs. (2) suffice to determine all the stresses if the structures are statically determinate. It will be found, however, that the determination of stresses in some structures requires additional equations of conditions. These equations are usually based on the laws of elasticity of the materials employed as expressing either deflections or the expenditure of work in producing those deflections. Such structures are called statically indeterminate; they will be considered later.

Art. 35.—General Types of Bridge-trusses.

The members constituting a truss figure are designated either as chord members or web members. Chord members are those mainly defining the outlines of the structure, and they are termed lower or upper chord members according as they are found at the bottom or top of the structure. In Fig 1 the members L_0L_1, $L_1L_2 \ldots L_5L_6$ are the lower chord members, and the members L_0U_1, U_1U_2

. . . U_5L_6 the upper chord members. The members L_0U_1 and U_5L_6 are also called end posts.

The members of a truss included between the chords are the web members. They are called posts or ties if they sustain compression or tension respectively. If

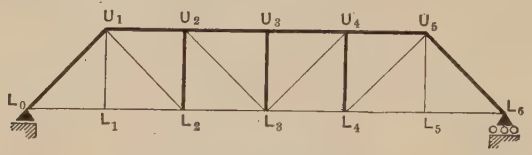


FIG. 1.—Pratt Truss.

members are capable of sustaining both tension and compression, they are said to be counterbraced.

The types of simple bridge-trusses at present most generally in use are the Pratt truss (Fig. 1), in which the

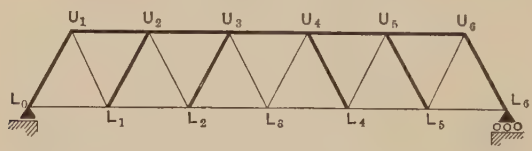


FIG. 2.—Warren Truss.

web members are alternately vertical and inclined, the inclined members being in tension; the Warren truss (Fig. 2), in which the web members are usually all inclined

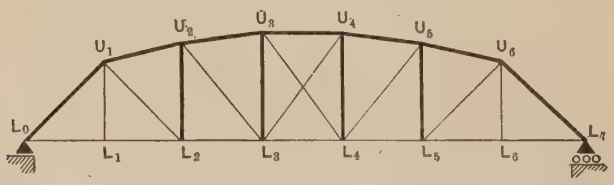


FIG. 3.—Broken Upper-chord Truss.

equally to a vertical; and the broken or inclined upper-chord truss (Fig. 3), generally used for spans exceeding about 200 feet, in which the web members are of the same

general type as in the Pratt truss. The web members in the Howe truss (Fig. 4) are alternately vertical and in-

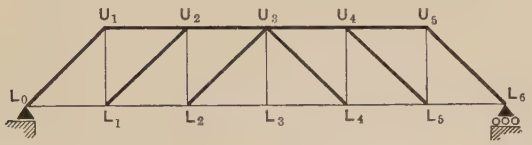


FIG. 4.—Howe Truss.

clined, as in the Pratt truss, but the vertical members are in tension, and the inclined in compression. In all these figures the heavy lines indicate compression members.

In trusses having long panels the latter may be sub-divided either in the manner shown in Fig. 5, or by some

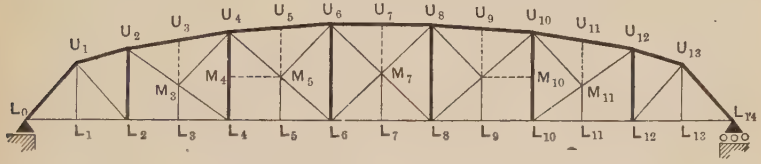


FIG. 5.—Broken Upper-chord Truss with Sub-divided Panels.

other system. In this way the advantage of few and large truss members is combined with a floor system in which the stringer lengths are not too great.

Art. 36.—Notation or Numbering of Members.

The method of naming the members of trusses is shown in the preceding figures, in which each lower panel-point is designated by *L* with the proper subscript, and the upper panel-points by *U*. In subdivided systems the panel-points not found in the chord lines are denoted by the letter *M* with subscripts. Since bridge-trusses are usually symmetrical about a vertical line midway between the points of support, it is frequently the practice to number one half the truss only, repeating in the right half of the

structure the lettering of the left half. Both methods of numbering will be used.

Art. 37.—Deck, Through, and Pony Trusses.

Without regard to types of construction, truss-bridges are divided into

Deck-bridges, with load carried on the upper chord line, and

Through-bridges, with load carried on the lower chord line.

Pony trusses are trusses insufficiently high (or deep) to need overhead cross-bracing. They have seldom been erected for spans of over about 80 feet, although there are examples of 100 feet (and even more) in length. They are now (1905) practically abandoned, but plate girders of a corresponding type are frequently built.

The lateral stability of pony trusses is precarious, but complete stability of pony plate girders is easily attained.

Art. 38.—Distribution of Fixed and Moving Loads for Bridge-trusses.

The total load on a bridge-truss always consists of two parts, the *fixed* load and the *moving* load. The fixed load consists of the entire weight of the bridge, including tracks, flooring, etc. The moving load, as its name indicates, consists of that load (whether concentrated or continuous) which moves over the bridge.

The truss is, of course, always subjected to the action of the fixed load.

If the truss is of uniform depth, the panel-fixed loads may always be taken uniform in amount for one chord; and even if the depth is variable, it will seldom be necessary to make a varying distribution of the weight of the trusses and lateral bracing. The amount and rate of such a

variation can only be determined by the circumstances of each particular case.

Moving loads are so supported on bridge-trusses that they are usually applied at panel-points only. This is accomplished by means of a system of longitudinal and transverse girders called stringers and floor-beams respectively. The stringers support the ties and rails directly and are supported by the floor-beams, which in turn are framed into the main members of the bridge-trusses.

In earlier bridge practice floor-beams were hung from the pins at panel-points, but this construction has entirely disappeared. It is now considered essential to build the floor system as rigidly as possible, and all girders are firmly riveted to their supports.

Art. 39.—Fixed or Dead Load.

Every truss bridge is composed of the following parts:

Upper and Lower Chords.
Web Members.
Upper Sway-bracing.
Transverse Bracing.
Lower Sway-bracing.
Floor system, including beams, stringers, and either rails and ties or some type of continuous floor with or without ballast.

The sum of the weights of these parts is the fixed or dead load of the bridge.

Art. 40.—Specifications for Floor Systems of Railroad Bridges.

The following extracts from modern standard specifications will illustrate the factors which must be considered in determining the weight of the floor system of railroad bridges:

American Bridge Company's General Specifications for Steel Railroad Bridges, 1900.

§ 8. *Wooden Floor.*—The floors shall consist of cross-ties 8×8 inches if the stringers are placed 6½ feet between centers. For greater distances they are to be proportioned to carry the maximum wheel load distributed over three ties, the fiber strain on the timber not to exceed 1000 pounds per square inch. The ties shall be spaced with openings not to exceed 6 inches, and shall be notched down ½ inch and have a full and even bearing on the stringers. Every fifth tie shall be fastened to a stringer by a ¾-inch bolt.

§ 10. There shall be guard-timbers 6×8 inches on each side of each track, with their inner faces not less than 3 feet 3 inches from the center of track. They shall be notched down one inch over every tie, and shall be fastened to every third tie and at each splicing by a ¾-inch bolt. The splices shall be over floor-timbers, with half-and-half joints of 6 inches lap.

§ 11. The floor-timbers and guards must be continued over piers and abutments.

§ 12. On curves the outer rails shall be elevated as may be required.

§ 13. In determining the weight of the structure for the purpose of calculating the dead-load stresses, the weight of timber shall be assumed at 4½ pounds per foot B.M., and the weight of rails, spikes, and joints at 100 pounds per linear foot of track.

The General Specification for Steel Railroad Bridges, by Theodore Cooper, C.E., 1901, contain provisions in addition to those quoted above, as follows:

§ 16. The floor-timbers from center to each end of span must be notched down over the longitudinal girders, so as to reduce the camber in the track, as directed by the engineer.

§ 17. All the floor-timbers shall have a full and even bearing upon the stringers; no open joints or shims will be allowed.

§ 23. The structure shall be proportioned to carry the following loads: first, the weight of metal in the structure and floor; second, the weight of rails, fastenings, ties, guards, foot-walk, and ballast, when

used. The rails and fastenings are to be assumed at 100 pounds per foot of track; timber, at $4\frac{1}{2}$ pounds per foot B.M.; and ballast, at 110 pounds per cubic foot. The minimum weight will be assumed at 400 pounds per foot of track.

The Standard Specifications for Steel Bridges issued by the Pennsylvania Railroad Company, January 1, 1901, require that (§ 7) the dead load shall be assumed as uniformly distributed and made up of (1) the net suspended weight of metal in the trusses and members; (2) the weight of the metal floor system (if any); (3) the weight of the wooden cross-ties or floor-beams; (4) 160 pounds per linear foot of track, covering the weight of rails, splices, guard-rails, etc., the above items of dead load to be properly distributed between the panel-points of the loaded and the unloaded chords.

The Specifications of the N. Y. C. & H. R. R.R. Company dated January 1, 1904, relating to the floor system of steel bridges are not very different from those quoted above. Fig. 6 shows in detail the construction of their standard floor system.

The N. Y. C. & H. R. R.R. Co.'s Specifications relating to solid floors are of interest

§ 19. When solid steel floors are used, they shall preferably be made of rolled beams with a plate not less than $\frac{7}{16}$ inch thick on top of same, riveted to each beam, rivets being staggered and spaced not over 12 inches apart in each gauge-line.

§ 20. When trough-floors must be used, on account of the limited space between base of rail and clearance-line of structure, they shall be made rectangular in shape, with plates and angles, unless otherwise specified.

§ 21. Solid steel floors shall be connected to main girders or trusses by angle-irons not less than $\frac{1}{2}$ inch thick or less than $3\frac{1}{2} \times 3\frac{1}{2}$ inches in size, one angle on each side of the web of the I beams, and of the vertical plates of troughs, where practicable to do so. Gusset-plates, not less than 16 inches wide at top of floor, shall be placed at intervals

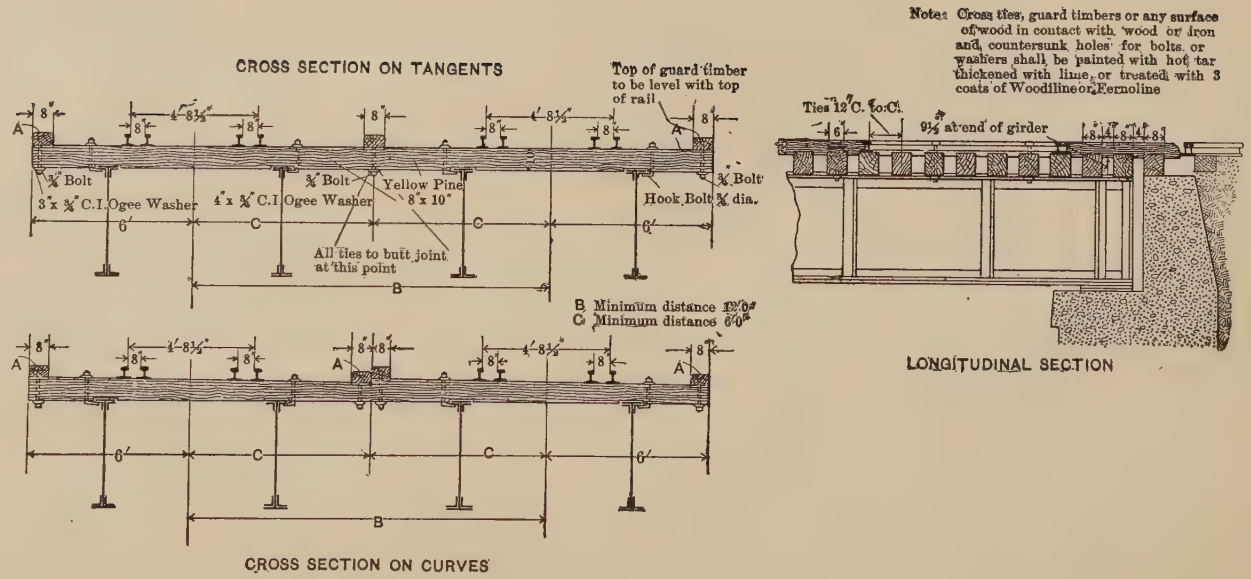


FIG. 6.—Standard Floor System of N. Y. C. & H. R. R.R. Co.

of not over 12 feet, when floors are riveted to through plate girders. These gusset-plates shall be well riveted to floors and girders and have the edge reinforced by a $3\frac{1}{2}\times3\frac{1}{2}\times\frac{3}{8}$-inch angle.

§ 22. Longitudinal solid steel floors shall, where they rest on masonry, have a continuous bearing-plate $\frac{3}{4}$ inch thick the whole width of the bridge and anchored to the masonry with $1\frac{1}{4}$-inch-diameter bolts extending 12 inches into the same.

§ 23. Drainage-gutters and down-spouts are generally furnished for solid bridge floors when directed. The gutters shall be not less than No. 22 galvanized iron, well supported; the down-spouts shall be of cast iron and not less than 4 inches in diameter.

Art. 41.—Weights of Stringers and Floor-beams and their Design.

It will be seen that the foregoing specifications cover parts of the floor system only. The probable weights of stringers, floor-beams, or trusses are not given. These must be assumed in every case by the designer and used as tentative quantities in the design of the structure. After the completion of the design, the weights calculated therefrom are to be compared with those first estimated and any necessary corrections can then be made. The assumption of closely approximate tentative weights is not difficult, since the design of the structure advances progressively from stringers to floor-beams and then to main members.

The dead weight of one pair of stringers and the included bracing may be estimated by means of the empiric formula $w=al$, where w is the weight in pounds per linear foot, l the length of the stringer in feet, and a is an empirical quantity depending upon the live loading, the span length, and the allowed intensities of stress. The value of a may vary between the limits of 12 and 20.

The weight per foot of a floor-beam may be estimated very closely after a rough, tentative design.

Table I is taken from the Specifications of the N. Y. C.

& H. R. R.R. Co., dated January 1, 1902. It furnishes the values of the estimated weights of the steelwork for floor-beams and stringers, including laterals, for the lengths of span there indicated. The table includes double-track as well as single-track spans.

TABLE I.—ESTIMATED WEIGHTS OF FLOOR-BEAMS AND STRINGERS, INCLUDING LATERALS.

FLOOR CARRYING ONE TRACK.

Depth Floor-beams.	Length Floor-beams.	Depth Stringers.	Length Stringers.	Weight per Linear Foot 1 Track.
1 ft. 8¾ ins.	14 ft. 0 ins.	1 ft. 1¼ ins.	7 ft. 8 ins.	455 lbs.
2 " ¼ "	13 " 0 "	1 " 9 "	13 " 1 "	555 "
2 " 3⅜ "	14 " 0 "	1 " 7⅝ "	10 " 10 "	465 "
2 " 3¼ "	14 " 0 "	1 " 9¼ "	14 " 10 "	498 "
2 " 6¼ "	15 " 0 "	2 " ¼ "	14 " 9 "	518 "
2 " 7⅛ "	15 " 0 "	2 " ¼ "	13 " 0 "	498 "
2 " 9¼ "	16 " 0 "	2 " 2¼ "	15 " 10½ "	484 "

FLOOR CARRYING TWO TRACKS.

Depth Floor-beams.	Length Floor-beams.	Depth Stringers.	Length Stringers.	Weight per Linear Foot 2 Tracks.
2 ft. 10¼ ins.	27 ft. 6 ins.	2 ft. 3⅜ ins.	12 ft. 5½ ins.	1540 lbs.
3 " 0 "	26 " 9 "	2 " 4¾ "	12 " 1¾ "	1560 "
3 " 1¾ "	27 " 3 "	2 " 6½ "	13 " 5½ "	1565 "
3 " 5¾ "	29 " 0 "	2 " 8¼ "	15 " 9¼ "	1250 "
4 " ¼ "	28 " 9 "	3 " 5¼ "	17 " 8 "	1430 "

Note.—The weights include the steelwork only.

Art. 42.—Weights of Bridges and Formulæ for the Determination of such Weights.

Figs. 7, 8, and 9 are taken from the same specifications of the N. Y. C. & H. R. R.R. Co. and they show in detail the approximate weights, per linear foot of span, of the different classes of bridges there indicated for one, two, and four tracks, from spans between 10 and 110 feet. These diagrams include, within the proper limits of length, various

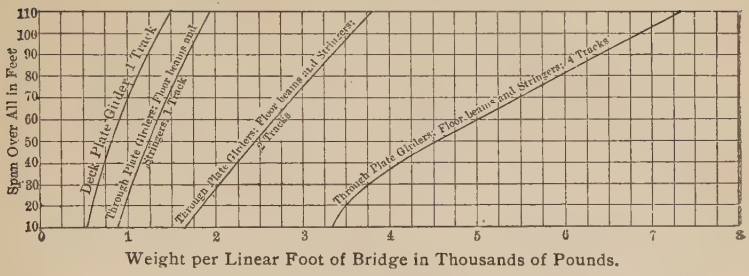


FIG. 7.—Diagram for Deck Plate Girders, and Through Plate Girders with Floor Beams and Stringers.

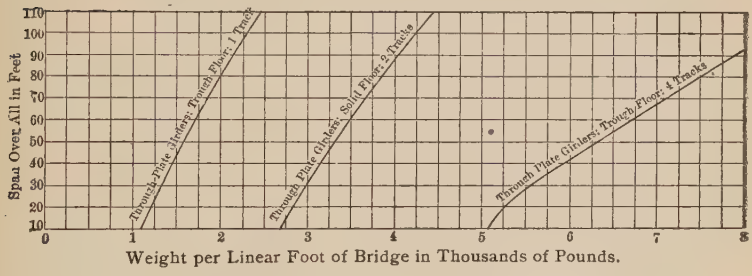


FIG. 8.—Diagram for Through Plate Girders, with Through Floor.

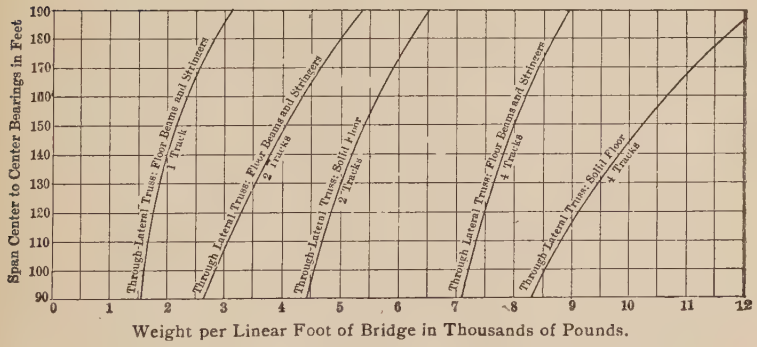


FIG. 9.—Diagram for Trusses; the Weights include the Steelwork only.

types of structures, such as deck plate girders, through plate girders, and trusses with both solid and open floors.

The live load for which these structures were designed consisted of two consolidation locomotives, Class E 40 of Cooper's Specifications, followed by a uniform live load of 4500 pounds per linear foot of track.

Attempts* have been made to express accurately by means of an equation the dead weight of the trusses and bracing of a bridge structure, but the results have not been satisfactory. The quantities appearing in such an equation would include the variation in type of the structure, the depth of trusses, the intensity of moving load, the allowed unit stresses, the degree of rigidity desired in the structure, etc.

It seems proper to neglect all such quantities and to adopt for use, instead, an equation similar to the one used for stringers, tempering the value of the constant a by the engineer's judgment. The equation would again be represented by $w = al$, where w represents the weight per linear foot of two trusses and included bracing, l the span length in feet, and a an empirical constant. The extreme limits of the value of a may lie between 6.5 and 10, 7 frequently occurring.

Art. 43.—Specifications for Floor Systems of Highway Bridges.

The Specifications of the American Bridge Company for steel highway bridges, 1901, include the following provisions:

GENERAL DESCRIPTION.

Classification. § 1. Bridges under these specifications are divided into six classes, viz.:

Class A.—For city traffic.

Class B.—For suburban or interurban traffic with heavy electric cars.

* Transactions American Society of Civil Engineers, Vol. XV, p. 105; Vol. XVI p. 191; Vol. XVIII, p. 179.

Class C.—For country roads with light electric cars or heavy highway traffic.

Class D.—For country roads with ordinary highway traffic.

Class E 1.—For heavy electric street railways only.

Class E 2.—For light electric street railways only.

Spacing of Trusses. § 8. The width between centers of trusses shall in no case be less than one twentieth of the span between centers of end pins or shoes.

Floor-beams. § 11. All floor-beams in through-bridges shall be riveted to the main girders.

Stringers. § 12. Steel stringers shall preferably be riveted to the web of the floor-beams.

Wooden joists shall not be less than 3 inches thick, shall be spaced not more than $2\frac{1}{2}$ feet between centers, and shall be dapped over the seat-angles or floor-beams to exact level. In the latter case they shall lap by each other over the full width of the floor-beam, and shall be separated $\frac{1}{2}$ inch for free circulation of air.

Roadway Planks. § 13. For single thickness the roadway planks shall not be less than 3 inches thick, nor less than one twelfth of the distance between stringers, and shall be laid transversely with $\frac{1}{4}$-inch openings.

When an additional wearing surface is specified for the roadway, it shall be $1\frac{1}{2}$ inches thick, and the lower planks, of a minimum thickness of $2\frac{1}{2}$ inches, shall be laid diagonally and with $\frac{1}{2}$-inch openings.

Wheel-guards. § 15. Wheel-guards of a cross-section not less than 6 inches by 4 inches on each side of the roadway shall be provided. They shall be blocked up from the floor-plank with blocks 2 inches by 6 inches by 12 inches long, not over 5 feet apart center to center, held in place by one $\frac{3}{4}$-inch bolt passing through the center of each blocking-piece and securely fastened to the stringer below. The wheel-guards shall be spliced with half-and-half joints with 6 inches lap over a blocking-piece.

Footwalk Planks. § 16. The footwalk-planks shall not be less than 2 inches thick nor more than 6 inches wide, spaced with $\frac{1}{2}$-inch openings.

§ 17. All plank shall be laid with heart side down, shall have full and even bearing on, and be firmly attached to, the stringers.

Solid Floor. § 18. For bridges of Classes A and B a solid floor, consisting of stone, asphalt, etc., on a concrete bed is recommended. For this case the flooring will consist of buckle-plates or corrugated sections, and the concrete bed shall be at least 3 inches thick for the roadway, and 2 inches thick for the footwalk, over the highest point to be covered, not counting rivet- or bolt-heads.

Buckle-plates. § 19. Buckle-plates shall not be less than $\frac{5}{16}$ inch thick for the roadway and $\frac{1}{4}$ inch thick for the footwalk.

Curbs. § 20. For solid floor the curb holding the paving and acting as a wheel-guard on each side of the roadway shall be of stone or steel projecting about 6 inches above the finished paving or gutter. The curb shall be so arranged that it can be removed and replaced when worn or injured. There shall also be a metal edging-strip on each side of the footwalks to protect and hold the paving in place.

Drainage. § 21. Provision shall be made for drainage clear of all parts of the metal-work.

Floor of Classes E 1 and E 2. § 22. The floor of bridges of Classes E 1 and E 2 shall consist of cross-ties not less than 6 inches by 6 inches, spaced with openings not exceeding 6 inches and securely fastened to the stringers by bolts There shall be guard-timbers not less than 6 inches by 6 inches on each side of each track, with their inner faces not less than 9 inches from center of rail. They shall be notched 1 inch over every tie, and fastened to every fourth tie.

Dead Load. § 23. In determining the weight of the structure for the purpose of calculating stresses, the weight of timber shall be assumed at 4 pounds per foot B.M., the weight of concrete and asphaltum at 130 pounds, of paving-brick

at 150 pounds, and of granite stone at 160 pounds per cubic foot. The rails, fastenings, splices, and guard-timbers of street-railway tracks, resting on cross-ties, shall be assumed as weighing 100 pounds per lineal foot of track.

Art. 44.—Moving or Live Loads for Railroad Bridges.

The moving load on a railway bridge may be taken as continuous or as a series of single weights as actually applied at the wheels under the locomotives and cars. The assumption of continuity or uniformity of moving load was formerly always made, a larger amount per lineal foot being taken to represent the extra locomotive weight. In such a case, if the moving load extends from the end of the bridge to the center of any panel or to the end of that panel, the panel-point immediately in front of the train will not sustain a full panel load; but if it be assumed that this panel-point does sustain the *full* load, then a small error on the side of safety will be committed. Such an assumption may be made, and the consequent method of computation will be given in some of the articles which will follow this chapter.

At the present time (1905), however, the demands of the best practice, particularly in the treatment of short spans, require the moving load to be taken at the actual points of application of the locomotive wheels, followed by a uniform load representing car weights.

If the span is short, one or two locomotives may cover the entire bridge, thus causing the moving load, per foot, to be very great for the whole span. If the span is long, the probability of the whole bridge being covered with an excessively heavy moving load is slight.

Thus it is seen that the moving load, per foot, may decrease as the span increases.

Floor-beams and stringers are really bridges of short

spans equal to their lengths, consequently they must be designed for the heavy loads belonging to those short spans.

Railroad companies naturally differ in their choice of locomotive loads, but the present tendency is toward the use of standard loadings. Those devised by Theodore Cooper are at present much employed, and they are shown in Fig. 10.

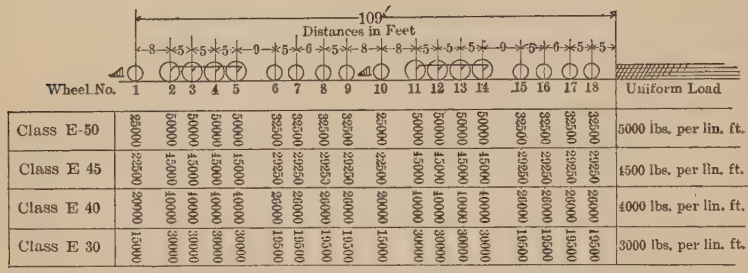


FIG. 10.—Cooper's Standard Loadings. Weights are Axle Loads in Pounds.

The length of wheel-bases for all these locomotives is the same, but the weights on the axles all vary in the same proportion in any two classes. Hence, if the stresses in a structure be found for any one class of locomotive, the stresses in the same structure for a different class may be determined by multiplying by the proper ratio.

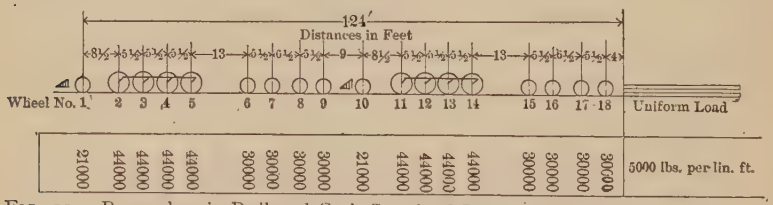


FIG. 11.—Pennsylvania Railroad Co.'s Standard Loading, Jan. 1, 1901. Weights are Axle Loads in Pounds.

At the present time (1905) locomotives E 40, E 45, and E 50 are frequently used, the general tendency being toward the heavier types.

The standard loading required by the N. Y. C. & H. R. R.R. Co. is the same as Cooper's E 40 locomotive, followed,

however, by a uniform load of 4500 pounds per linear foot.

Fig. 11 illustrates the standard loading specified by the Pennsylvania Railroad Co., Jan. 1, 1901.

Fig. 12 shows a locomotive built for the Atchison, Topeka & Santa Fé Railroad, and Fig. 13 shows one built for the Pittsburg, Bessemer & Lake Erie Railroad. These figures illustrate types of the heaviest locomotives yet

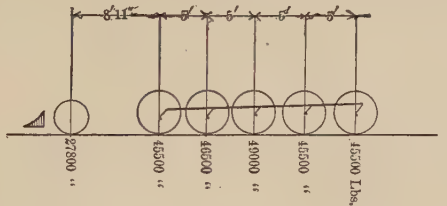


FIG. 12.—A., T. & Santa Fé R.R. Co.'s Locomotives.

built. The A., T. & Santa Fé locomotive weighs, without tender, 232,000 pounds on five driving-axles, with a driving-wheel base of 20 feet, or 11,600 pounds per linear foot. The P., B. & L. E. locomotive weighs 225,200 pounds on four driving-axles, on a driving-wheel base of 15 feet 7 inches, or 14,450 pounds per linear foot.

Mr. H. W. Hodge, in a discussion before the American Society of Civil Engineers, June 11, 1903, states that the

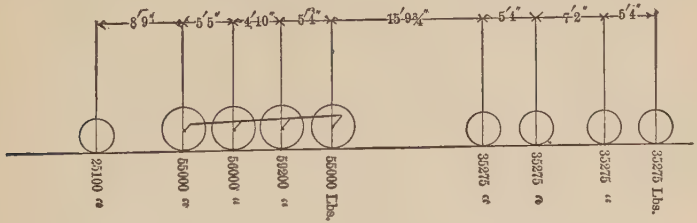


FIG. 13.—P. B. & L. E. R.R. Co.'s Locomotive.

heaviest loaded steel cars of which he has knowledge are those in use on the Algoma Central Railroad, which weigh 44,000 pounds each and carry a net load of 100,000 pounds, their over-all length being 24 feet. This loading is equivalent to 6000 pounds per linear foot.

Mr. A. J. Hymes, in the same discussion, records a loading on the Monongahela Connecting Railroad of Pittsburg, which uses cars having a capacity of 200,000 pounds and weigh 40,000 pounds. The length of the shortest of these cars is 33 feet over all, making a total weight of about 7300 pounds per linear foot.

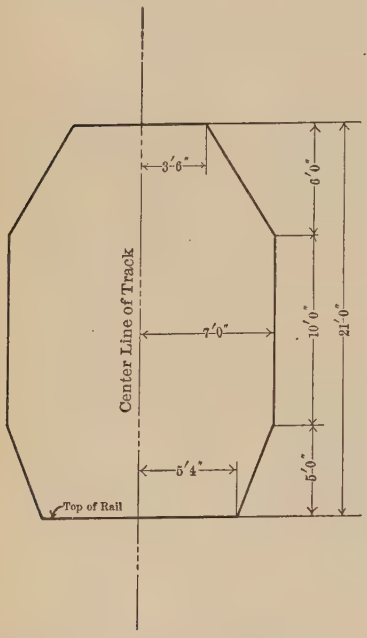


FIG. 14.—Clearance Diagram. N. Y. C. & H. R. R.R. Co. Standard Distance between Tracks = 12 feet.

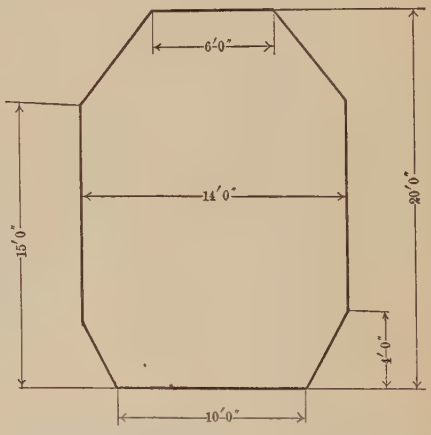


FIG. 15.—Clearance Diagram, Pennsylvania R.R.

Such loadings are perhaps extreme at the present date, but 5000 pounds per linear foot is not an excessive uniform load for trunk lines.

At the International Engineering Congress held at St. Louis, 1904, Mr. H. W. Hodge stated that in his opinion the largest steam-locomotive it would be reasonable to expect on a standard-gauge railway, with the present clear-

ance, would probably have four driving-axles on 5-ft. centers, each axle loaded with 35 tons.

The clearances required by the locomotives of the N. Y. C. & H. R. R.R. Co., and the Pennsylvania R.R. Co. are illustrated in Figs. 14 and 15.

A powerful and successful electric locomotive recently constructed is the type known as 6000 of the N. Y. C. & H. R. R.R. Co. This locomotive has a weight of 69 tons, equally distributed on four axles having a rigid wheel-base of 13 feet.

It is seen that the weight of the electric locomotive is much less than that of the steam-locomotive.

In order to provide for specially heavy concentrations which may affect the stresses in short spans or in the stringer system of a large span engineers sometimes specify a special loading. Cooper, for instance, specifies the use of a loading of 100,000 pounds, equally distributed on two pairs of driving-wheels spaced $7\frac{1}{2}$ feet center to center, when these loads furnish higher stresses than by using the usual locomotive concentrations.

Art. 45.—Equivalent Loads.

Attempts have been made to replace locomotive wheel concentrations by uniform or other loads producing equal moments or shears and therefore called equivalent loads. Such loads, however, will vary not only with the span length, but also as they are employed for determining moments or shears.

This is well illustrated by the following table showing the uniform load per linear foot which will furnish the same moments and shears in a given span as the actual concentrations of Cooper's Class E 40 locomotive. In spans of 250 feet length even, the equivalent uniform loads for moments and shears differ appreciably.

MAXIMUM MOMENTS, *M*, END SHEARS, *S*, AND FLOOR-BEAM REACTIONS, *R*, PER TRACK, PRODUCED BY COOPER'S STANDARD LOADING E 40, ON SPANS FROM 10 TO 250 FEET.

Span in Feet.	Maximum Moment, *M*, in Foot-pounds.	Maximum End Shear, *S*, in Pounds.	Maximum Floor-beam Reaction, *R*, in Pounds.	Equivalent Uniform Load.		
				M.	*S*.	*R*.
10	112,500	60,000	82,000	9,000	12,000	8,200
12	169,100	68,000	93,200	9,340	11,330	7,830
15	254,000	80,000	110,000	9,030	10,670	7,330
18	341,600	92,000	122,000	8,430	10,240	6,780
20	400,000	100,000	130,000	8,000	10,000	6,500
25	616,500	113,500	151,000	7,890	9,080	6,040
30	820,800	126,500	171,500	7,300	8,430	5,720
35	1,052,500	138,000	195,000	6,870	7,890	5,570
40	1,312,500	151,500	216,000	6,560	7,570	5,400
50	1,908,300	176,400	257,000	6,110	7,060	5,140
60	2,580,000	196,000	305,000	5,730	6,530	5,080
70	3,372,000	222,000	354,000	5,510	6,340	5,060
80	4,320,000	250,000	396,000	5,400	6,250	4,950
90	5,320,000	276,000	437,000	5,250	6,130	4,860
100	6,436,000	301,200	474,000	5,140	6,020	4,740
125	9,960,000	360,900	.....	5,100	5,770	
150	14,100,000	417,600	.....	5,010	5,570	
175	18,720,000	467,800	.....	4,890	5,350	
200	23,700,000	524,400	.....	4,740	5,240	
250	35,220,000	629,300	.....	4,510	5,030	

A compromise is sometimes effected between the method of locomotive concentrations and the method of the uniform load, by specifying that the loading shall consist of a uniformly distributed load, together with one or two concentrated weights, the latter not necessarily at the head of the uniform load, but so placed as to cause the greatest stresses. These excess loads are of such value and so spaced that they represent closely the differences between one or two locomotives and the uniform load of the same length. This class of loads may sometimes be advantageously employed, but under the modern methods of bridge computations no appreciable gain is found in this method except in special cases.

At the present time locomotive loadings have been so standardized and the quantities used in the numerical

calculations have been so arranged that there is but little advantage in using an equivalent uniform load in place of concentrations. Graphic methods also tend to do away with any advantage in favor of the equivalent uniform live-load system.

Art. 46.—Moving or Live Loads for Highway Bridges.

The following provisions relating to Highway Bridges have been extracted from the Specifications of the American Bridge Co., 1901; the classes referred to have already been explained.

§ 24. *Live Load.*—The bridges of the different classes shall be designed to carry, in addition to their own weight and that of the floor, a moving load, either uniform or concentrated, or both, as specified below, placed so as to give the greatest stress in each part of the structure.

Class A.—*City Bridges.*

For the floor and its supports, on each street-car track or on any part of the roadway, a concentrated load of 24 tons on two axles 10 feet centers and 5 feet gauge (assumed to occupy a width of 12 feet), and upon the remaining portion of the floor, including footwalks, a load of 100 pounds per square foot.

For the trusses, for spans up to 100 feet, 1800 pounds per lineal foot of each car-track (assumed to occupy 12 feet in width), and 100 pounds per square foot for the remaining floor surface; for spans of 200 feet and over, 1200 pounds for each lineal foot of track and 80 pounds per square foot of floor; proportionally for intermediate spans.

Class B.—*Suburban or Interurban Bridges.*

For the floor and its supports, on any part of the roadway, a concentrated load of 12 tons on two axles 10 feet centers and 5 feet gauge (assumed to occupy a width of 12 feet), or on each street-car track a concentrated load of 24 tons on two axles 10 feet centers; and upon the remaining portion of the floor, including footwalks, a load of 100 pounds per square foot.

For the trusses, for spans up to 100 feet, 1800 pounds per lineal foot of each car-track and 80 pounds per square foot for the remaining floor surface; for spans of 200 feet and over, 1200 pounds for each lineal foot of track and 60 pounds per square foot of floor; proportionally for intermediate spans.

Class C.—*Heavy Country Highway Bridges.*

For the floor and its supports, on any part of the roadway, a concentrated load of 12 tons on two axles 10 feet centers and 5 feet gauge (assumed to occupy a width of 12 feet), or on each street-car track a concentrated load of 18 tons on two axles 10 feet centers; and upon the remaining portion of the floor, including footwalks, a load of 100 pounds per square foot.

For the trusses, the same as for Class B, except that the load on car-tracks for spans up to 100 feet will be 1200 pounds, and for spans of 200 feet and over, 1000 pounds.

Class D.—*Ordinary Country Highway Bridges.*

For the floor and its supports, a load of 80 pounds per square foot of total floor surface, or 6 tons on two axles 10 feet centers and 5 feet gauge.

For the trusses, a load of 80 pounds per square foot of total floor surface for spans up to 75 feet; and 55 pounds for spans of 200 feet and over; proportionally for intermediate spans.

Class E 1.—*Bridges for Heavy Electric Street Railways Only.*

For the floor and its supports on each track a load of 24 tons on two axles 10 feet centers.

For the trusses, a load of 1800 pounds per lineal foot of each car-track for spans up to 100 feet; and a load of 1200 pounds for spans of 200 feet and over; proportionally for intermediate spans.

Class E 2.—*Bridges for Light Electric Street Railways Only.*

For the floor and its supports, on each track a load of 18 tons on two axles 10 feet centers.

For the trusses, a load of 1200 pounds per lineal foot of each car-track for spans up to 100 feet; and a load of 1000 pounds for spans of 200 feet and over; proportionally for intermediate spans.

In the Specifications of J. A. L. Waddell, C.E., highway bridges are divided into three classes, viz., Class A, which includes those subject to the continued application of heavy loads; Class B, which includes those subject to the occasional application of heavy loads; and Class C, which includes those for ordinary light traffic. Class A covers bridges in densely populated cities; Class B, those in smaller cities and manufacturing districts; and Class C, country road bridges. Mr. Waddell specifies that the uniformly distributed live loads per square foot of floor, including the entire clear widths of both main roadway and footwalks, shall be taken from the diagram shown in Fig. 16.

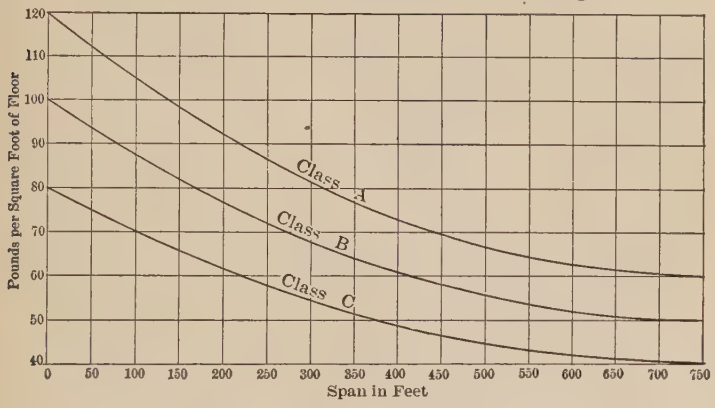


FIG. 16.—Waddell's Live Loads for Highway Bridges.
From Waddell's Specifications.

"In addition to these loads, the floor, joists, floor-beams, beam hangers, and primary truss members are to be proportioned for the following concentrated loads, which are, however, supposed to occupy a whole panel length of the main roadway to the exclusion of the other live loads there (excepting only the electric-railway live load).

"Class A.

"A road roller weighing 30,000 pounds, of which 12,000 pounds are concentrated upon the roller in front of the

machine, and 9000 pounds on each of the wheels at the rear, the distance between the central planes of these wheels being 5 feet and that between their axis and the axis of the front roller 11 feet. The width of the front roller is to be 4 feet and that of each rear wheel 1 foot 8 inches.

"Class B.

"A concentrated load of 16,000 pounds, equally distributed upon two pairs of wheels, the axes of which are 8 feet apart, and the central planes of the wheels 6 feet apart.

"Class C.

"A concentrated load of 10,000 pounds, distributed in the same manner as for Class B.

"The road-roller load is assumed to be equally divided between all of the joists that it can cover, and the wheel loads for Classes B and C equally between two joists."

The preceding loads do not cover electric road concentrations; these must be found in addition, and are assumed to occupy 10 feet in width of the clear roadway to the exclusion of all other live load on said 10 feet, except in the cases of floor-beams and primary truss members, when they are to be considered as passing either the road-roller or the heavy-wagon load.

Weights of Crowds of People.

The latest and most authentic tests concerning the weight of a crowd of people have been given by L. J. Johnson, in a paper printed in the Journal of the Association of Engineering Societies, January, 1905. The conclusions there stated are that loads of 180 pounds per square foot may occur in exceptional circumstances; that 160 pounds must frequently occur; that 140 pounds must be common on station platforms, in corridors, and many other

places frequented by throngs of people, and that 80 pounds per square foot must be common in gatherings in private houses. It is probable, however, that loads greater than 80 pounds per square foot occur so seldom in practice that they need be considered only in exceptional cases.

Art. 47.—Stresses Caused by Other than Fixed and Moving Loads.

In addition to the stresses in framed structures due to the fixed and moving loads acting upon them, it will be necessary to consider the following:

(*a*) Stresses caused by impact.
(*b*) " " " the traction of the moving load.
(*c*) " " " centrifugal forces.
(*d*) " " " wind.
(*e*) " " " temperature changes.
(*f*) " " " the initials tresses in some of the members.
(*g*) " " " snow load.

Art. 48.—Impact Stresses.

In designing the members of trusses it is necessary to distinguish between the unit stresses allowed for the dead and for the live load, on account of the difference in the conditions under which those classes of stress are caused. The dead-load stresses are constant in kind and amount, but those caused by the moving load vary not only in character and amount, but are applied with more or less suddenness of shock due to imperfections of track or rolling stock, lack of complete balance of locomotive driving-wheels, or to the inherent conditions necessarily associated with any rapidly moving train.

Two methods of treatment are in use: one specifies definite but separate allowed intensities of stress for live-

and dead-load stresses without any reference to the length of bridge covered by the moving load. The other method specifies the same intensity of stress for both classes of load, but adds to the live-load stress a certain percentage of itself, to allow for impact or sudden application.

The first method is that advocated by Theodore Cooper, who specifies the allowed dead-load stresses at one half the values of the live load. This method is also that adopted by the N. Y. C. & H. R. R.R. Co. The second method employs a formula of the following form:

$$I = S\left(\frac{a}{L+b}\right),$$

in which I is the impact stress to be added to the computed maximum live-load stress S, L is the length of loaded distance in feet which produces the maximum stress in the member, and a and b are constants, being lengths expressed in feet. The American Bridge Co., 1900, for instance, specifies for its railroad bridges $a = b = 300$, and J. A. L. Waddell specifies $a = 400$ and $b = 500$ for railway bridges, and $a = 100$ and $b = 150$ for highway bridges. In the latter case, however, L is taken as the total span length. For highway bridges, the American Bridge Co. (1900) specifies that 25 per cent of the maximum live-load stresses shall be added thereto, to compensate for impact and vibration.

The specifications for impact, as furnished by the Pennsylvania R.R. Co., are given in full below; they involve in the impact formula the dead-load as well as the live-load stresses, and they also provide for reversals of stress in the same member.

From the Standard Specifications of the Pennsylvania Railroad Company.

§ 13. The calculation of stresses produced by the live and dead loads will determine the following values for each member:

M = maximum calculated stress in the member (compression or tension);

m = (1) minimum calculated stress in members subjected to one kind of stress only (all compression or all tension); or (2) maximum calculated stress of lesser kind, in members subjected to reversal of stress.

Note.—Minimum stress is understood to mean the absolute minimum; i.e., in a main diagonal or post of a simple span m equals the calculated dead-load stress minus the maximum calculated counter-stress due to live load. Then

$$R = \frac{m}{M}.$$

§ 14. The maximum calculated stress (M) in each member shall then be multiplied by the coefficient $(1+k)$, and the resultant quantity, $M(1+k)$, shall be regarded as the equivalent static stress in the member. (For the value of k see Tables A and B.)

TABLE A.—COEFFICIENTS OF STRESS INCREMENT.

CASE 1.—STRESSES OF ONE KIND ONLY—ALL COMPRESSION OR ALL TENSION.

$$k = \frac{1-R}{1+R}.$$

R.	k.	R.	k.	R.	k.	R.	k.	R.	k.
1.00	0.000	0.79	0.117	0.58	0.266	0.37	0.460	0.16	0.724
.99	.005	.78	.124	.57	.274	.36	.471	.15	.739
.98	.010	.77	.130	.56	.282	.35	.482	.14	.754
.97	.015	.76	.136	.55	.290	.34	.493	.13	.770
.96	.020	.75	.143	.54	.299	.33	.504	.12	.786
.95	.026	.74	.149	.53	.307	.32	.515	.11	.802
.94	.031	.73	.156	.52	.316	.31	.527	.10	.818
.93	.036	.72	.163	.51	.325	.30	.538	.09	.835
.92	.042	.71	.170	.50	.333	.29	.550	.08	.852
.91	.047	.70	.176	.49	.342	.28	.563	.07	.869
.90	.053	.69	.183	.48	.351	.27	.575	.06	.887
.89	.058	.68	.190	.47	.361	.26	.587	.05	.905
.88	.064	.67	.198	.46	.370	.25	.600	.04	.923
.87	.070	.66	.205	.45	.379	.24	.613	.03	.942
.86	.075	.65	.212	.44	.389	.23	.626	.02	.961
.85	.081	.64	.220	.43	.399	.22	.639	.01	.980
.84	.087	.63	.227	.42	.408	.21	.653	.00	1.000
.83	.093	.62	.235	.41	.418	.20	.667		
.82	.099	.61	.242	.40	.429	.19	.681		
.81	.105	.60	.250	.39	.439	.18	.695		
.80	.111	.59	.258	.38	.449	.17	.709		

TABLE B.—COEFFICIENTS OF STRESS INCREMENT.

CASE 2.—STRESSES SUBJECT TO REVERSAL.

$$k=\frac{2+R}{2-R}.$$

R.	k.	R.	k.	R.	k.	R.	k.	R.	k.
0.00	1.009	0.21	1.235	0.42	1.532	0.63	1.920	0.84	2.448
.01	1.010	.22	1.247	.43	1.548	.64	1.941	.85	2.478
.02	1.020	.23	1.260	.44	1.564	.65	1.963	.86	2.509
.03	1.030	.24	1.273	.45	1.581	.66	1.985	.87	2.540
.04	1.041	.25	1.286	.46	1.597	.67	2.007	.88	2.571
.05	1.051	.26	1.299	.47	1.614	.68	2.030	.89	2.603
.06	1.062	.27	1.312	.48	1.632	.69	2.053	.90	2.636
.07	1.073	.28	1.325	.49	1.649	.70	2.077	.91	2.670
.08	1.083	.29	1.339	.50	1.667	.71	2.101	.92	2.704
.09	1.094	.30	1.353	.51	1.685	.72	2.125	.93	2.738
.10	1.105	.31	1.367	.52	1.703	.73	2.150	.94	2.773
.11	1.116	.32	1.381	.53	1.721	.74	2.175	.95	2.809
.12	1.128	.33	1.395	.54	1.740	.75	2.200	.96	2.846
.13	1.139	.34	1.409	.55	1.759	.76	2.226	.97	2.884
.14	1.150	.35	1.424	.56	1.778	.77	2.252	.98	2.922
.15	1.162	.36	1.439	.57	1.797	.78	2.279	.99	2.960
.16	1.174	.37	1.454	.58	1.817	.79	2.306	1.00	3.000
.17	1.186	.38	1.469	.59	1.837	.80	2.333		
.18	1.198	.39	1.484	.60	1.857	.81	2.361		
.19	1.210	.40	1.500	.61	1.878	.82	2.390		
.20	1.222	.41	1.516	.62	1.899	.83	2.419		

§ 15. All members shall be so proportioned that the stress $M(1+k)$ shall not cause the tensile unit stress to exceed 15,000 pounds, nor the compressive unit stress to exceed 15,000 pounds properly reduced in accordance with § 16.

§ 16. For compression members the unit stress of 15,000 pounds per square inch shall be reduced in proportion to the ratio of the length of the least radius of gyration of the section by the following formula:

$$p=\frac{15000}{1+\frac{l^2}{13500r^2}},$$

where p=permissible working stress per square inch in compression;
l=length of piece in inches between centers of connections;
r=least radius of gyration of section in inches

§ 25. To insure the stability of bridges under increased live loads, a live load shall be assumed 100 per cent greater than that previously provided for in this specification. If the resultant stress $M(1+k)$

produces a stress per square inch in any member more than twice the permissible unit stress previously specified, additions must be made to the sections until that limit is not exceeded. Counters having in no case less than $1\frac{1}{2}$ square inches of section must be provided where required by the increased live load, and in case of reversal of stress the member must be properly designed to resist such reversal.

Art. 49.—Traction Stresses.

Traction stresses are due to the effect of friction of wheels on the rails, the greatest value of the friction being found when the brakes are applied. In the general design of bridge trusses this force is generally neglected; in special cases it may, however, be necessary to investigate its effect. In viaduct construction, the stresses in the cross-bracing caused by this traction force must be determined.

The following provisions taken from the specifications of the N. Y. C. & H. R. R.R. Co., 1904, fully cover this matter:

§ 97. The stresses produced in the *bracing* of the trestle towers, in any members of the trusses, or in the attachments of the girders or trusses to their bearings, by the greatest tractive force of the engines, or by suddenly stopping the trains on any part of the bridge, must be provided for. The coefficient of friction of the wheels on the rails will be assumed at 0.20. In all structures provision shall be made for trains going in either direction and on any track. The stresses produced in *the trestle legs* from this force need not be considered.

Art. 50.—Stresses Due to Centrifugal Load Stresses.

When the track on a structure is curved, the moving load produces a centrifugal effect which constitutes a horizontal load in a radial direction. For any given weight W, moving with a velocity v, expressed in feet per second, and on a curve whose radius is r, this centrifugal effect is

$\frac{Wv^2}{gr}$, g being the acceleration of gravity and equal to 32.2 feet per second^2.

The specifications under which any structure is designed will indicate the velocity and curvature for which the design is to be made. Mr. Theodore Cooper specifies a speed of $(60-3D)$ miles per hour, D being the degree of curvature, and the resulting force is to be taken as a live load acting 5 feet above the base of rail.

The Pennsylvania R.R. Co. determines the centrifugal force by means of the formula $F = 220D\left(1+\frac{13}{S}\right)$, F being expressed in pounds per linear foot of one track, D being the degree of curvature, and S the span in feet. The coefficient 220 is reduced by 12 for each degree of curvature above 6°. Table C furnishes the values of this centrifugal force for various degrees of curvature and various span lengths.

TABLE C.—CENTRIFUGAL FORCE IN POUNDS PER LINEAR FOOT OF ONE TRACK.

Span in Feet.	1-degree Curve.	2-degree Curve.	3-degree Curve.	4-degree Curve.	5-degree Curve.	6-degree Curve.	7-degree Curve.	8-degree Curve.	9-degree Curve.	10-degree Curve.	11-degree Curve.	12-degree Curve.
20	360	730	1090	1450	182c	2180	240c	259c	2730	2840	2900	2930
25	330	670	1000	1340	1670	2010	2210	2380	2520	2610	2680	2700
30	320	630	950	1260	1580	1890	2090	2250	2370	2470	2520	2550
40	300	580	880	1170	1460	1750	1930	2080	2190	2230	2330	2350
60	270	540	800	·1070	1340	1610	1770	1910	2020	2090	2140	2160
100	250	500	750	1000	1240	1490	1650	1770	1870	1940	1990	2010
200	230	470	700	940	1170	1410	1550	1670	1760	1830	1870	1890
300	230	460	690	920	1150							
500	230	450										

The N. Y. C. & H. R. R.R. Co. (1904) specifies as follows:

§ 96. When the bridge is in a curve, the centrifugal force shall be considered as a live load assumed to act 5 feet above the base of rail, and computed by the following formula for a maximum train load on

each track moving at a speed of 60 miles per hour for curves of 4 degrees or under, and at a speed of $(60-2D)$ miles per hour for curves exceeding 4 degrees:

$$F=\frac{Wv^2D}{85700},$$

where F = centrifugal force;
W = live load;
v = velocity in miles per hour;
D = degree of curve.

The American Bridge Co. (1900) employs the following formula:

$$F=0.03WD,$$

the symbols having the same meaning as above. The coefficient 0.03 is, however, to be reduced 0.001 for every degree of curvature above 5 degrees.

The method of application of the centrifugal force in finding the stresses in a structure will be shown later.

Art. 51.—Stresses Due to Wind.

The wind blowing against an exposed framed structure causes stresses in its members. In order to design rationally the requisite wind-bracing, it becomes necessary to determine the probable maximum force of the wind.

If the wind were directed as a finite stream against an infinitely large surface, so that the direction of the air is completely changed, an equation expressing the force against that surface may be obtained from the laws of mechanics.

Let W = the weight of air directed against any normal surface in a given time;
w = the weight in pounds of one cubic foot of air;
v = the velocity of the wind in feet per second;
a = the area of cross-section of the wind stream.

Then $W=wav$.

Let M = the mass of the air of the weight W;
g = the acceleration due to gravity = 32.2 feet per second per second;
F = the force acting on the area a.

Then
$$F = Mv = \frac{Wv}{g} = \frac{wav^2}{g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If a be taken at 1 square foot, and w at 0.0807 pound per cubic foot, for a temperature of 32° F. and a barometric pressure of 760 mm., and if v be replaced by V, the velocity in miles per hour, then

$$F = 0.0054V^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Since the weight of air changes both with the temperature and the barometric pressure, the constant in eq. (2) may be subjected to small changes, viz., it decreases for increase of temperature, and it increases with the barometric pressure. In structural practice, it is unnecessary to consider such refinements.

In actual cases of wind-pressures on structures, the ideal conditions of a small stream impinging against an infinite surface do not exist and therefore the change of direction is not complete. The air always tends to cushion in front of the surface, and to form a vacuum behind it, the latter effect increasing the resultant pressure against the body. These conditions involve, however, merely the determination by experiment of the proper coefficient for V, since the form of the equation itself is correct.

John Smeaton, in 1759, in discussing some experiments by Rouse, deduced the value of the coefficient as 0.00492, and Colonel Duchemin, in France, published in 1842 the same value for the ordinary air-pressures on thin stationary plates. Colonel Duchemin, in experimenting with

inclined flat thin plates, expressed the normal pressure as

$$p_n = 0.00492 V^2 \frac{2 \sin \alpha}{1 + \sin^2 \alpha},$$

the pressure parallel to the wind as

$$p_p = 0.00492 V^2 \frac{2 \sin^2 \alpha}{1 + \sin^2 \alpha},$$

and for the lifting or depressing pressure perpendicular to the wind as

$$p_v = 0.00492 V^2 \frac{2 \sin \alpha \cos \alpha}{1 + \sin^2 \alpha}.$$

In all cases α is the acute angle between the wind and the plane of the plate, V is expressed in miles per hour and p in pounds per square foot.

A commission of French officers, Piobert, Morin, and Didion, in 1835–38, found a formula $p = 0.0073 + 0.0034 V^2$.

The Tay Bridge Commission of 1881 determined the value of the coefficient at 0.01, if the velocity of the wind be found by anemometers.

H. Allen Hazen, in 1886, from his own and other experiments concluded (see p. 241, vol. 134, Am. Jour. of Science, 1887) that $p = 0.0034 V^2$. In 1888–89, W. H. Dines, at Hersham, England, deduced the formula $p = 0.0035 V^2$, and in 1890 C. F. Marvin, at Mount Washington, N. H., determined the coefficient to be 0.004.

Lieut. O. T. Crosby, in a paper, "An Experimental Study of Atmospheric Resistance," read before the West Point Branch of the United States Military Service Institution in 1890, deduced the formula $p = \frac{V}{7}$ for velocities from 30 to 130 miles per hour and with a surface 1 square foot in area. This result is such a radical departure from

the hitherto accepted values of p that further tests are desirable, although it is difficult to detect any ground of error in Mr. Crosby's paper. In 1888–90 S. P. Langley, from experiments at the Allegheny, Pa., Observatory, deduced $p=0.0039V^2$ from velocities 10 to 70 miles per hour.

In addition to Duchemin's formulæ for pressure on inclined surfaces, Hutton, in England, in 1787–88 deduced the ratio of the resistance between inclined and normal plates as $\sin \alpha^{1.842 \cos \alpha}$, α as before being the angle between the surface and the direction of the wind. Table I furnishes the values of the ratios of the normal pressure p_n for any surface for the angles indicated.

TABLE I.

Angle in Degrees.	Duchemin.	Hutton.	$p_n=p \sin \alpha$.
0	0.00	0.00	0.00
10	0.34	0.24	0.17
20	0.61	0.46	0.34
30	0.80	0.66	0.50
40	0.91	0.84	0.64
50	0.97	0.95	0.76
60	0.99	1.00	0.87

Hagen, 1874, deduced the formula

$$p=0.00306(1+0.048C)V^2,$$

which involves both the area and the perimeter of the surface, against which the wind is directed; C is the perimeter.

T. E. Stanton, in Proc. Inst. Civ. Eng., Vol. CLVI, 1903, found $p=0.0036V^2$ to be a general value for the pressure on a plate, this pressure being the algebraic sum of the suction pressure on the leeward side and the positive pressure on the windward side.

If the preceding equations be expressed by an average result of $p=0.004V^2$, Table II furnishes the values in

pounds per square foot for the given velocities of the wind in miles per hour.*

TABLE II.

Velocity of Wind in Miles per Hour.	Pressure in Pounds per Square Foot.
20	1.6
40	6.4
50	10.0
60	14.4
80	25.6
100	40.0

It is also to be noted that the intensity of wind-pressures measured by small gauges is always greater than when measured on large exposed surfaces, for the maximum intensity of pressure is probably confined to a small surface. For instance, the highest pressure actually observed and measured at a very exposed point on the site of the Forth Bridge, over a period of six years, on a surface of 300 (15×20 feet) square feet, was 27 pounds per square foot, although 41 pounds per square foot was at the same time observed on a small area of 1.5 square feet immediately adjacent to the large one.

Mr. C. Shaler Smith records in the Trans. Am. Soc. Civ. Engrs., Vol. X, a pressure of 93 pounds per square foot, which derailed a locomotive at East St. Louis, Mo., in 1871.

At the Bidston Observatory, near Liverpool, England, a pressure of 80 pounds per square foot was recorded in 1868, and 90 pounds pressure in 1871, these pressures being measured by spring pressure anemometers.

The following abstracts from standard bridge specifications illustrate modern (1905) practice.

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* For variation of wind-pressure above the surface of the ground, see Ency. Brit., Vol. XII (Hydromechanics).

(From Cooper's Specifications for Steel Railroad Bridges, 1901.)

§ 24. To provide for wind stresses and vibrations from high-speed trains, the top lateral bracing in deck-bridges, and the bottom lateral bracing in through-bridges, shall be proportioned to resist a lateral force of 600 pounds for each foot of span; 450 pounds of this is to be treated as a moving load, and as acting on a train of cars, at a line 6 feet above the base of rail.

The bottom lateral bracing in deck-bridges, and the top lateral bracing in through-bridges, shall be proportioned to resist a lateral force of 150 pounds for each lineal foot for spans up to 300 feet, and 10 pounds additional for each additional 30 feet.

§ 25. In trestle towers the bracing and columns shall be proportioned to resist the following lateral forces, in addition to the stresses from dead and live loads: (1) With either one track loaded with cars only, or with both tracks loaded with maximum train load, the lateral forces specified in § 24; and a lateral force of 100 pounds for each vertical lineal foot of the trestle bends; or

(2) With both tracks unloaded, a lateral force of 500 pounds for each longitudinal foot of the structure, acting at the center line of the girders, and a lateral force of 200 pounds for each vertical lineal foot of the trestle bents.

§ 26. For determining the requisite anchorage for a loaded structure, the train shall be assumed to weigh 800 pounds per lineal foot.

(From the Osborn Co.'s Specifications, 1896.)

Trusses.

§ 33. The bottom lateral bracing in deck-bridges and the top lateral bracing in through-bridges shall be proportioned to resist a moving load of 150 pounds per lineal foot for spans of 200 feet and under, and 0.4 pound per lineal foot for each additional foot in length of span over 200 feet.

The bottom lateral bracing in through-bridges and the top lateral bracing in deck-bridges shall be proportioned to resist a moving load of 450 pounds per lineal foot for spans of 200 feet and under, and 0.4 pound per lineal foot for each additional foot in length over 200 feet.

Deck Plate Girders. § 34. The lateral bracing of deck plate girders shall be proportioned to sustain a moving load per lineal foot of bridge of 330 pounds with 30 pounds additional for each foot in depth of girder. When laterals are used both at the top and bottom, with diagonals at each apex, this force may be considered to be equally divided between the two systems.

Through Plate Girders. § 35. The lateral bracing of through plate girders shall be proportioned to sustain a moving force per lineal foot of bridge of 300 pounds with 30 pounds additional for each foot in depth of girder.

Trestles. § 36. In trestle towers the cross-section of bracing and posts shall be proportioned to resist lateral pressures in addition to the stresses from dead load, live load, and centrifugal force as follows:

First. At post-cap a force of 1000 pounds with 30 pounds per lineal foot of structure in addition for each foot in depth of floor and girder measured from top of rail to the bottom of girder, and when girders are of different depths, a mean depth will be taken.

Second. A force of 300 pounds per lineal foot of structure applied 8.5 feet above base of rail.

Third. A force of 2700 pounds applied at each intermediate transverse strut.

(From the Pennsylvania Railroad Co.'s Specifications, 1901.)

Wind Loads. § 9. The wind-pressure shall be assumed acting horizontally in either direction:

(1) At 30 pounds per square foot on the exposed surface of all trusses and the floor as seen in elevation, and on the side of a train 10 feet high, beginning at $2\frac{1}{2}$ feet above the base of rail and moving across bridge.

(2) At 50 pounds per square foot on all exposed surfaces of the unloaded structure.

The greater calculated stress will be used in proportioning the wind-bracing.

Anchorage. § 10. For determining the requisite anchorage for the loaded structure, the train shall be assumed to weigh 800 pounds per lineal foot.

(From the American Bridge Co.'s Specifications for Steel Railroad Bridges, 1900.)

Wind-pressure. § 16. The wind-pressure shall be assumed acting in either direction horizontally.

First. At 30 pounds per square foot on the exposed surface of all trusses and the floors, as seen in elevation, in addition to a train of 10 feet average height, beginning 2 feet 6 inches above the base of rail, moving across the bridge.

Second. At 50 pounds per square foot on the exposed surface of all trusses and the floor system. The greatest result shall be assumed in proportioning the parts.

(From the American Bridge Co.'s Specifications for Steel Highway Bridges, 1901.)

Wind-pressures. § 26. The wind-pressure shall be assumed acting in either direction horizontally.

First. At 30 pounds per square foot on the exposed surface of all trusses and the floor as seen in elevation, in addition to a horizontal live load of 150 pounds per lineal foot of the span moving across the bridge.

Second. At 50 pounds per square foot on the exposed surface of all trusses and the floor system. The greatest result shall be assumed in proportioning the parts.

Art. 52.—Temperature Stresses.

In the case of simply supported framed structures, changes of length * due to temperature changes are provided for by placing one end of the structure on rollers or by sup-

* A large number of determinations of the thermal expansion of iron and steel per degree Fahrenheit may be found in the U. S. Report of Tests of Metals and

porting it in such a way as freely to permit expansion and contraction of the structure as a whole. For those classes of structure in which the ends are fastened, so that this method is inapplicable, it is generally specified that a range of temperature of 150° F. (American Bridge Co., 1900; Cooper, 1901; Pennsylvania R.R. Co., 1901) must be provided for.

In general the ends of bridges which exceed about 80 feet in length are provided with nests of turned friction-rollers. The ends of bridges less than about 80 feet in length are usually free to move upon planed surfaces.

Art. 53.—Stresses Due to Initial Tension.

When members of framed structures are made adjustable in length by means of turnbuckles it is sometimes specified that they be given an initial tension, in order to avoid vibration. Such initial tension must be determined by the judgment of the designer, and the stresses caused by it must be investigated. A value frequently used is 10,000 pounds. It is to be noted that initial stresses for simply supported trusses occur only with redundant members, viz., with members whose stresses cannot be determined by simple statical equations of condition. The structure is therefore erected with these members entirely slackened, and after the structure is self-supporting these members are given their adjustment.

If redundant members of the kind indicated are built without adjustment, as in the stiff diagonals of wind-bracing, the stresses caused by improper length or by

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other Materials 1887. The maximum, minimum, and mean for steel bars are as follows:

.000006756, .000006466, .00000617.

Other coefficients of thermal expansion are also given as follows:

Wrought iron	.00000673
Cast iron	.000005926
Copper	.000009129

temperature can be determined only with great difficulty, and consequently they are usually neglected. It would be necessary to treat them by using either the principle of least work or of virtual displacements. For an excellent discussion of this matter, the reader is referred to a paper by F. H. Cilley, in the Technology Quarterly, June, 1897, on "Some Fundamental Propositions Relating to the Design of Frameworks."

Art. 54.—Snow-load Stresses.

Since the majority of railroad bridges have open floors, it is not necessary to consider snow loads for them, but for highway bridges, for railroad bridges with solid floors, and for roof-trusses the stresses caused by snow loads must be determined.

The amount of snow load carried by a roof-truss depends not only on its latitude, but also on the pitch of its surface. Ordinarily the snow is specified as a load in pounds per square foot of horizontal projection. In the vicinity of New York a value of 20 pounds per square foot for roofs flat or nearly so is usually taken, but this is decreased for roofs with steep pitch. If the pitch is 60° or more, no snow loads need be taken, although a minimum weight of 10 pounds per square foot, due to sleet, is sometimes specified. For cold climates the highest value of the snow load may be taken at 30 pounds per square foot, but for southern latitudes the snow disappears.

In general, snow loads are considered only for roof-trusses.

CHAPTER III.

MOMENTS AND SHEARS. DESIGN OF PLATE GIRDERS.

Art. 55.—External Bending Moments and Shears in General.

A beam is said to be *non-continuous* if its extremities simply rest at each end of a span without constraint.

A beam is said to be *continuous* if it extends over more than one span, or one or both ends, in any case, suffer constraint.

A cantilever is a beam which overhangs its span, one end being constrained or fixed and the other being in no manner supported. Each of the overhanging portions of an open swing-bridge is a cantilever truss.

Fig. 1 represents a beam simply supported at each

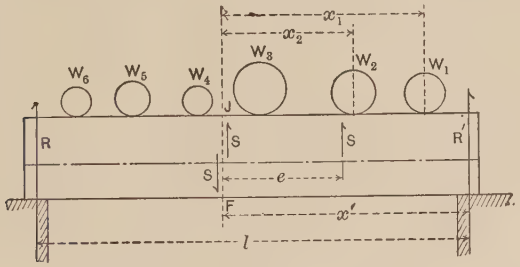


FIG. 1.

end, carrying the loads W_1, W_2, W_3, etc. Let bending moments be taken for any section, as JF, at the distance x' from the right-hand abutment, at which location the reaction R' acts. The load W_1 is at the distance x_1 from

the section, W_2 at the distance x_2, and W_3 at the distance x_3 from the same section, the last distance not being shown in the figure. The desired bending moment will be

$$M = R'x - W_1x_1 - W_2x_2 - W_3x_3. \quad . \quad . \quad . \quad (1)$$

This equation is typical of all external bending moments for a beam simply supported at each end, whatever may be the system of loading or its amount, or whatever may be the location of the section. This equation is frequently written in the folowing form:

$$M = \Sigma Wx. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The summation sign indicates that the sum is to be taken of the products formed by multiplying each external force, whether loading or reaction, by its lever-arm or normal distance from the section in question. It is a common and convenient mode of expressing the general value of the bending moment in any case whatever.

In eq. (1) the differentials of x', x_1, x_2, and x_3 are all equal, so that if the equation be differentiated, the first derivative of M will have the following form:

$$\frac{dM}{dx} = R' - W_1 - W_2 - W_3 = \Sigma W = S. \quad . \quad . \quad . \quad (3)$$

It will at once be evident that S in eq. (3) is the total transverse shear in the section for which the bending moment M is written, since the algebraic sum of R' and the loads between the end of the beam and the section constitute that shear. Indeed, the usual manner of determining the total transverse shear is the simple operation of summing up all the external forces acting on one of the portions of the beam formed by the section in question; the external forces, such as the reaction, acting in one direction being given one sign and those like the loading, acting in the

other direction, being given the opposite sign. The shear, therefore, becomes the numerical difference of the two sets of forces having opposite directions.

Eq. (3) thus establishes the following important principle: *The total transverse shear at any section is equal to the first differential coefficient of the bending moment considered a function of x.*

As is well known, the analytical condition for a maximum or minimum bending moment in a beam is

$$\frac{dM}{dx} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From eqs. (3) and (4) is to be deduced the following principle: *The greatest or least bending moment in any beam is to be found in that section for which the shear is zero.*

The greatest bending moment obviously is of importance in the design of beams and trusses, and eq. (4) shows that the section in which it will be found can be located by simple inspection of the loading. It is only necessary to sum up the reaction at one end and the loads adjacent to it, until the point is reached where the summation is zero. This point will usually be found where a load is supported. In that case the single load may arbitrarily be divided into two parts, supposed to act indefinitely near to each other, so that one of the parts may be just sufficient to make the zero summation desired. A single practical operation will make this feature perfectly clear and simple.

If the loading is uniformly continuous and of the intensity p, in each of the equations (1), (2), and (3) pdx is to be used for each of the separate loads W_1, W_2, W_3, etc. The bending moment thus becomes

$$M = R'x' - \Sigma Wx = R'x' - \int_0^x x \cdot pdx = R'x' - \tfrac{1}{2}px^2. \quad (5)$$

The expression for the shear then becomes

$$\frac{dM}{dx} = S = R' - px. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

A second differentiation gives

$$\frac{d^2M}{dx^2} = p. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Or, the second differential coefficient of the moment considered a function of x is equal to the intensity of the continuous load.

This method of passing from formulæ for concentrated loads to those for continuous loads is perfectly simple and frequently employed.

Art. 56.—Intermediate and End Shears.

The beam shown in Fig. 2 is supposed to carry any loading whatever, and the figure is consequently intended

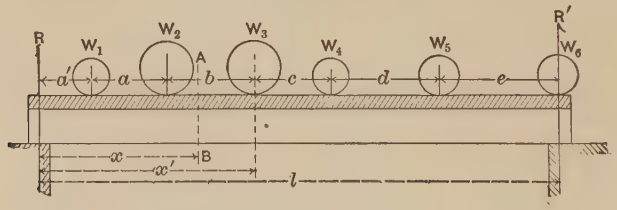


FIG. 2.

to exhibit a uniform load in addition to a load of concentrations. The amount of uniform loading per linear unit, usually a foot, is represented by p, while the concentrations, as heretofore, are represented by W_1, W_2, etc.

The determination of the transverse shear at any section of a beam or truss is one of the most simple and frequent as well as one of the most important computations

required in the design of structures. It is first necessary, after knowing the position of the loading, to find the reactions at both ends of the span. In Fig. 2 the various weights or loads are separated by the distances shown, a' being the distance from W_1 to the reaction R or end of the span. W_6 is supposed to rest at the right end of the span, for a purpose that will presently appear. The reaction R'' at the left end of the span (not shown) resulting from the concentrated loads only will have the following value:

$$R'' = W_1\left(\frac{a+b+\ldots+e}{l}\right) + W_2\left(\frac{b+c+\ldots+e}{l}\right) + W_3\left(\frac{c+d+e}{l}\right) + W_4\left(\frac{d+e}{l}\right) + W_5\frac{e}{l}. \quad (1)$$

The reaction R''' at the other end of the span (not shown) can be expressed by a similar equation, but it is simpler and more direct to write it as follows:

$$R''' = W_1 + W_2 + W_3 + W_4 + W_5 - R''. \quad . \quad . \quad (2)$$

Obviously the sum of the two reactions R'' and R''' must be equal to the total concentrated loading.

That part of the reaction due to the uniform load extending over the span l will clearly be one half of that load or

$$R_1 = \tfrac{1}{2}pl = R_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The reaction R_1 is supposed to be found at the left end of the span and R_2 at the right end. The total reactions then will be as follows. At the left end of the span

$$R = R'' + \tfrac{1}{2}pl. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

At right end of the span

$$R' = R''' + \tfrac{1}{2}pl. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The transverse shear at any intermediate section of the beam whatever may now readily be written. Let the section AB at the distance x from the left end of the span first be considered. The total loading between that section and the end of the span is $W_1 + W_2 + px$, and it acts downward. As the reaction R acts upward the expression for the shear will be

$$S = R - W_1 - W_2 - px. \quad . \quad . \quad . \quad . \quad (6)$$

In this case the section considered has been taken between two weights. Let the section at the weight W_3 be considered, that weight being at the distance x' from the end of the span. The amount of uniform load over the length x' is simply px', but inasmuch as the weight W_3 is located at the section under consideration, the portion of that weight which may be taken as resting on the left of the section considered is indeterminate. In such cases it is proper and customary to take any portion or all of the weight as resting on either side of the section, but indefinitely near to it. If it is a case where the maximum shear is desired, the single weight should be taken in such a position as to make the transverse shear as great as possible. If the case is one where it is desired to find the section at which the total load from that section to the end of the span is equal to the reaction, any portion may be taken which is found necessary to make the equality. If, for instance, $px' + W_1 + W_2$ is less than R, while $Px' + W_1 + W_2 + W_3$ is greater than R, then that portion of W_3 which would be considered on the left of the section but indefinitely near to it would be $R - px' - W_1 - W_2$. The remaining portion of W_3 would be considered as resting at the right of the section but indefinitely near to it. In such a case the transverse shear is zero at the weight W_3.

Again, let it be desired to find the greatest upward shear at W_3, it being supposed that R is greater than the total load between W_3 and the left end of the span. In this case no portion of W_3 would be considered as acting to the left of the section, but the expression for the shear would be

$$S = R - px' - W_1 - W_2. \quad . \quad . \quad . \quad . \quad (7)$$

It can be seen from the preceding statements that the maximum transverse shear in the beam will occur at the ends of the span where the value of the shear is the end reaction. Inasmuch as the end reaction R or R' is thus the greatest shear in the entire span, it is a most important quantity to determine in the design of beams and trusses, as it mainly controls the amount of material required at the end sections of both beams and trusses. The value of this end shear is given by the values for R and R' in eqs. (4) and (5).

Since the total transverse shear in any section of a beam is simply the summation of all the external loads, including the reactions from one end of the span up to the section considered, it is evident, first, that the summation may be made from either end of the span, and second, that the amount so found will be equal numerically but affected by opposite signs. In determining the shear, therefore, in any given case, it is usual to make the summation from that end of the span which can be used with the greatest convenience in computation.

Fig. 3 exhibits a graphical representation of the preceding treatment of intermediate and end shears, MN being the length of span shown in Fig. 2; MF is the reaction R laid off at a convenient scale. The weights or loads W_1, W_2, W_3, etc., are laid off vertically downward in their proper locations at the same scale as shown. The vertical distance of G below F is the amount of uni-

form load pa' between R and W_1 in Fig. 2, also laid down by the same scale. GG_1 is, therefore, the shear in the beam of Fig. 2 immediately to the left of W_1, and H_1G_1 is the shear immediately to the right of the same load. Similarly H_1H being drawn horizontally, HK is the amount of uniform loading pa between W_1 and W_2. The remainder of the diagram is drawn in the same manner.

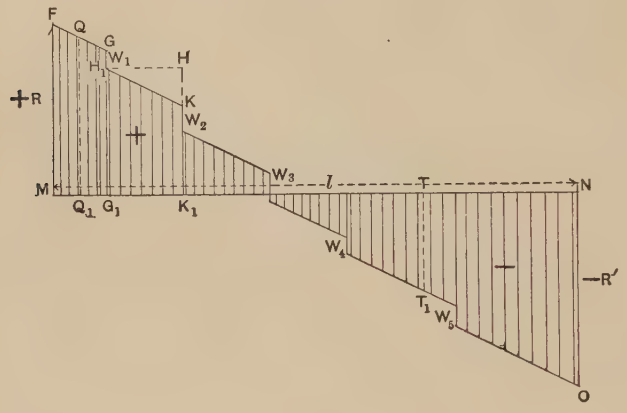


FIG. 3.

Any vertical ordinate drawn from MN either up or down to the broken line $FGH_1K \ldots O$ represents the shear at the corresponding point in the span at the same scale used in laying off the reactions and loads. QQ_1 is the shear at the point or section of beam at Q_1, while TT_1 is the shear at the section T. The shear is zero at W_3, where it changes its sign. At that point also will be found the greatest bending moment in the beam.

As the diagram is drawn the shears on the left of W_3 and above MN are positive, those on the right of W_3 and below MN being negative; but the diagram might have been drawn with equal propriety so as to have made R' and the shears between it and W_3 positive and those between that load and R negative.

A glance at the diagram shows that the end shears, equal to the reactions, are the greatest in the span.

If a beam carries a load of concentrations only, its shear diagram will be illustrated by Fig. 4, in which there

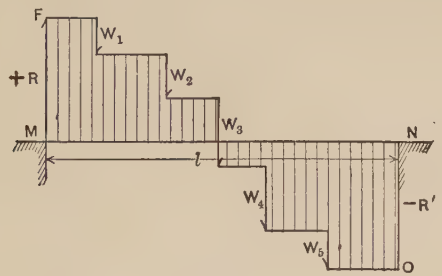


FIG. 4.

are five loads, the diagram being composed of rectangles only. If, again, the load is wholly uniform, Fig. 5 will represent the shear diagram composed of two triangles with their apices at *C*, the center of the span and point of no shear. Any vertical ordinate drawn from *MN* in

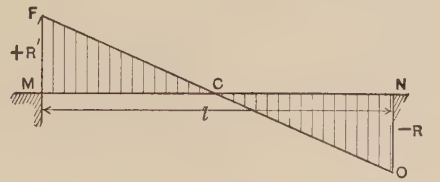


FIG. 5.

either figure to the stepped line in the one case and to the straight line in the other will represent the shear at the section of beam from which the ordinate is drawn. Those diagrams represent completely the graphical treatment of shears in all cases.

Art. 57.—Greatest Bending Moment in a Non-continuous Beam Produced by Concentrated Loads.

The position of the moving load for the greatest bending moment at any section of a non-continuous beam may be very simply determined. In Fig. 6, let FG represent any

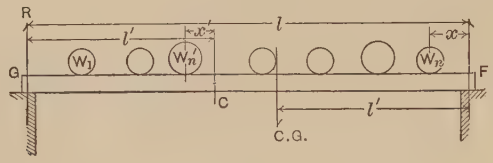


FIG. 6.

such beam with the span l, and let any moving load whatever, as $W_1 \ldots W_n' \ldots W_n$, advance from F toward G. Let C be the section at which it is desired to determine the maximum bending moment, and let n' loads rest to the left of C, while n is the total number of loads on the span. Finally, let x' represent the distance of W_n' from C and to the left of that point, while x is the distance of W_n to the left of F. If a is the distance between W_1 and W_2, b the distance between W_2 and W_3, c the distance between W_3 and W_4, etc., the reaction R at G will be

$$R = \left\{ \begin{array}{l} W_1 \dfrac{a+b+c+ \ldots +x}{l} \\ +W_2 \dfrac{b+c+ \ldots +x}{l} \\ \ldots\ldots\ldots\ldots\ldots \\ +W_n \dfrac{x}{l} \end{array} \right\} \quad \ldots \quad (1)$$

The bending moment M about C will then take the value

$$M = Rl' - \begin{cases} W_1(a+b+c+ \ldots +x') \\ +W_2(b+c+ \ldots +x') \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ +W_n'x'. \end{cases}$$

Or, after inserting the value of R from above,

$$\begin{aligned} M = \frac{l'}{l}[&W_1 \cdot a + (W_1+W_2)b + (W_1+W_2+W_3)c \\ &+ \ldots + (W_1+W_2+W_3+ \ldots +W_n)x] \\ &- W_1a - (W_1+W_2)b - W_1+W_2+W_3)c - \ldots \\ &- (W_1+W_2+W_3+ \ldots +W_n')x'. \quad \ldots\ldots \quad (2) \end{aligned}$$

If the moving load advances by the amount Δx, the moment becomes, since $\Delta x = \Delta x'$,

$$\begin{aligned} M' = M + \frac{l'}{l}(W_1+W_2+W_3+ \ldots +W_n)\Delta x \\ -(W_1+W_2+ \ldots +W_n')\Delta x. \quad (3) \end{aligned}$$

Hence for a maximum the following value must never be negative:

$$\begin{aligned} M' - M = \Delta x \Big\{ \frac{l'}{l}(W_1+W_2+W_3+ \ldots +W_n) \\ -(W_1+W_2+ \ldots +W_n') \Big\} = 0. \quad (4) \end{aligned}$$

Or, the desired condition for a maximum takes the form

$$\frac{l'}{l} = \frac{W_1+W_2+ \ldots +W_n'}{W_1+W_2+W_3+ \ldots +W_n}. \quad \ldots \quad (5)$$

It will seldom or never occur that this ratio will exactly exist if W_n' is supposed to be a *whole* weight; hence W_n' will usually be that part of a whole weight at C which

is necessary to be taken in order that the equality (5) may hold.

It is to be observed that if the moving load is very irregular, so that there is great and arbitrary diversity among the weights W, there may be a number of positions of the moving load which will fulfill eq. (5), some one of which will give a value greater than any other; this is the absolute maximum desired.

From what has preceded, it follows that W_n' may always be taken at the point C in question; hence x' in eq. (2) may always be taken equal to zero when that equation expresses the greatest value of the moment. The latter may then take either of the two following forms:

$$\left.\begin{aligned} M=\frac{l'}{l}[&W_1a+(W_1+W_2)b+\ldots+W_1+W_2+\ldots+W_n)x] \\ &-W_1a-(W_1+W_2)b-\ldots \\ &-(W_1+W_2+\ldots+W_{n'-1})(?), \end{aligned}\right\} \quad (6)$$

$$\left.\begin{aligned} M=\frac{l'}{l}[&W_1(a+b+\ldots+x)+W_2(b+c+\ldots+x) \\ &+W_3(c+d+\ldots+x)+\ldots+W_n\,.\,x] \\ &-W_1(a+b+\ldots+?)-W_2(b+\ldots+?)-\ldots-W_{n'-1}(?). \end{aligned}\right\} \quad (6a)$$

In these equations x corresponds to the position of maximum bending, while the sign (?) represents the distance between the concentrations $W_{n'-1}$ and $W_{n'}$.

It has already been shown that for any given condition of loading the greatest bending moment in the beam will occur at that section for which the shear is zero. But if the shear is zero, the reaction R must be equal to the sum of the weights $(W_1+W_2+\ldots+W_n')$ between G and C, the latter now being the section at which the greatest moment in the span exists.

Hence for that section eq. (5) will take the form

$$\frac{l'}{l}=\frac{R}{W_1+W_2+W_3+\ldots+W_n}. \quad \ldots \quad (7)$$

But if cg is the center of gravity of the load in Fig. 6, and if its distance from F is l',

$$R = \frac{l'}{l}(W_1 + W_2 + \ldots + W_n). \quad . \quad . \quad . \quad (8)$$

This equation in connection with eq. (7) shows that the center of gravity of the load is at the same distance from one end of the beam as the section or point of greatest bending is from the other. In other words, *the distance between the point of greatest bending for any given system of loading and the center of gravity of the latter is bisected by the center of span.*

If the load is uniform, therefore, it must cover the whole span.

It is to be observed that eq. (6) is composed of the sums W_1, $W_1 + W_2$, etc., multiplied by the distances a, b, c, etc. Again, as in the equation immediately preceding eq. (2), the expression for the moment M may be taken as composed of the positive products of each of the single weights W_1, W_2, etc., multiplied by its distance from any point distant x to the right of W_n and of the negative products similarly taken in reference to the section located by x', as shown by eq. (6a).

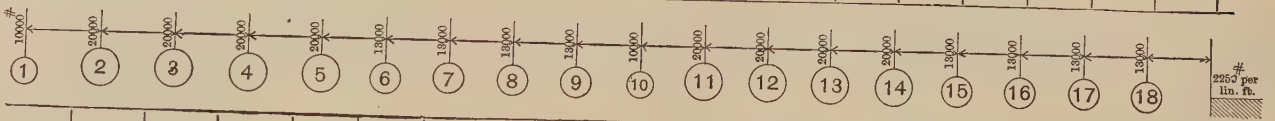
The practical application of the preceding formulæ can therefore best be effected by means of a tabulation of moments like that shown in Table I, taken from the standard specifications of the N. Y. C. & H. R. R.R. Co. for 1902. The wheel weights and train loads shown in the table are for one rail only, i.e., they are half those for one track. By comparing the weights and spacings with those in Fig. 6 and eq. (6) it will be seen that $W_1 = 10{,}000$ pounds, $W_2 = 20{,}000$ pounds, $W_3 = 20{,}000$ pounds, etc., and that $a = 8$ feet, $b = 5$ feet, $c = 5$ feet, etc.

In using tables similar to Table I for shears as well as for moments it is convenient to have the summations

TABLE I.

CONSOLIDATION ENGINE, N. Y. C. & H. R. R. R.—1902.

10	30	50	70	90	103	116	129	142	152	172	192	212	232	245	258	271	284	1
284	274	254	234	214	194	181	168	155	142	132	112	92	72	52	39	26	13	2
8	13	18	23	32	37	43	48	56	64	69	74	79	88	93	99	104	109	3
109	101	96	91	86	77	72	66	61	53	45	40	35	30	21	16	10	5	
8′	5′	5′	5′	9′	5′	6′	5′	8′	8′	5′	5′	5′	9′	5′	6′	5′	5′	4



16364	15274	13254	11334	9514	7794	6793	5857	4999	4206	3676	2776	1976	1276	676	403	195	65	5
14944	13904	11984	10164	8444	6824	5888	5017	4224	3496	3016	2216	1516	916	416	208	65		6
13589	12599	10779	9059	7439	5919	5048	4242	3514	2851	2421	1721	1121	621	221	78		65	7
12041	11111	9411	7811	6311	4911	4118	3390	2740	2155	1785	1205	725	345	65		78	221	8
10816	9936	8336	6836	5436	4136	3408	2745	2160	1640	1320	840	460	180		65	208	416	9
8728	7938	6518	5198	3978	2858	2247	1701	1233	830	600	300	100		117	299	559	884	10
7668	6928	5608	4388	3268	2248	1702	1221	818	480	300	100		100	282	529	854	1244	11
6708	6018	4798	3678	2658	1738	1257	841	503	230	100		100	300	547	859	1249	1704	12
5848	5208	4088	3058	2148	1328	912	561	288	80		100	300	600	912	1289	1744	2264	13
4632	4072	3112	2252	1492	832	520	273	104		160	420	780	1240	1656	2137	2696	3320	14

Line 1. Summation of loads from left end. 2. Summation of loads from right end. 3. Summation of distances from left end. 4. Summation of distances from right end. 5. Moments about end of uniform load. 6 to 14. Moments about wheels 18 to 10 respectively. Figures to left of heavy line are moments of loads to left of wheel indicated. Figures to right of heavy line are moments of loads to right of wheel indicated. Moments in thousands of Ft.-lbs. Summations of loads in thousands of pounds.

of weights, distances between weights or spacings of loads, and the moments already described from each extremity of the system or series. An examination of the table with the explanatory notes at the bottom will make its composition perfectly clear. The figures on the right of the heavy stepped line are found by taking moments of each load or weight proceeding from load 10 to the right. For example: $20{,}000 \times 8 = 160{,}000$; $20{,}000 \times (8+5) + 20{,}000 \times 8 = 420{,}000$; $20{,}000 \times (5+5+8) + 20{,}000 \times (5+8) + 20{,}000 \times 8 = 780{,}000$. Also in line 10, $13{,}000 \times (5+9) + 13{,}000 \times 9 = 299{,}000$, etc. Again, beginning at the right end of line 5, $13{,}000 \times 5 = 65{,}000$; $13{,}000 \times (5+5) + 13{,}000 \times 5 = 195{,}000$, etc. All the lines from 5 to 14 are made up in a similar manner.

The method of finding shears set forth in Art. 56 and of finding the greatest bending moments by eqs. (5), (6), and (6*a*) of this article by the aid of Table I leads to the numerical results shown in Table II. The series of concentrated loadings exhibited in Tables I and II are identical and they are given in the specifications of Mr. Theodore Cooper, C.E., as the locomotive loading E 40. It will be observed that the entire train for Table II is composed of two coupled locomotives followed by a uniform train of 2,000 pounds per linear foot for each rail; in Table I, however, the train load is 2,250 pounds per linear foot.

Table II is taken from a paper by Mr. O. E. Selby, Jun. Am. Soc. C. E., in the Trans. Am. Soc. C. E., Vol. XLII, 1899. In general the computations of maximum shears at arbitrary points like quarter-points and the center is largely tentative by the use of the general methods of Art. 56, but Mr. Selby obtained his shears (of Table II) by means of equivalent loads, which he explains in his paper.

When a uniform train load is a part of the system of loading it is only necessary to consider any desired portion

TABLE II.

MAXIMUM MOMENTS AND SHEARS IN PLATE GIRDERS.

Moments in Thousands of Ft.-lbs. Shears in Thousands of Lbs. Weights on One Rail.



Span, Feet.	Moment.	Shears. End.	Shears. ‡ P't.	Shears. Center.	Span, Feet.	Moment.	Shears. End.	Shears. ‡ P't.	Shears. Center.	Span, Feet.	Moment.	Shears. End.	Shears. ‡ P't	Shears. Center.	Span, Feet.	Moment.	Shears. End.	Shears. ‡ P't.	Shears. Center.
....	.....	.....	.....	.....	36	548.6	70.6	44.1	20.6	66	1539.5	105.0	63.6	29.7	96	2993.9	144.8	83.3	38.3
....	.....	.....	.....	.....	37	574.3	71.9	44.8	21.0	67	1581.5	106.4	64.2	30.0	97	3049.0	146.2	84.2	38.5
8	37.8	27.5	17.5	10.0	38	600.0	73.1	45.4	21.3	68	1623.5	107.8	64.8	30.2	98	3106.5	147.5	85.0	38.8
9	46.9	28.9	18.9	10.0	39	626.6	74.3	46.0	21.7	69	1665.5	109.2	65.4	30.5	99	3162.3	148.8	85.8	39.1
10	56.3	30.0	20.0	10.0	40	655.6	75.4	46.8	22.0	70	1706.9	110.5	65.9	30.7	100	3219.9	150.0	86.6	39.4
11	65.7	32.7	20.9	10.9	41	684.6	76.8	47.5	22.3	71	1749.3	111.8	66.5	31.1	101	3277.6	151.2	87.3	39.6
12	80.0	35.0	21.7	11.7	42	713.6	78.1	48.2	22.6	72	1792.2	113.3	67.0	31.4	102	3335.9	152.4	88.1	39.9
13	95.0	36.9	22.3	12.3	43	742.6	79.4	48.9	22.9	73	1833.9	114.8	67.5	31.7	103	3410.6	153.7	88.8	40.1
14	110.0	38.6	23.6	12.9	44	771.6	80.6	49.5	23.2	74	1879.2	116.3	68.0	32.0	104	3475.2	154.9	89.5	40.4
15	125.0	40.0	25.0	13.3	45	800.6	81.7	50.1	23.4	75	1025.8	117.7	68.6	32.3	105	3537.6	156.1	90.3	40.6
16	140.0	42.5	26.3	13.8	46	829.6	82.8	50.7	23.7	76	1972.0	119.1	69.2	32.6	106	3600.3	157.3	90.9	40.9
17	155.0	44.7	27.4	13.8	47	858.6	83.8	51.4	23.9	77	2019.1	120.4	69.9	32.9	107	3666.6	158.5	91.7	41.1
18	170.0	46.7	28.3	13.9	48	887.6	84.8	52.1	24.2	78	2064.4	121.7	70.5	33.2	108	3745.3	159.6	92.4	41.3
19	186.6	48.4	29.2	14.0	49	918.8	86.0	52.8	24.5	79	2112.3	123.0	71.1	33.4	109	3818.4	160.8	93.2	41.6
20	206.3	50.0	30.0	14.0	50	950.9	87.1	53.5	24.9	80	2160.5	124.2	71.7	33.7	110	3886.8	162.0	93.9	41.8
21	226.0	51.4	31.4	14.5	51	983.1	88.2	54.1	25.2	81	2207.7	125.6	72.3	34.0	111	3958.2	163.2	94.6	42.0
22	245.7	52.7	32.7	15.0	52	1015.2	89.2	54.8	25.5	82	2256.7	126.9	73.0	34.4	112	4026.9	164.4	95.3	42.2
23	265.4	53.9	33.9	15.4	53	1047.4	90.2	55.4	25.8	83	2306.5	128.2	73.7	34.7	113	4099.0	165.5	96.0	42.5
24	285.2	55.0	35.0	15.8	54	1081.4	91.2	56.1	26.1	84	2356.3	129.5	74.4	35.0	114	4172.0	166.7	96.8	42.8
25	305.0	56.5	36.0	16.2	55	1116.9	92.1	56.8	26.4	85	2406.9	130.7	75.1	35.3	115	4245.0	167.9	97.5	43.1
26	324.8	57.9	36.9	16.5	56	1152.4	93.0	57.5	26.6	86	2459.6	132.1	75.8	35.6	116	4318.8	169.0	98.3	43.4
27	344.6	59.2	37.8	16.9	57	1187.9	94.2	58.2	26.9	87	2510.6	133.4	76.5	35.9	117	4389.5	170.2	99.0	43.7
28	365.5	60.4	38.6	17.1	58	1223.4	95.4	58.8	27.2	88	2564.2	134.7	77.1	36.2	118	4463.8	171.4	99.7	43.9
29	388.0	61.6	39.3	17.4	59	1261.0	96.5	59.5	27.5	89	2615.9	136.0	77.9	36.5	119	4538.8	172.5	100.4	44.2
30	410.5	63.0	40.0	17.7	60	1299.6	97.6	60.1	27.9	90	2670.5	137.2	78.7	36.7	120	4614.1	173.7	101.1	44.5
31	432.9	64.4	40.7	18.2	61	1338.3	98.7	60.7	28.2	91	2723.0	138.5	79.5	37.0	121	4686.5	174.8	101.8	44.7
32	455.4	65.7	41.3	18.8	62	1377.0	100.0	61.3	28.5	92	2775.9	139.8	80.3	37.3	122	4762.7	176.0	102.5	45.0
33	477.9	66.9	42.0	19.2	63	1415.6	101.3	61.8	28.8	93	2831.5	141.1	81.0	37.5	123	4836.2	177.1	103.2	45.3
34	500.4	68.1	42.8	19.7	64	1455.5	102.6	62.4	29.1	94	2885.3	142.4	81.7	37.8	124	4917.4	178.3	104.0	45.7
35	523.0	69.2	43.5	20.1	65	1497.5	103.8	63.0	29.4	95	2939.5	143.6	82.5	38.0	125	4996.4	179.4	104.7	46.0

of it as acting through its center of gravity, i.e., through its mid-point. Taking that center as its point of application, the separating space is the distance from that point to the nearest concentration. If in Table II 20 feet of train load be taken, that train weight will be 20,000 pounds applied at the distance $10+5=15$ feet from load 18. This simple operation is all that is needed for any uniform load or for a series of sections of uniform load.

Art. 58.—Computations for a Railroad Plate Girder.

Let a railroad plate girder with an effective span of 80 *feet be traversed from right to left by the moving load shown in Table II, except that there will be taken but one locomotive followed by the uniform train load of* 2,000 *pounds per linear foot. It is required to find the greatest bending moments and shears at the center and quarter-points of the span, the dead load or own weight of the span and track being taken at* 1,400 *pounds per linear foot.*

Dead Load.

By eq. (6) of Art. 57 the bending moments at the quarter-point and center are, since the reaction R is $40\times 700=28{,}000$:

	Quarter-point, $x=20$ feet.	Center, $x=\frac{1}{2}l=40$ feet.
$M=$	420,000 ft.-lbs.	560,000 ft.-lbs.

By eq. (7) of Art. 56 the shears at end, quarter-point, and center are

	End, $x=0$.	Quarter-point, $x=20$ feet.	Center, $x=40$ feet.
Shear	28,000 lbs.	14,000 lbs.	Zero

Moving Load.

If wheel 3 be placed at the quarter-point, there will be one locomotive and 20 feet of uniform train load on the

span, the latter being 5 feet from the last tender wheel, which may be represented by wheel 9. As $\frac{l'}{l}=\frac{1}{4}$, the criterion eq. (5) gives either $\frac{l'}{l}=\frac{30,000}{182,000}$ or $\frac{50,000}{182,000}$, the first being too small and the second too large. Hence wheel 3 at the quarter-point is the proper position for the maximum bending moment. In eq. (6) or (6*a*), $W_n=40,000$ pounds and $x=10$, also $W_n'_{-1}=W_2=20,000$ pounds. Hence the bending moment has the value $M=\frac{1}{4}(4,206,000+142,000\times 20+40,000\times 10)-230,000=1,631,500$ ft.-lbs.

If wheel 5 be placed at the center of the span, 10 feet of uniform train load will rest on the girder, making a total of 162,000 pounds. The criterion eq. (5) then gives

$$\frac{l'}{l}=\frac{70,000}{162,000} \quad \text{or} \quad \frac{90,000}{162,000},$$

the first value being less than $\frac{1}{2}$ and the second greater. Hence the trial position is correct, and the corresponding greatest moment will be

$$M=\tfrac{1}{2}(4,206,000+142,000\times 10+20,000\times 5)-830,000$$
$$=2,033,000 \text{ ft.-lbs.}$$

As the span is 80 feet, the uniform load giving the same center moment as the actual load, i.e., the equivalent uniform load, is

$$p=\frac{8}{80\times 80}\times 2,033,000=2,541 \text{ pounds per linear foot;}$$

the total bending moment at the quarter-point will then be

$$1,631,500+420,000=2,051,500 \text{ ft.-lbs.,}$$

and at the center

$$2{,}033{,}000 + 560{,}000 = 2{,}593{,}000 \text{ ft.-lbs.}$$

The end shear will be found by placing W_2 at the left end of the span, requiring 35 feet of train load to be on the girder. The reaction, i.e., the end shear, at the left end of the span will then be written by the aid of Table I:

$$\text{Reaction} = \text{end shear} = \frac{4{,}206{,}000 + (142{,}000 - 10{,}000) \times 35 + 70{,}000 \times 17.5 - 10{,}000 \times 53}{80}$$

$$= 119{,}013 \text{ lbs.}$$

The equivalent uniform load producing the same shear is

$$p = \frac{119013}{40} = 2975 \text{ lbs. per linear foot.}$$

The shear at the quarter-point will be found by placing W_2 at that point, requiring 10 feet of train load to be on the girder. The reaction at the left end of the span will then be

$$R = \frac{4{,}206{,}000 + 142{,}000 + 10 + 20{,}000 \times 5}{80} = 71{,}575 \text{ lbs.}$$

The shear at the quarter-point will therefore be

$$71{,}575 - 10{,}000 = 61{,}575 \text{ lbs.}$$

The center shear will be found by placing W_2 at the center of the span, requiring W_9 to be at the right-hand extremity of the span. The reaction at the left-hand end of the span will then be

$$R = \frac{4{,}206{,}000 - 142{,}000 \times 5}{80} = 43{,}700 \text{ lbs.}$$

The center shear will then be

$$43,700 - 10,000 = 33,700 \text{ lbs.}$$

The total shear at the end, quarter-point, and center will have the values:

At end. ..... 119,013 + 28,000 = 147,013 lbs.
At quarter-point... 61,575 + 14,000 = 75,575 lbs.
At center..... = 33,700 lbs.

There may be more than one position of concentrated loadings satisfying the criterion eq. (5), as may be seen by trial. Such maxima must frequently be sought, and they can be found, if existing, with little trouble.

Art. 59.—Moments and Shears in Special Cases.

Certain special cases of beams are of such common occurrence, and consequently of such importance, that a somewhat more detailed treatment than that already given may be deemed desirable. The following cases are of this character:

CASE I.

Let a non-continuous beam supporting a single weight P at any point be considered, and let such a beam be represented in Fig. 7. If the span RR' is represented by

$$l = a + b = RP + R'P,$$

the reactions R and R' will be

$$R = \frac{b}{l}P \quad \text{and} \quad R' = \frac{a}{l}P. \quad . \quad . \quad . \quad . \quad (1)$$

Consequently, if x represents the distance of any section in RP from R, while x' represents the distance of any section

of $R'P$ from R', the general values of the bending moments for the two segments a and b of the beam will be

$$M = Rx \quad \text{and} \quad M' = R'x'. \quad . \quad . \quad . \quad . \quad (2)$$

These two moments become equal to each other and represent *the greatest bending moment in the beam* when

$$x = a \quad \text{and} \quad x' = b,$$

or *when the section is taken at the point of application of the load* P.

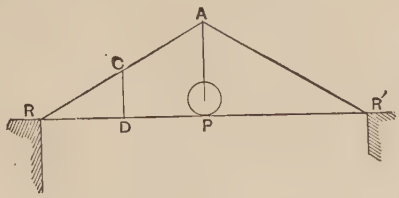


FIG. 7.

Eq. (2) shows that the moments vary directly as the distances from the ends of the beam. Hence if AP (normal to RR') is taken by any convenient scale to represent the greatest moment, $\frac{ab}{l}P$, and if RAR' is drawn, any intercept parallel to AP and lying between RAR' and RR' will represent the bending moment for the section at its foot by the same scale. In this manner CD is the bending moment at D.

The shear is uniform for each single segment; it is evidently equal to R for RP and R' for $R'P$. It becomes zero at P, where is found the greatest bending moment.

CASE II.

Again, let Fig. 8 represent the same beam shown in Fig. 7, but let the load be one of uniform intensity, p, ex-

tending from end to end of the beam. Let C be placed at the center of the span, and let R and R', as before, represent

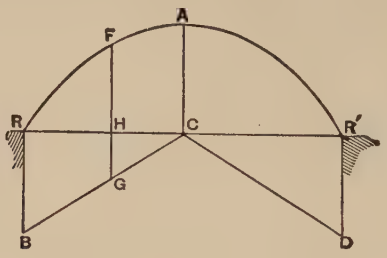


FIG. 8.

the two reactions. Since the load is symmetrical in reference to C,

$$R = R'.$$

For the same reason the moments and shears in one half of the beam will be exactly like those in the other; consequently reference will be made to one half of the beam only. Let x and x_1 then be measured from R toward C. The forces acting upon the beam are R and p, the latter being uniformly continuous. Applying the formulæ for the bending moment at any section x, remembering that x_1 has all values less than x,

$$M = Rx - p\int_0^x (x - x_1)\,dx_1;$$

$$\therefore M = Rx - \frac{px^2}{2} \quad \ldots\ldots\ldots \quad (3)$$

If l is the span, at C, M becomes

$$M_1 = \frac{Rl}{2} - \frac{pl^2}{8} \quad \ldots\ldots\ldots \quad (4)$$

But because the load is uniform

$$R = \frac{pl}{2}.$$

Hence

$$M_1=\frac{pl^2}{8}=\frac{Wl}{8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

if W is put for the total load. Placing $R=\frac{pl}{2}$ in eq. (3),

$$M=\frac{p}{2}(lx-x^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The moments M, therefore, are proportional to the abscissæ of a parabola whose vertex is over C, and which passes through the origin of coordinates R. Let AC, then, normal to RR', be taken equal to M_1, and let the parabola RAR' be drawn. Intercepts, as FH, parallel to AC, will represent bending moments in the sections, as H, at their feet.

The shear at any section is

$$S=\frac{dM}{dx}=R-px=p\left(\frac{l}{2}-x\right), \quad . \quad . \quad . \quad . \quad (7)$$

or *it is equal to the load covering that portion of the beam between the section in question and the center.*

Eq. (7) shows that the shear at the center is zero; it also shows that $S=R$ at the ends of the beam. It further demonstrates that *the shear varies directly as the distance from the center.* Hence take RB to represent R and draw BC. The shear at any section, as H, will then be represented by the vertical intercept, as HG, included between BC and RC.

The shear being zero at the center, the greatest bending moment will also be found at that point. This is also evident from inspection of the loading.

Eq. (2) of Case I shows that if a beam of span carries a weight $\frac{W}{2}$ at its center, the moment M at the same point will be

$$M_1 = \frac{W}{4} \cdot \frac{l}{2} = \frac{Wl}{8}. \quad \ldots \ldots \quad (8)$$

The third member of eq. (8) is identical with the third member of eq. (5). It is shown, therefore, *that a load concentrated at the center of a non-continuous beam will cause the same moment at the center as double the same load uniformly distributed over the span.*

Eqs. (5) and (8) are much used in connection with the bending of ordinary non-continuous beams, whether solid or flanged; and such beams are frequently found.

CASE III.

The third case to be taken is a cantilever uniformly loaded; it is shown in Fig. 9. Let x be measured from the free end A, and let the uniform intensity of the load be represented by p. The load px acts with its center at the distance $\frac{1}{2}x$ from the section x. Hence the desired moment will be

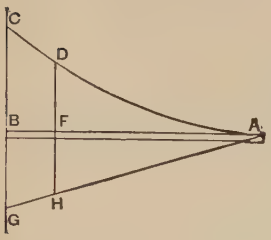


FIG. 9.

$$M = -px \cdot \frac{x}{2} = -\frac{px^2}{2}. \quad \ldots \ldots \quad (9)$$

If $AB = l$, the moment at B is

$$M_1 = -\frac{pl^2}{2}. \quad \ldots \ldots \quad (10)$$

The negative sign is used to indicate that the *lower* side of the beam is subjected to compression. In the two preceding cases, evidently the *upper* side is in compression.

The shear at any section is

$$S = \frac{dM}{dx} = -px. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Hence the shear at any section is the load between the free end and that section.

Eq. (9) shows that the moments vary as the *square* of the distance from the free end; consequently the moment curve is a parabola with the vertex at A, and with a vertical axis. Let BC, then, represent M_1 by any convenient scale, and draw the parabola CDA. Any vertical intercept, as DF, will represent the moment at the section, as F, at its foot.

Again, let BG represent the shear pl at B, then draw the straight line AG. Any vertical intercept, as HF, will then represent the shear at the corresponding section F.

Art. 60.—General Formulæ of the Common Theory of Flexure.

It is necessary to express in a perfectly general manner the fundamental formulæ of the common theory of flexure for direct use in treating some types of bridge structures, such as arched ribs. It will be assumed that the ordinary laws of flexure and their usual applications are known.* The general formulæ desired may then be established in the following manner:

In Fig. 10, let ABC represent the center line of any bent beam; AF a vertical line through A; CF a horizontal line through C, while A is the section of the beam at which the deflection (vertical or horizontal) in reference to C, the bending moment, the shearing stress, etc., are to be determined. As shown in the figure, let x

* The subject of flexure will be found fully treated in "The Elasticity and Resistance of Materials," by Wm. H. Burr.

be the horizontal coordinate measured from A, and y the vertical one measured from the same point; then let z be the horizontal distance from the same point to the point of application of ny external vertical force P. To complete the notation, let D be the deflection desired; M_1 the moment of the external forces about A; S the shear at A; P' the strain (extension or compression) per

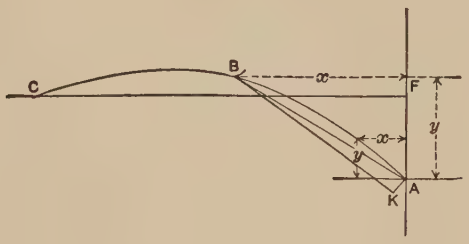


FIG. 10.

unit of length of a fiber parallel to the neutral surface and situated at a normal distance of unity from it; I the general expression of the moment of inertia of a normal cross-section of the beam, taken in reference to the neutral axis of that section; E the coefficient of elasticity for the material of the beam, and M the moment of the external forces for any section, as B.

Again, let Δ be an indefinitely small portion of any normal cross-section of the beam, and let y' be an ordinate normal to the neutral axis of the same section. By the "common theory" of flexure the intensity of stress at the distance y' from the neutral surface is $(y'P'E)$. Consequently the stress developed in the portion Δ of the section is $EP'y'\Delta$, and the resisting moment of that stress is $EP'y'^2\Delta$.

The resisting moment of the whole section will therefore be found by taking the sum of all such moments for its whole area.

Hence

$$M = EP' \Sigma y'^2 \Delta = EP' I.$$

Hence, also,

$$P' = \frac{M}{EI}.$$

If n represents an indefinitely short portion of the neutral surface, the strain for such a length of fiber at unit's distance from that surface will be nP'.

If the beam were originally straight and horizontal, n would be equal to dx.

P' being supposed small, the effect of the strain nP' at any section, B, is to cause the end A of the chord BA to move vertically through the distance $nP'x$.

If BK and BA (taken equal) are the positions of the chords before and after flexure, $nP'x$ will be the vertical distance between K and A.

By precisely the same kinematical principle, the expression $nP'y$ will be the horizontal movement of A in reference to B.

Let $\Sigma nP'x$ and $\Sigma nP'y$ represent summations extending from A to C, then will those expressions be the vertical and horizontal deflections respectively of A in reference to C. It is evident that these operations are perfectly general, and that x and y may be taken in any direction whatever.

The following general but strictly approximate equations relating to the subject of flexure may now be written:

$$S = \Sigma P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$M_1 = \Sigma Pz. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$P' = \frac{M}{EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\Sigma nP' = \Sigma n\frac{M}{EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$D = \Sigma nP'x = \Sigma\frac{nMx}{EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$D_h = \Sigma nP'y = \Sigma\frac{nMy}{EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

D_h represents the horizontal deflection.

The summation ΣPz must extend from A to a point of no bending, or from A to a point at which the bending moment is M_1'. In the latter case

$$M_1 = \Sigma Pz + M_1'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

M_1 may be positive or negative.

Art. 61.—The Theorem of Three Moments.

The theorem of three moments is expressed by an equation showing the relations between the bending moments existing in a continuous beam at three adjacent points of support, the loading of the beam being any whatever. If the two adjoining spans under a continuous beam or truss are represented by l_a and l_c, as shown in Fig. 11, and

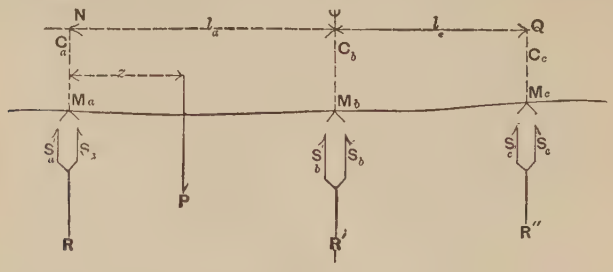


FIG. 11.

if the three consecutive bending moments are M_a, M_b, and M_c, the ordinary or special form of the theorem is

$$M_a l_a + 2M_b(l_a + l_c) + M_c l_c + \frac{1}{l_a}\overset{a}{\Sigma} P(l_a^2 - z^2)z + \frac{1}{l_c}\overset{c}{\Sigma} P(l_c^2 - z^2)z = 0. \quad (1)$$

The summations $\overset{a}{\Sigma}$ and $\overset{c}{\Sigma}$ belong to the spans l_a and l_c respectively. The horizontal ordinate z is measured from the left end of the left-hand span or from the right end of the right-hand span.

Equation (1) is a special and simplified form* of the general value of the theorem of three moments. As it is generally employed, any weight P, only, is assumed to rest on one span, as l_a, with nothing on the other. Eq. (1) would then take the form

$$M_a l_a + 2M_b(l_a + l_c) + M_c l_c + \frac{1}{l_a} P(l_a^2 - z^2)z = 0. \quad (2)$$

The general demonstration of this theorem, reference to which is made above, shows that eqs. (1) and (2) are strictly true only for a beam or truss straight between points of support, and that those points of support may or may not be in the same level if they belong to a configuration of no bending in the beam.

Art. 62.—Shears and Reactions under Continuous Beams and Trusses.

The reactions at each point of support is composed of two shears, as shown in Fig. 11 of the preceding article. These shears may readily be found by taking moments about the ends of spans. Giving the positive sign to upward forces and taking moments about B,

$$M_a + S_a l_a - P(l_a - z) = M_b. \quad . \quad . \quad . \quad . \quad (1)$$

* The complete general demonstration of the theorem may be found in "The Elasticity and Resistance of Materials," by Wm. H. Burr.

Again, taking moments about A,

$$M_b + S_b' l_a - Pz = M_a. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Eqs. (3) and (4) then give at once

$$S_a = P\frac{l_a - z}{l_a} - \frac{M_a - M_b}{l_a}, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$S_b' = P\frac{z}{l_a} + \frac{M_a - M_b}{l_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Similar values of shears may be written for other spans by simply changing the subscripts. If a be changed to c in the second members of eqs. (3) and (4),

$$S_b = P\frac{z}{l_c} + \frac{M_c - M_b}{l_c}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$S_c' = P\frac{l_c - z}{l_c} - \frac{M_c - M_b}{l_c} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The reactions at the various points of support may then be expressed as follows by referring to Fig. 11 of the preceding article:

$$R = S_a' + S_a, \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$R' = S_b' + S_b, \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$R'' = S_c' + S_c, \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$\text{etc.} = \text{etc.} + \text{etc.}$$

If there are a number of weights concurrently on the span, the sign Σ is to be written before each term containing P in eqs. (3), (4), (5), and (6).

Continuous Loading with Uniform Intensity.

If the loading over the different spans is of uniform intensity, then, in general, $P = wdz$, w being the intensity. Consequently

$$\Sigma P(l^2 - z^2)z = \int_0^l w(l^2 - z^2)zdz = w\frac{l^4}{4}. \quad . \quad . \quad (1)$$

In all equations, therefore, for $\frac{1}{l_a}\overset{a}{\Sigma}P(l_a{}^2 - z^2)z$ there is to be placed the term $w_a\frac{l_a{}^3}{4}$; and for $\frac{1}{l_c}\overset{c}{\Sigma}P(l_c{}^2 - z^2)z$ the term $w_c\frac{l_c{}^3}{4}$. The letters a and c mean that reference is made to the spans l_a and l_c.

Art. 63.—The Design of a Plate Girder.

The total length of a plate girder is materially more than the length of clear span over which the girder is designed to carry load. The ends of the girders are carried by blocks, or pedestals of masonry or metal, resting on the masonry or other supporting masses or members. The distance between the centers of these blocks or pedestals is called the effective span of the girder, as it is the span length used in computing bending moments, shears, or reactions. Plate girders must evidently be somewhat longer than the effective span. In Fig. 12 is shown the detailed drawing of an 81-foot railroad deck plate-girder span, with end pin and roller bearing, together with the relations of the various parts of the end of the girder. As shown, the ties rest directly on the upper flange. Where headroom is limited under the structure it is necessary to build through-girders with a complete floor system of longitudinal stringers and cross floor-beams, or with a

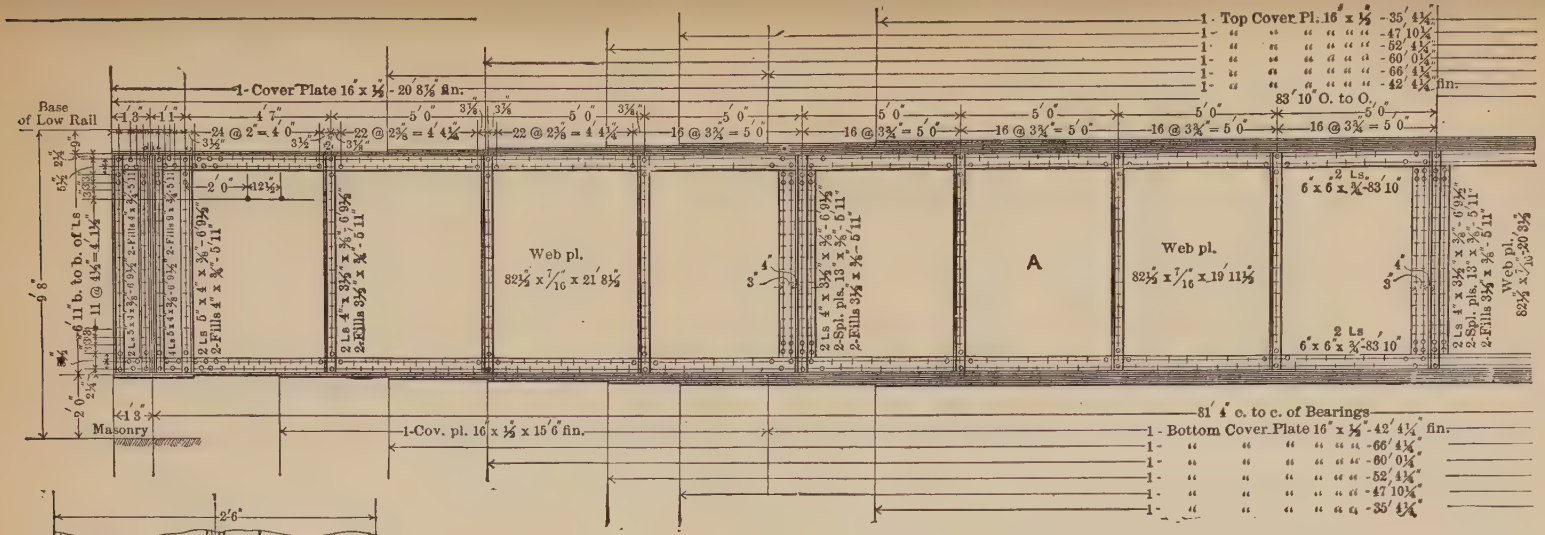
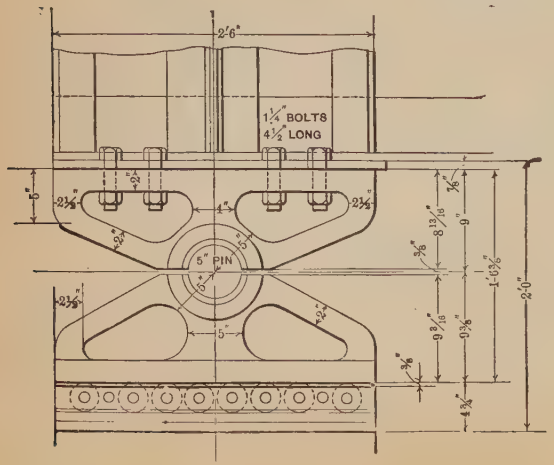


FIG. 12.—Detail Drawing of an 81-foot Railroad Plate-girder Span with End Pin and Roller Bearing.



End Pin and Roller Bearing.

trough floor frequently ballasted. In some special cases it may be permissible to support the ends of the ties on the inner projections of the lower flanges. The case of a complete floor system belongs properly to paneled truss construction, which will be fully treated later. Trough-floor plate girders, except for the rivets in the angles of the upper flange, are treated precisely as deck-girders.

The deck plate-girder span shown in Plate I will be designed in detail as an illustration of general plate-girder design. The main girders will have an effective span of 87 feet 9 inches and a limiting length, out to out of the steel, of 90 feet.

The specifications to be employed will be those of the American Bridge Co. for 1900 for medium open-hearth steel. The rivets are to be $\frac{7}{8}$ inch in diameter. The live loading employed is Cooper's E 50.

The effective depth of the girder is the vertical distance or depth between the centers of gravity of the two flanges. When the girder has cover-plates, this effective depth may be greater than the depth of web-plate at the center of span, and less than that at the ends, even when the web-plate is of uniform depth. It is always customary, however, to take the effective depth of a plate girder with uniform depth of web as constant. Frequently that depth is taken equal to the depth of the web-plate, as in the specification of the N. Y. C. & H. R. R.R. Co. for 1900 (§ 100), which requires that "the depth shall be taken as the distance between the centers of gravity of the flange areas, except that in no case shall it exceed the distance, back to back, of flange angles."

In case the web-plate is not of uniform depth, the effective depth might still be taken as the depth of the web-plate at the various sections of the girder, or it may be taken as the depth between centers of gravity of the flanges at the same sections.

The differences between the effective span and the clear span and total length are obviously dependent upon the length of the span. For short spans those differences are relatively small, and relatively large for long spans.

It is usual to specify that the depth of plate girders, wherever possible, shall be not less than one tenth, and in no case less than one twelfth, of the span. The depth of web-plate will, therefore, be taken at 90 inches, and it will be found later that at, and in the vicinity of, the center of span 4 cover-plates will be needed.

The dead load or own weight of the girder and track will depend somewhat upon whether the girder is of the through or deck type. The only difference in computation arising in those two types is due to the fact that in deck-girders the rivets connecting the upper flanges with the web must be assumed to carry the wheel concentrations in addition to their other duties, and this will be illustrated in the example chosen. The assumed dead load will be taken as follows:

Track (rails, ties, etc.). .	500	lbs. per linear foot of bridge
Main girders.	1080	" " " " " " "
Total.	1580	" " " " " " "

Inasmuch as there are two girders, each will carry one half of the moving load and one half of the above dead load or own weight.

Bending Moments.

The first computations necessary are those required to determine the bending moments, and from them the flange stresses at different points on the span. For the fixed load, the bending moments vary throughout the length of span, as the ordinates of a parabola having its vertex over the center. In order to simplify the work, it is generally assumed that the moving-load bending moments follow a

similar curve, the ordinate at the center representing the maximum bending at that point. This is not exact, but it is sufficiently accurate for ordinary purposes. In more exact work the actual maximum values of the bending moments at sections 6, 8, or 10 feet apart should be computed. The closer together the sections are taken, the greater will be the degree of accuracy attained. In the present instance the parabolic law will be assumed. After the bending moments are obtained, the flange stresses at once result by dividing the former by the effective depth.

As each girder will carry 790 pounds of dead load per linear foot, and as the effective span is 87.75 feet, the expression for the dead-load bending moment in foot-pounds at any point will be as follows:

$$M = \frac{790}{2}(87.75x - x^2). \quad . \quad . \quad . \quad . \quad . \quad (1)$$

At the center, then, $M = 760{,}400$ foot-pounds $= 9{,}125{,}000$ inch-pounds.

Reference to the table on page 96 will show that the center maximum live-load moment for Cooper's E 40, however, is 2,551,000 foot-pounds, or 30,612,000 inch-pounds. For Cooper's E 50 loading, this quantity becomes $\frac{50}{40} \times 30{,}612{,}000 = 38{,}265{,}000$ inch-pounds. The criterion which must be used to determine this position requires the center of span to divide in half the distance between the point of maximum bending and the center of gravity of the loading. No uniform train load will rest on the bridge.

The specifications (§ 37) require that "girders shall be proportioned on the assumption that one eighth of the gross area of the web is available as flange area. The compressive flange shall have the same sectional area as

the tension flange, but the unsupported length of flange shall not exceed sixteen times its width."

Further, § 15 requires the addition to the dead- and live-load stress of an impact stress I determined by the formula $I = S\left(\frac{300}{L+300}\right)$, where S is the calculated maximum live-load stress and L the length of loaded distance in feet producing that stress.

In the present case, since the entire bridge is covered by the moving load,

$$I = 38{,}265{,}000\left(\frac{300}{387.75}\right) = 29{,}580{,}000 \text{ inch-pounds.}$$

A tensile live-load working stress of 17,000 pounds per square inch is specified in § 21 for medium steel.

The resultant moment is

$$9{,}125{,}000 + 38{,}265{,}000 + 29{,}580{,}000 = 76{,}970{,}000 \text{ inch-pounds.}$$

The effective depth, will later be found to be 90.5 inches. Usually, however, a tentative value would first be employed. Dividing the moment by the product $17{,}000 \times 90.5$, the net required area is 50.0 square inches.

The specifications permit one eighth of the area of the web-plate section, i.e. 4.9 square inches in this case, to be considered as flange area. Therefore only $50 - 4.9 = 45.1$ square inches of net section in the tension flange is required.

It should be stated that the bending resistance of a plate girder is sometimes computed by the general formula involving the moment of inertia of the section $M = \frac{kI}{d}$, d being the distance from the center of gravity of the section, i.e. usually the mid-depth, to the extreme fiber, while k is the greatest permitted intensity of stress in that fiber.

Sectional Area of Flange.

The net section of the flange must be made up of cover-plates and angles, but before making the actual design the arrangement of rivets connecting the flange angles with the web-plate should be outlined. In this case angles with legs 6 inches long will be used, since it is advisable that the angles of the flanges should always form as large a part of the flange section as practicable; it is even sometimes specified that that part shall preferably be not less than one half. It is then certain that two lines of rivets in each leg will be employed, but it does not follow that two rivet-holes should be taken out of each leg to find the net section of these angles, for by staggering the rivet-holes a less amount of material may be wasted. There are not sufficient results of experiments with staggered rivet-holes on record to determine the degree of staggering of two lines of rivets to make it proper to deduct one hole only for the net section. In this case two rivet-holes will be taken from each angle to find the net area of section.

The specified diameter of rivet, $\frac{7}{8}$ inch, belongs to the cold rivet as delivered at the shop. The increased diameter caused by driving is not recognized in rivet computations, but the reamed rivet-hole must be taken $\frac{1}{8}$ inch larger in diameter than the rivet, or one inch in the present case.

Two $6 \times 6 \times \frac{3}{4}$-inch angles with a gross area of 16.9 square inches will be taken for each flange. The metal punched from the angles in each flange will then be $4 \times 1 \times \frac{3}{4} = 3$ square inches, and the remaining net section will be 13.9 square inches. This net sectional area of the two angles runs throughout the entire length of the tension flange. As a total net section of 45.1 square inches is required at the center cover-plates must be added at that point to complete the flange. The lengths of those plates are to be determined by the additional areas required at other points.

As already stated, the flange stresses and hence the flange areas are supposed to vary as the ordinates of a parabola when the depth of girder is constant. The proper lengths of the cover-plates may then be found by the following procedures.

Determination of Length of Cover-plates.

(*a*) *Depth of Girder Uniform.*

The following notation will be employed, it being assumed that *one* cover-plate only is required:

l = effective length of span either in feet or inches;
L = length of cover-plate required in the same unit as l;
A = total net flange area at center;
a = net cover-plate area required.

Since the flange and cover-plate areas, like the flange stresses, vary as the ordinates of a parabola when the depth of girder is constant, the following equation will result:

$$\frac{L^2}{l^2} = \frac{a}{A},$$

or

$$L = l\sqrt{\frac{a}{A}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Eq. (1) will give the length of the cover-plate whose area of section is a. Any convenient unit may be taken for a and A, but the square inch is ordinarily employed.

If there are several cover-plates, a is to be taken successively as the area of the first, second, third, etc., cover-plates in summation; i.e., it will first be taken as the net sectional area of the top cover, then as the net sectional area of the top cover added to that below it, and so on.

(*b*) *Depth of Girder Varying.*

Let w = uniform load per linear foot, or "equivalent uniform load" per linear foot;

d and d' represent the effective depths of girder in feet at the center of span and at the end of the cover-plate respectively;

$A - a = a'$ = area of flange section at the end of cover-plate;

T = permissible flange stress per square inch.

The bending moment at the end of the cover-plate will then be

$$M = w\frac{l^2}{8} - \frac{w}{2}\left(\frac{L}{2}\right)^2 = AdT - w\frac{L^2}{8} = d'a'T. \quad . \quad . \quad (2)$$

By solving the second and third members of the preceding equation there will result

$$L = 2\sqrt{2}\sqrt{\frac{(Ad - a'd')T}{w}} = 2.83\sqrt{\frac{(Ad - a'd)T}{w}}. \quad . \quad (3)$$

It must be remembered that the application of either of the two preceding methods will give the *net* length of the cover-plate. There must be added 12 to 18 inches at each end with rivets closely pitched so that the cover-plate may certainly take its stress at the points where its effectiveness should begin.

The selection of the cover-plates for girders so as to best make up the areas required is largely a matter of judgment. There are, however, some general considerations always to be observed. No cover-plate should be used thicker than the one underneath it, and as a rule, the width of cover-plates should be but little, if any, less than the width of the two flange angles and the web-plate be-

tween them, but the cover-plate may have a somewhat greater width. In the present instance the cover-plates should not be less than 12 inches wide. The width of 16 inches will be adopted, as it gives well-proportioned plates. As there will be four lines of staggered rivets, the net width of the plates will be taken at 14 inches. Four plates, each $\frac{9}{16}$ inch thick, will then furnish an additional net area of $4 \times 7.9 = 31.6$ square inches, which, together with the 13.9 square inches of the angles, will afford a total net section of 45.5 square inches. This is but slightly more than the exact amount required, as it should be.

The length of the top cover-plate is then found by means of eq. (1) to be $L = 87.75\sqrt{\frac{7.9}{50.4}} = 34.0$ feet; that of the second, $87.75\sqrt{\frac{15.8}{50.4}} = 49$ feet; that of the third, 60 feet; and that nearest the angles, 69.5 feet. It is sometimes specified that the first flange-plate of the top and bottom flange should extend the full length of the girder, except for girders less than 75 feet long, on which the bottom flange-plate need not extend the full length. This provision is not found in the American Bridge Co.'s specifications. As at least 12 to 18 inches must be added to each end of each cover-plate, the actual lengths of those under consideration will be taken at 41, 54, 64, and 73 feet.

Obviously at the ends of the flanges there will be some excess of net section in the angles, just as there will also be found some excess of section at and near the end of each cover-plate, but those conditions are inevitable.

In designing and arranging the cover-plates of a plate girder, care should be taken to use plates neither too thick nor too thin. It is seldom necessary to use a plate as thick as 1 inch, and no plate less than $\frac{3}{8}$ inch thick should be used for a railroad plate girder. In general, it is the custom to punch and ream rivet-holes in plates $\frac{5}{8}$ inch thick

or less and to drill holes in thicker plates; it is therefore generally preferable not to exceed $\frac{5}{8}$ inch thickness of plate.

The distance between the centers of gravity of the flanges at the center of span is 90.5 inches, which checks the value of the effective depth used in determining the flange stresses. It is to be noted that the distance back to back of the upper and lower flange angles is always made $\frac{1}{4}$ inch greater than the depth of web-plate, so that the web-plate may not project between the flange angles and interfere with the riveting of the cover-plates.

Rivets between Web and Flanges and End-stiffeners.

Stresses are given to the flanges of a plate girder through the rivets which bind the flange angles to the web-plate. It is necessary, therefore, to have a sufficient number of rivets through the legs of the flange angles lying against the web-plate to carry to the flange metal those flange stresses which have been computed. Each of those rivets will obviously be in double shear, but the bearing capacity of the rivet can be determined only after the thickness of the web-plate is known, and it will next be necessary to ascertain that thickness. In the present case the web-plate is assumed to carry both transverse shear and a part of the longitudinal stress. The specification, § 39, permits an allowed intensity of shearing stress of 10,000 pounds per square inch, but no web-plate less than $\frac{3}{8}$ of an inch in thickness is allowed.

The maximum transverse shear will be at the end of the span, i.e., it will be the maximum reaction. In the present instance it will exist when the first driver (wheel No. 2) is at one end, or, strictly speaking, at an indefinitely short distance from one end. Assuming that position of the moving load, therefore, the reaction will have the value of 134,700 pounds for Cooper's E 40 loading, for a

span of 88 feet, as shown by the table on page 96. For Cooper's E 50 the end shear becomes $\frac{5}{4} \times 134{,}700 = 168{,}000$ pounds. Similarly, at the quarter- and center-points of the span the maximum shears will be 96,300 and 45,300 pounds. The end reaction due to dead load will be $\frac{1}{2}(790 \times 87.75) = 34{,}700$ pounds; the dead-load shears at the quarter- and center-points will be 17,400 and 0 pounds respectively. The impact shear to be added to the end reaction is 130,200 pounds; at the center, 39,500 pounds; and at the quarter-points, 89,600 pounds.

The final shears will then be 332,900, 203,300, and 84,800 pounds at the end-, quarter-, and center-points respectively. Hence the required sectional area at the end will be $\frac{332{,}900}{10{,}000} = 33.3$ square inches, and a plate $90 \times \frac{7}{16} = 39.4$ square inches will be chosen. It will not prove advisable for this length of span to change the web section. Since plates of this size are not ordinarily rolled in greater lengths than 340 inches,* it will be necessary to splice the web at three points, viz., at the center and the quarter-points, thus using 4 plates, each about 270 inches long.

In determining the number of rivets required to convey stresses from the web to the flanges, it must be observed that each rivet passing through the vertical legs of the flange angles and the web-plate is subjected to double shear at the two surfaces of the latter, while it bears against the $\frac{7}{16}$-inch web-plate.

The specifications (§ 34) fix the allowable shearing stress on medium-steel rivets at 12,000 pounds per square inch and the allowable bearing pressure at 24,000 pounds per square inch. Again, § 38 requires that "the whole of the shear acting (in the web-plate) on the side next the abutment is to be considered as being transferred into the flange angles in a distance equal to the depth of the girder."

* See Carnegie Handbook.

This last requirement is based upon the following simple method of finding the pitch of rivets, which is in turn found from the general but elementary expression for the bending moment:

$$\Sigma Px = M.$$

By differentiating this equation

$$\Sigma \cdot P \cdot dx = Sdx = dM,$$

where S represents the total transverse shear.

If dM is the change of bending moment for the distance along the flange represented by the pitch of rivets, p, the change of flange stress for the same distance will be found by dividing dM by the effective depth of girder, h. If the pitch of rivets, p, be placed in the preceding equation in place of dx, the corresponding change of flange stress will represent the amount of stress transferred to the flange by one rivet. Representing that variation of flange stress by v, the last of the preceding equations may be written

$$S'p = hv. \quad \therefore \; p = \frac{hv}{S}.$$

In this equation v represents either the bearing capacity of one rivet against the web-plate or against the two flange angles, or the double shearing value of the same rivet, i.e., the least of those three values. The total transverse shear S is readily determined at any section, and has already been found for the girder under consideration.

The double shearing value of one $\frac{7}{8}$-inch rivet at 12,000 pounds per square inch is 14,440 pounds, whereas the bearing value of each $\frac{7}{8}$-inch rivet against a $\frac{7}{16}$-inch plate at 24,000 pounds per square inch is 9,180 pounds. Hence the bearing value of the rivet against the web-plate, being a little less than two thirds of its double shearing value, will

necessarily govern the number of rivets needed to secure each flange to the web-plate. The pitch at the ends of the girder will therefore be $\frac{9,180 \times 90.5}{332,900} = 2.49$ inches; at the quarter-point, $\frac{9,180 \times 90.5}{203,300} = 4.12$ inches; and at the center, $\frac{9,180 \times 90.5}{84,800} = 9.8$ inches.

This spacing, as found above, is the greatest which may be employed in securing the vertical legs of the flange angles to the web-plate. As a matter of fact, the actual number of rivets used is considerably larger than indicated by the preceding computations, for the reason that the pitch of rivets (§ 56 of the specifications) "must not exceed 6 inches in the direction of the stress, nor sixteen times the thickness of the thinnest outside plate connected, and not more than forty times that thickness at right angles to the stress," in order that the different parts of the girder may be secured to each other with sufficient rigidity or stiffness. At and near the ends of the flanges the rivets must be pitched closer together than in the intermediate portions so as to give the ends of the girder sufficient solidity, and for the further reason that the flange stresses increase much more rapidly in that portion of the flange than in other portions.

When the moving load rests directly upon the upper flanges of the plate girder, as in the present case, the rivets through the vertical legs of the flange angles and the web-plate carry the driving-wheel concentrations as well as the flange stresses, the latter acting in a horizontal direction, while the former are vertical. It is uncertain in such a case over what length of flange a single driving-wheel concentration is distributed, but it is reasonable to assume that the distribution is essentially uniform over a distance not less than 2 feet. If that distance be chosen, the amount of flange stress near the ends of the girder given to the flange

by the rivets found within it, if they are pitched $2\frac{1}{2}$ inches apart, will be about $9{,}180 \times \frac{24}{2\frac{1}{2}} = 88{,}300$ pounds. The greatest driving-wheel concentration in this case is 25,000 pounds. The rivets within a length of 2 feet therefore should carry the resultant force $\sqrt{(88{,}300)^2 + (25{,}000)^2} = 91{,}600$ pounds. Hence the required number of rivets would be $91{,}600 \div 9{,}180 = 10$, nearly. It will therefore be preferable to fix the rivet spacing at $2\frac{1}{4}$ inches in the two rows or $4\frac{1}{2}$ inches in each row at the ends of the flanges. The rivet spacing at other points of the flanges, as already found, must be similarly modified, although in the central portions of the girder the computed spacing will be so large as to cause the provisions of § 56, already quoted, to govern. As an illustration Plate I may be consulted, where the spacing is seen to vary from 2 inches at the end to $3\frac{3}{4}$ inches at the center of the span

Rivets in Cover-plates.

The number of rivets required in the cover-plates must next be determined. Obviously the rivets at and near the end of a cover-plate must be sufficient to give to the latter the total stress which it is designed to carry. The general arrangement of these rivets will be as shown in Fig. 13. There are four rows of staggered rivets, the pitch in each row being 6 inches for the first 20 inches of the cover-plate and 9 inches for the remainder. With these rivet pitches it is reasonable to assume that the net sections of the cover-plate will be found by deducting the metal taken out of two rivet-holes only. The net width of the 16-inch cover-plate will therefore be 14 inches; consequently the total stress carried by each $\frac{9}{16}$-inch cover-plate will be $14 \times \frac{9}{16} \times 17{,}000 = 134{,}300$ pounds. In this detail each rivet will be subjected to

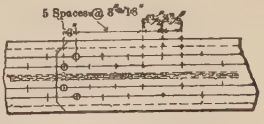


FIG. 13.

single shear and that single-shear value is 7,220 pounds, much less than the bearing value against the $\frac{5}{16}$-inch plate. The total number of rivets required for a $\frac{5}{16}$-inch cover-plate will then be $134{,}300 \div 7{,}220 = 19$ rivets. By referring to Fig. 13 it will be seen that this number is much more than afforded by the arrangement shown. It is not feasible, however, for the requisite stiffness and solidity of the flanges of the girder to make rivet pitches greater than those shown. Each $\frac{5}{16}$-inch cover-plate will obviously have the same arrangement of rivets at and near the ends as well as throughout other portions of their lengths.

Compression Flanges or Chords.

The design of the tension flange being completed, including all its details, except such splice-joints as may be needed, the compression flange might be designed in precisely the same manner. Gross sections would then be considered instead of net sections, since all rivet-holes are filled with the driven rivets. The unit working intensity of compressive stress would be somewhat less than the tensile intensity already used. Hence it is customary, as required by the present specifications, although not the invariable practice, first to design the tension flange completely and then make the gross section of the compression flange precisely the same. Indeed not only the gross section of the compression flange is frequently made identical with the tension flange, but the dimensions also of its various component parts. The only exception from this procedure arises from the fact that it is advisable to carry the cover-plate immediately adjacent to the flange angles in the compression flanges throughout its entire length. In this manner the angles near the ends of the girder are bound more solidly together and the flange as a whole in that vicinity is in better condition to resist compression. These observations acquire added significance in those cases where the ties of

the railroad track rest directly on the upper flange of the girder. The inevitable deflection of the ties tends to bend downward the horizontal leg of one flange angle in each girder, and this tendency is effectively resisted if those angles carry a cover-plate throughout their entire length. It is better practice, therefore, to make the first cover-plate extend the whole length of the compression chord, although this is not done in this case.

End-stiffeners.

The end-stiffeners shown in Plate I immediately over the pedestal at the end of the effective span must be considered as taking the reaction of the girder and its load. The lower ends of the end-stiffeners resting closely against the lower flange angles sustain the entire reaction. That reaction, however, is gradually given to the web of the girder in passing to the upper ends of the stiffeners through the rivets securing the latter to the web-plate. The number of rivets required, therefore, in an end-stiffener must be sufficient to convey the entire reaction to the web-plate in the manner indicated. It has already been shown that the maximum reaction at the end of the girder is 332,900 pounds. It is clear from what has been stated that the cross-section of the end stiffener must be sufficient to sustain the compression produced by the reaction somewhat as in a short column, but obviously with little or no tendency to flexure. It is reasonable, therefore, to assume under the circumstances outlined that compression in the end-stiffeners may reach 17,000 pounds per square inch as the greatest permissible working stress. The sectional area required for the stiffeners will therefore be

$$332{,}900 \div 17{,}000 = 19.6 \text{ square inches.}$$

Four angles $5 \times 3\frac{1}{2} \times \frac{5}{8}$ inches will give an aggregate area of 19.7 square inches, which happens to be essentially equal

to the area required. Hence the end-stiffener on each side of the web-plate will be taken as composed of a pair of those steel angles the 5-inch legs of which must be placed against a $10 \times \frac{3}{4}$-inch filler. These fillers on the two sides of the web-plate are required so that the end-stiffeners may be used without being bent around the vertical legs of the flange angles.

The rivets piercing the end-stiffeners will evidently be in double shear on the two sides of the web-plate, each rivet having the capacity of 14,440 pounds. The bearing value of each $\frac{7}{8}$-inch rivet on the $\frac{7}{16}$-inch web is 9,180 pounds. The number of rivets required, therefore, must be determined by dividing 332,900 by 9,180, and that division shows 37 rivets as the number required. This would give less than 19 rivets to each pair of stiffening angles, far too few to make a satisfactory joint between the end-stiffeners and the web-plate. The end-stiffeners take the immediate effect of all the hammering at the end of the span produced by heavy and rapidly moving loads, and they should be held to the plate by a riveted joint especially rigid and strong. The pitch in each line of rivets in each pair of angles will therefore be made 3 inches. As the depth of girder is 90 inches, this arrangement will call for a total of about 60 rivets, and it is none too many for the purpose. Fig. 12 shows the number of end-stiffeners and rivets and their arrangement for an 81-foot girder. The ends of the end-stiffeners, and indeed all of the stiffeners, should be machine-finished or faced to fit exactly the fillets of the flange angles in the chords or flanges.

Intermediate Stiffeners.

The dimensions and the location of the intermediate stiffeners Plate I are chiefly matters of judgment. It may be shown * in considering the internal stresses of a beam

* Burr's "Resistance of Materials," p. 141.

or girder that in directions at right angles to each other, and at 45 degrees with the neutral axis, two equal stresses of tension or compression exist, and hence that those compressive stresses tend to buckle the web of a plate girder in directions at 45 degrees to the neutral axis. It may also be shown that the common intensity of the two stresses is the same as that of either the transverse or longitudinal shear at the same point, which is also known to be $\frac{3}{2}$ the mean for the whole section in the case of a solid rectangular beam. Since the intensity of shear at the neutral surface of such a beam is a maximum and is zero at the top and bottom surfaces, it being assumed that the web takes shear only, it follows that if the shear be assumed uniformly distributed throughout any transverse section of the web the latter may be supposed to be composed of an indefinitely great number of columns, each of which is an indefinitely thin strip of the web, making an angle of 45 degrees with the axis of the beam. In a direction normal to these columns an equal intensity of tension will, of course, exist.

In order to prevent most effectively this tendency to buckle the web-plate, the intermediate stiffeners should be placed on that plate also at 45 degrees with the neutral axis. That procedure, however, involves some inconvenience in construction, and ample stiffening can be secured by placing the intermediate stiffeners at right angles to the neutral axis or, in the ordinary position of girders, in a vertical direction. Hence that arrangement is always followed in plate-girder construction. It has been usual to place these intermediate stiffeners at distances apart of once to once and a half the depth of the girder. Inasmuch as the transverse shear increases towards the ends of the girder, and is relatively small at the center of the span, the stiffeners might be placed farther apart as the center of span is approached, and that is sometimes done. In any case the specifications under which the design is being made must

be followed. It is required in § 40 that "the web shall have stiffeners riveted on both sides, with a close bearing against upper and lower flange angles at the ends and inner edges of bearing-plates, and at all points of local and concentrated loads, and also when the thickness of the web is less than $\frac{1}{60}$ of the unsupported distance between flange angles, at points throughout the length of the girder, generally not farther apart than the depth of the full web plate, with a maximum limit of 5 feet." Since the thickness of the web-plate is less than $\frac{1}{60}$ of the unsupported distance between flange angles, the intermediate stiffeners must be placed at intervals of 5 feet, as shown in Plate I. There will be 8 pairs of intermediate stiffeners in each half of the girder. The cross-section of the intermediate stiffener angles is a matter of judgment, and in this case they will be taken as $5 \times 3\frac{1}{2} \times \frac{3}{8}$-inch angles, each with one line of rivets spaced $4\frac{1}{2}$ inches apart.

No fillers will be used, making it necessary to crimp the stiffening angles to fit the flanges and web, although obviously filling-plates under stiffening angles add materially to the effectiveness of the stiffeners.

Filling-plates will, however, be required at the connection of the cross (X) frames to the main girders.

As illustrating a different method of designing the intermediate stiffeners, Theodore Cooper, in his Specifications for Steel Railroad Bridges, 1901, specifies (§ 44) that "the webs of plate girders must be stiffened at intervals not exceeding the depth of the girders, or a maximum of 5 feet, wherever the shearing strain per square inch exceeds the stress allowed by the following formula:

$$\text{Allowed shearing stress} = 10{,}000 - 75H,$$

where H = ratio of depth of web to its thickness; but no web-plate shall be less than $\frac{3}{8}$ inch thick;" and (§ 45) that "all

stiffeners must be capable of carrying the maximum vertical shear without exceeding the allowed unit stress,

$$P = 10{,}000 - 45\frac{l}{r}.$$

Each stiffener must connect to the webs by enough rivets to transfer the maximum shear to or from the webs."

Web-plate Joint.

The chief function of the splicing of the web-plate is to resist shear, although in this example the splice must also be designed to resist flexure, as one eighth of the web section is taken as flange area. In no case, however, should a splice have less than two rows of rivets on each side of the joint, and that arrangement will be used in this case, although in a large girder like that under consideration there might be three. The splice will naturally be a butt-joint with two covers. The thickness of each cover should be substantially more than one half the thickness of the web-plate. Since no plate less than $\frac{3}{8}$ inch thick may be used, and as these plates also serve as fillers for the stiffening angles placed at the same points, each splice-plate will be made $14 \times \frac{7}{16} \times 60$ inches deep. These splice-plates do not extend the full depth of the web-plate, but near the flanges longitudinal plates $9 \times \frac{7}{16} \times 28$ inches long will be placed, to aid in transmitting the longitudinal stress of flexure past the splice. The maximum shear at the quarter-point has already been found to be 203,300 pounds, and as the bearing capacity of one $\frac{7}{8}$-inch rivet in a $\frac{7}{16}$-inch plate is 9180 pounds, 22 rivets are required simply to transmit the shear.

The two rows of rivets on each side of the joint in the 60-inch plate, with rivets staggered 3 inches apart, provide for 40 rivets in the manner shown in Fig. 14.

The rivets in the longitudinal splice-plates, whose duty

is to transmit the flexural stresses of tension and compression assumed to be carried by the web, may be approximately found as follows:

Let k = the allowed intensity in pounds per square inch of the stress in the flange (17,000 pounds);
a = one eighth area of the web section in square inches (4.9 square inches);
S = the stress transmitted by the splice-plates in pounds.

Then $S = ka = 83{,}200$ pounds approximately,

and since the bearing capacity of one rivet is 9,180 pounds, 10 rivets will be required. These will be placed in three

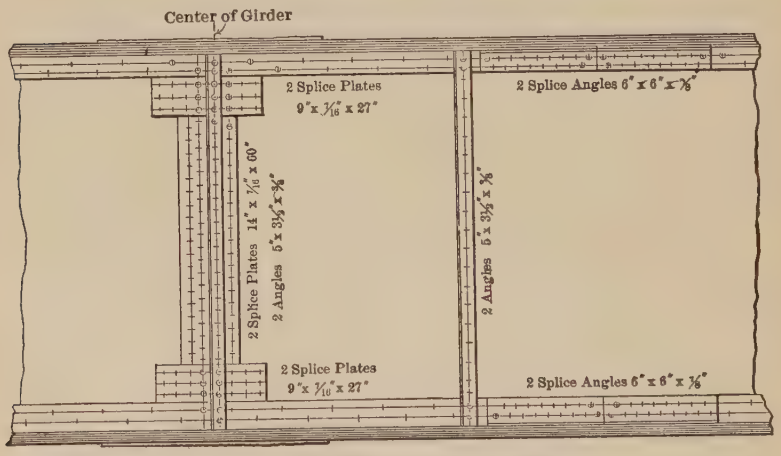


FIG. 14.

horizontal rows in each plate, furnishing 12 rivets on each side of the splice.

The entire arrangement of the web-joint with the rivets distributed as indicated is shown in Fig. 14. The web-plate splice at the center of the girder will be reinforced by

an additional cover-plate 5 feet long on both upper and lower flanges.

Joints in Flanges or Chords.

Although it is usually practicable to secure flange angles 100 feet long, as an illustration a splice will be made at some intermediate point. This point might be taken at the center of span, offering the advantage of making all the angle bars of one length, but there would result the disadvantage of cutting the angles at the point of maximum required flange area. The splice-joint will therefore be placed $7\frac{1}{2}$ feet from the center, the greatest lengths of angles required being then $52\frac{1}{2}$ and $37\frac{1}{2}$ feet. As the longest cover-plate required, 73 feet in length, can usually be secured, no splice will be made. When such a splice does occur, it is placed where the end of the next cover can act as the splice-plate. Material economy in the splice-joint is thus secured. This is illustrated in Fig. 12, where the first cover-plate on the lower flange is cut 15 feet 6 inches from the left end, and the splice is made by continuing one of the other cover-plates a few feet beyond that point.

If splices are made both in the angles and cover-plates they should be placed at different points.

The net section of the two flange angles has been found to be 13.9 square inches. At 17,000 pounds per square inch, the total stress carried by them will be 236,000 pounds. The rivets used at the splice will be in double shear with a capacity of 14,440 pounds each. The bearing capacity of a $\frac{7}{8}$-inch rivet in a $\frac{3}{4}$-inch angle leg will be 15,760 pounds. The shearing value will therefore control the number of rivets required, that number being $236{,}000 \div 14{,}440 = 17$. The splicing of each $\frac{3}{4}$-inch angle will be done by a single piece of $6 \times 6 \times \frac{7}{8}$-inch angle with its corner fitted to the fillet of the flange angle, thus giving a net section a little

larger than that of the flange angle itself. The arrangement of the rivets will be as shown in Fig. 14, in which it is seen that there are 8 and 9 rivets respectively in each staggered row, making 17 on either side of the joint.

It is proper that a joint should have a little greater section of metal than the members spliced and show a little calculated excess of resistance.

It will be supposed that the corresponding splice in the compression flange is made precisely as in the tension flange, although the ends of the flange angles and spliced cover-plates, being machine-finished, as they should be, may abut. The process of riveting may draw the abutting ends apart to a slight degree and thus render it advisable to make the resisting capacity of the rivets of the joint the same as in the tension flange.

General Considerations.

The girder proper, with its flanges, web, and stiffeners, has been designed without indicating the manner of connecting such lateral or cross bracing as would be required in the complete design of a railroad plate-girder span, although that entire bracing is indicated in the stress sheet.

The design of such bracing would be supplementary to the actual design of the girder as made, and it is the purpose here to illustrate only those principles belonging to the design of the girder proper. The design of the necessary cross-bracing and its details of connection with the girder follow easily after the treatment of wind-bracing as given in a later chapter.

It is to be remembered that large plate girders are not always built complete in the shop, although girders 100 feet in length are frequently and perhaps usually so completed at the present time (1905). When it is necessary to build them in parts and rivet those parts together in the field, the general principles governing the construction of the neces-

sary field joints are precisely the same as those illustrated in this article. They are simply adjusted or adapted to the exigencies of each particular case.

A closely approximated bill of material and estimated weight of a single girder and one half the bracing, as designed, is as follows, the weights of small gusset-plates, etc., being omitted.

			Pounds.
Four $6\times6\times\frac{3}{4}$″ angles, 28.7 lbs. per lin. ft., 90 ft. long	4×	28.7×90	=10,350
Two cover-plates, $16\times\frac{9}{16}$″, 73 ft. long	2×	30.6×73	= 4,470
" " $16\times\frac{9}{16}$″, 64 " "	2×	30.6×64	= 3,920
" " $16\times\frac{9}{16}$″, 54 " "	2×	30.6×54	= 3,300
" " $16\times\frac{9}{16}$″, 41 " "	2×	30.6×41	= 2,520
" " $16\times\frac{9}{16}$″, 5 " "	2×	30.6× 5	= 300
Web-plate, $90\times\frac{7}{16}$″, 90 ft. long	1×	133.9×90	=12,080
Eight $5\times3\frac{1}{2}\times\frac{5}{8}$″ angles (end-stiffeners) $7\frac{1}{2}$ ft. long	8×	16.8× 7.5	= 1,000
Four end-fillers, $10\times\frac{?}{?}$″, $7\frac{1}{2}$ ft. long	4×	25.5× 7.5	= 760
Thirty-four $5\times3\frac{1}{2}\times\frac{3}{8}$″ angles (interm. stiffeners) $7\frac{1}{2}$ ft. long	34×	10.4× 7.5	= 2,650
Six splice-plates, $14\times\frac{7}{16}$″, 5 ft. long	6×	20.8× 5	= 620
Twelve splice-plates, $9\times\frac{7}{16}$″, 2 ft. long	12×	13.4× 2	= 320
Four $6\times6\times\frac{7}{8}$″ angles (splices) 5 ft. long	4×	33.1× 5	= 660
Approximately 3500 rivets at 22.6 lbs. per 100 heads	35×	22.6× 2	= 1,580
Seven $4\times4\times\frac{3}{8}$″ angles (horizontal bracing) 8 ft. long	7×	9.8× 8	= 550
Two $5\times3\frac{1}{2}\times\frac{7}{16}$″ " " " 8 " "	2×	12× 8	= 190
Two $5\times3\frac{1}{2}\times\frac{1}{2}$″ " " " 8 " "	2×	13.6× 8	= 220
Two $5\times3\frac{1}{2}\times\frac{9}{16}$″ " " " 8 " "	2×	15.2× 8	= 250
Four $4\times4\times\frac{5}{16}$″ " (interm. frames) 7 ft. long	4×	8.2× 7	= 230
Four $4\times4\times\frac{5}{16}$″ " " " 9 " "	4×	8.2× 9	= 290
Two $4\times4\times\frac{3}{8}$″ " (end frames) 7 ft. long	2×	9.8× 7	= 160
Two $4\times4\times\frac{3}{8}$″ " " " 9 " "	2×	9.8× 9	= 180
Total			=46,580

The weight of bridge per linear foot, the above being for one girder, will therefore be $46,580 \div 45 = 1030$ pounds, which checks the value assumed.

For making the estimate of cost, the weight of shoes, rollers, etc., must also be included in the final weight.

End Fastenings.

There still remain to be designed the end bearing-plates and end rollers. Provision for expansion due to temperature changes require the latter. They are placed below a suitably constructed steel casting, as shown in Fig. 12, so that the weight brought upon them is uniformly distributed. The following specifications apply:

"§ 53. All bridges exceeding 80 feet in length shall have hinge bolsters on both ends, and at one end nests of turned friction-rollers running between planed surfaces. Rollers will not be less than 4 inches in diameter, and the pressure per linear inch of roller, including impact, shall not exceed $1{,}200\sqrt{d}$ for steel rollers between steel surfaces (d = diameter of roller in inches).

"§ 55. Bedplates shall be so proportioned that the pressure upon masonry (including impact) will not exceed 400 pounds per square inch."

The maximum end reaction has already been found to be 332,900 pounds. At 400 pounds per square inch, 832 square inches of bearing surface on the masonry will be required, and this will be furnished by a plate $\frac{7}{8}$ inch thick, 36×36 inches square, with a superficial area of 1,296 square inches. The intensity allowed on a 4-inch roller is, according to § 53, 2,400 pounds per linear inch. The pedestal requires, therefore, $332{,}900 \div 2{,}400 = 139$ linear inches of roller, and this will be furnished by eight rollers, each 18 inches long.

If a trough flooring composed of Z bars and plates or angles and plates, or of other shapes, be employed, that flooring must be designed by obtaining the maximum moment to which any section of it may be subjected and equating it to the expression

$$\frac{kI}{d},$$

where k is the allowable intensity of stress in the extreme fiber, I the moment of inertia of the section in inches, and d the distance from the neutral axis to the extreme fiber, all in inches. The quantity $\frac{I}{d}$ is usually termed the section odulus.

Art. 64.—Economic Depth of a Plate Girder.

That depth of plate girder which will give the least weight depends upon the manner and amount of the loading. With heavy concentrated loads, it is relatively large; on the other hand, with light loads it may be less than one twentieth of the span. As all bending moments may be supposed to be caused by some uniform load, either fictitious or real, the following analytical discussion may be of some value:

Economic Depth of Plate Girders with Uniform Flanges.

If a plate girder carries a uniform load, and is designed with flanges of uniform cross-sectional area, the depth which will give the least weight of girder mav easily be obtained.

Let l = span in feet;

d = depth in feet;
t = web thickness in inches,
p = allowed working stress in pounds per square inch for the flanges;
p' = allowed working stress in pounds per square inch for the end stiffeners;
a = sectional area of each intermediate stiffener in *square feet*;
n = number of stiffeners (intermediate);
w = total load per linear foot of girder (in pounds).

The flange stress at center will then be

$$F = \frac{wl^2}{8d}.$$

The volume of the web plate in cubic feet will be

$$\frac{ldt}{12}.$$

If one-sixth of the latter is taken to be concentrated in the flange, the volume of the two flanges in cubic feet will be

$$\frac{2Fl}{144p} - 2\frac{1}{6}\frac{ldt}{12} = \frac{wl^3}{576pd} - \frac{ldt}{36}.$$

The volume of the end stiffeners will be

$$\frac{wld}{144p'},$$

and that of the intermediate stiffeners

$$nad.$$

The volume of the entire girder will then take the value

$$V = \frac{wl^3}{576pd} + \frac{ldt}{18} + \frac{wld}{144p'} + nad. \quad . \quad . \quad . \quad . \quad (5)$$

By taking the first derivative,

$$\frac{dV}{d(d)} = -\frac{wl^3}{576pd^2} + \frac{lt}{18} + \frac{wl}{144p'} + na = 0.$$

Solving for d^2,

$$d^2 = \frac{wl^3}{576p\left(\frac{lt}{18} + \frac{wl}{144p'} + na\right)},$$

$$d = \frac{l}{2}\sqrt{\frac{wl}{p\left(8lt + \frac{wl}{p'} + 144na\right)}}. \quad . \quad . \quad . \quad . \quad (6)$$

If one sixth of the web is not concentrated in each flange, $12lt$ will take the place of $8lt$ in eq. (6).

If all stiffeners, both end and intermediate, are omitted, eq. (6) will take the form

$$d = \frac{l}{4}\sqrt{\frac{w}{2pt}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In reality p is seldom or never exactly the same for both flanges, since it is the working stress in reference to the *gross section*. It will be sufficiently near, however, for all usual purposes to make it a mean of the two actual working stresses for the gross sections.

It should be borne in mind that local circumstances frequently compel a different depth from that given by eq. (6). It will also be found that a considerable variation from that depth will cause a comparatively small variation in weight. Again, the difficulties of handling a deep girder, and the shop cost *per pound*, may, and usually does, make the economic depth a little less than that found by the aid of eq. (6).

CHAPTER IV.

TRUSSES WITH PARALLEL AND HORIZONTAL CHORDS.

Art. 65. Reactions and General Equations for Stresses in Simple Trusses.

Two distinct operations are required to determine the stresses in a structure which rests simply on two points of support and which carries loading.

First, there must be found the external forces or reactions acting at the points of support; and secondly, the internal forces or stresses in the members of the structure.

The three equations of condition of Art. 34, Chapter II, must therefore first be applied to the external forces without reference to the members of the structure, and secondly, they must be applied to those external and internal forces which balance each other.

The method of determining the reactions is precisely that already illustrated in the case of beams. Let Fig. 1 repre-

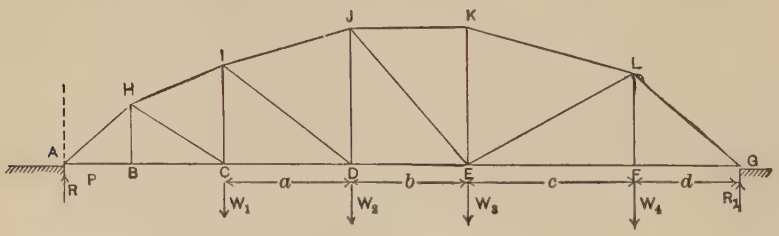


FIG. 1.

sent a truss of span l supporting the loads W_1, W_2, W_3, etc., at the panel-points shown. In order to determine the

reaction R, moments are taken about the right abutment G; there will then be obtained

$$Rl - W_1(a+b+c+d) - W_2(b+c+d) \ldots = 0.$$

Therefore

$$R = \frac{W_1(a+b+c+d) + W_2(b+c+d) + \ldots}{l}$$

The value of R_1 is then obtained by means of the equation of condition $\Sigma V = 0$, for

$$R + R_1 - W_1 - W_2 - W_3 \ldots = 0;$$

therefore

$$R_1 = W_1 + W_2 + W_3 + \ldots - R.$$

Both reactions are thus determined.

These reactions are vertical in direction, i.e., parallel to the loads, but these conditions do not always prevail and their modifications will be treated later.

The internal forces or stresses in a truss are next to be determined, making use of either of the two following methods of treatment. The stability of each joint of the structure with the members centering there may be investigated, or sections may be taken through the truss and the stresses in the members cut by these sections investigated. In the first of these methods only those panel-points may be chosen at which there are but two unknown forces; for in the case of concurrent coplanar forces the equations of condition for equilibrium reduce to two. In Fig. 1, for example, there are only two unknown forces at the abutments. Taking point A, therefore, at which the stresses in AB and AH are unknown, and assuming that directions upward and to the right are positive, it will be found (Fig. 2)

for $\Sigma H = 0$ that $AH \sin \alpha + AB = 0$, . . (3)

for $\Sigma V = 0$ that $AH \cos \alpha + R = 0$. . . (4)

Since R is a vertical force, it does not appear in eq. (3), and since AB is a horizontal force, it does not appear in eq. (4). By solution of eqs. (3) and (4),

$$AH = -R \sec \alpha, \quad \ldots \ldots \ldots \quad (5)$$
$$AB = -AH \sin \alpha = +R \tan \alpha. \quad \ldots \quad (6)$$

The minus sign in eq. (5) indicates that the direction of action of the force AH is toward A; consequently it is a compressive stress. Since AB is positive, the stress in that member is tension.

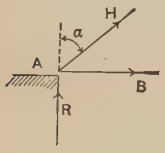


FIG. 2.

After having determined the stresses at A, panel-point B is next to be treated, for at that point only the stresses in HB and BC are unknown. BC, however, is identical with AB, and the stress in HB is simply the panel load supported at B. At the panel-point H, next in order, the two unknown forces are now HC and HI. They can therefore at once be found by two equations similar to eqs. (5) and (6). In the same manner there will be but two unknown forces to be found at each of the points C, I, D, J, etc., taken in that order.

The other and more generally used method for determining the stresses in a structure is termed the method of moments, and its application may also be illustrated by Fig. 1.

Pass a section, not necessarily plane, completely cutting the structure; then the forces or stresses acting in the members cut must balance all the external forces situated on one or the other side of the section, for those stresses and the external forces on one side of the section constitute a system in equilibrium.

By the proper choice of sections, it will generally be found possible to determine the unknown stresses by means of the single equation of condition $\Sigma M_z = 0$; for, if the section be passed through such members that the

moments of their stresses all disappear except one, the equation of condition will contain but one unknown quantity.

In general, therefore, if the section is made to pass through three members, the center of moments must be chosen at the intersection of two of these, since, in that case, the lever-arms of these two forces become equal to zero and their moments disappear. The stress in the third member is then easily found.

For instance, pass a section through IJ, ID, and CD, Fig. 3, which then illustrates the forces acting in the lines

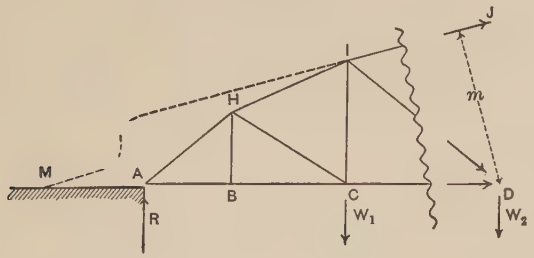


FIG. 3.

of the members cut. In order to determine the stress in IJ, the center of moments is taken at D, the intersection of ID and CD. Assuming that rotation in the direction of the hands of a clock is positive, the equation of moments may be written as follows:

$$IJ \cdot m + R \cdot AD - W_1 \cdot CD = 0.$$

$$\therefore IJ = \frac{-R \cdot AD + W_1 \cdot CD}{m}.$$

If the value of IJ is found to be negative, the stress in it will be compression, or tension if the sign is positive.

The stress in the member CD may then be found in a similar manner by taking moments about the point I, the

intersection of IJ and ID. The stress in the member ID may be found by taking moments about the point M, which is the intersection of IJ and CD.

In a precisely similar way other sections may be passed through the truss and the stresses in all the members successively found.

It is seen that this method offers an advantage over the first method employed, since the stress in any member does not depend upon any previously determined stress, and errors are not likely to be cumulative. This method is consequently valuable as a check on graphical work in which the stresses are found by a continuous operation. If, therefore, the last stress found graphically is checked by this analytic method of sections, it may be assumed that all previously determined stresses found graphically are correct.

Art. 66. Applications to Trusses with Parallel and Horizontal Chords. Web Members.

Let Fig. 4 represent a truss having parallel and horizontal chords and carrying loads W_1, W_2, W_3, etc., at the

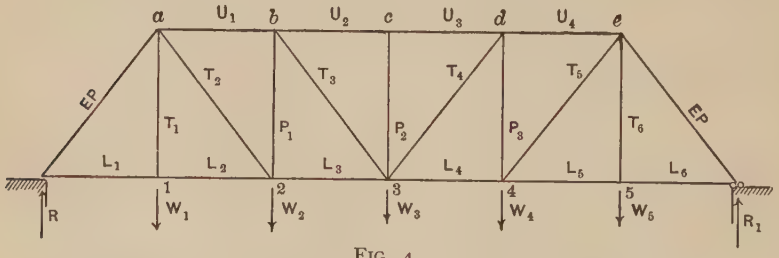


FIG. 4.

various panel-points as shown. The reactions are indicated by R and R_1. Applying the equation of condition $\Sigma V = 0$ to any section as shown in Fig. 5, if θ is the angle

between the web member T_3 and a vertical, there is found

$$R - W_1 - W_2 - T_3 \cos \theta = 0$$
$$T_3 = (R - W_1 - W_2) \sec \theta. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The quantity $R - W_1 - W_2$ is the shear in the truss at the section cut, the shear being the algebraic sum of all the

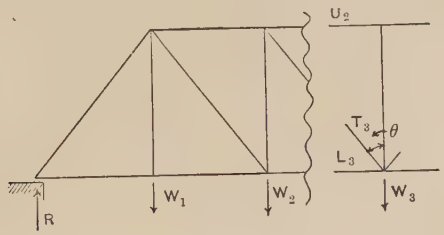


FIG. 5.

external vertical forces between the end of the truss and the section considered. If S, therefore, represents the shear at the section, eq. (7) will take the following form:

$$T = S \cdot \sec \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This equation has been deduced in the most general manner and is applicable to all web members whether the load rests on the upper or lower chord or on both. The equation may be expressed in the following rule:

The stress in any web member of a truss with parallel chords and loaded vertically is equal to the shear in the panel of which the web member is a part, multiplied by the secant of the angle which the member makes with a vertical line.

In order to determine whether the stress in a web member is tensile or compressive, it is only necessary to know in which direction the shear tends to slide the two parts of the truss cut by the section. In Fig. 5, if R be taken greater in amount than $W_1 + W_2$, the two parts tend to move away from one another; that is, the right side down and the left

side up. This condition, it is evident, tends to pull apart the member T_3; it is, therefore, in tension. The section for the member P_1 is that shown in Fig. 6. In this case the shear tends to force together the two parts of the truss, evidently causing a compression in the member P_1.

Art. 67. Applications to Trusses with Parallel and Horizontal Chords. Chord Members.

A similar rule may be formulated for determining the stresses in the chord members of trusses having parallel

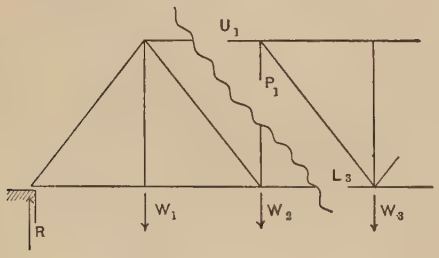


FIG. 6.

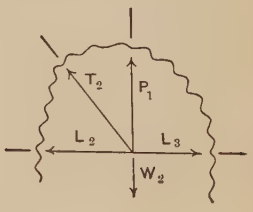


FIG. 7.

and horizontal chords. By applying the equation of condition $\Sigma H = 0$ to panel-point 2 of the truss of Fig. 4, as indicated in Fig. 7, the following equation will result:

$$-L_2 + L_3 - T_2 \cdot \sin\theta = 0.$$

But $T_2 = \text{shear} \cdot \sec\theta$.

$$\therefore\ L_3 = L_2 + (\text{shear in } T_2)\tan\theta. \quad . \quad . \quad . \quad (9)$$

Hence the stress in any chord member is equal to the stress in the preceding chord member plus an increment equal to the algebraic sum of the shears in the intersecting inclined web members multiplied by the tangents of their inclinations to a vertical line.

Beginning with the end chord member, therefore, the stress in each successive member, as the center is approached, is easily found by adding the proper increment to the preceding stress.

This method is particularly adapted to finding dead-load stresses, the loading being constant in amount per panel and fixed in position. The method of moments is, however, more convenient for moving-load stresses.

Attention should be drawn to the fact that the stress in the member L_3 is the same as that in the member U_1, but of opposite sign. This is easily shown by passing a section like that indicated in Fig. 8, and applying the equation of condition $\Sigma H = 0$. Since the only horizontal forces acting are the forces in the chord members, it follows that the stress in the one must balance that in the other; they must, therefore, be equal in amount and of opposite sign. This holds true for other pairs of panels similarly located in trusses with parallel chords.

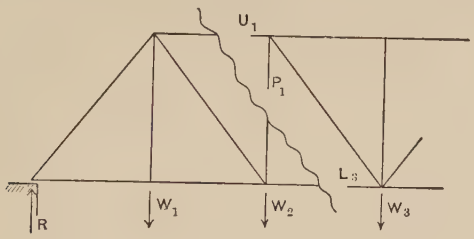


FIG. 8.

The preceding methods will be illustrated by an example.

Art. 68. Dead-load Stresses in a Six-panel Pratt Truss.

The principles set forth in the preceding articles may be illustrated by the following computations.

The truss whose stresses will be determined is shown in Fig. 9. It is a single-track through railroad bridge, con-

taining six panels, each having a length of 25 feet. The distance between centers of end pins is 150 feet, and the height of the truss is 28 feet.

The dead load of the track has been assumed at 400 pounds per linear foot, and that of the floor at 520 pounds

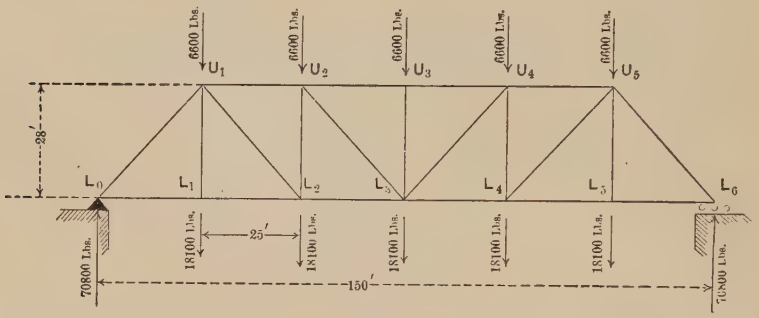


FIG. 9.

per linear foot. The weight of the track and floor is carried entirely at the lower panel-points, and a panel load per truss is $\frac{920 \times 25}{2} = 11{,}500$ pounds. The weight of the trusses and included bracing may be empirically determined by the means of the formula

$$w = al,$$

where w represents the weight per linear foot of trusses and bracing; a, a constant, assumed to be 7; and l, the span in feet; w therefore, in this case, becomes 1,050 pounds. It is known, however, that the actual weight of the trusses and bracing as constructed was 1,060 pounds per foot, and that amount will be taken. This weight is equally distributed between the upper- and lower-chord panel-points and is 6,600 pounds per panel-point per truss. The lower panel load is, therefore, $6{,}600 + 11{,}500 = 18{,}100$ pounds and the upper 6,600 pounds.

Fig. 9 illustrates the method generally used for lettering the panel-points of a truss and for naming the members. The letter L, with the proper subscripts, is used for denoting the lower panel-points, and the letter U those in the upper chord. The numbers are continuous from the left end of the truss to the center; the right half of the truss is then lettered symmetrically about the center line. In some cases it will be found necessary to letter the truss continuously from one end to the other.

For the loading given, the reactions at the ends are alike and equal to $\frac{1}{2}[(6\times 18{,}100)+(5\times 6{,}600)]=70{,}800$ pounds. It should be noted that the load at the end of the truss, equal to half the regular panel load, causes no stress in the structure, as it rests directly on the abutment. It must be used, however, in designing the end bearings, since it is a part of the load which they carry. The character of the end bearings will depend to some extent on the presence or absence of an end floor-beam. If an end floor-beam is used, it is ordinarily supported on the pedestals at the feet of the end posts. If there is no end floor-beam, the ends of the end stringers rest on the abutment masonry. The live-load reactions under the trusses will be correspondingly affected.

Stresses in Web Members.

The shear in the panel L_1L_2 is equal to $61{,}750-24{,}700=37{,}050$ pounds. The secant of the inclination of U_1L_2 to a vertical is 1.34; therefore the stress in U_1L_2 is equal to $37{,}050\times 1.34=50{,}000$ pounds.

Stresses are usually not determined more exactly than to the nearest thousand pounds of stress, since it is hardly possible to design a member with any greater degree of accuracy; slide-rule work will generally suffice.

In exactly the same way the shear for U_2L_3 is found to

be 12,350 pounds, and the stress in that member is $12,350 \times 1.34 = 17,000$ pounds. The character of the stress is at once determined by noting in which direction the shear tends to force the two portions of the truss cut by the section. In both cases it will be seen that the members tend to be lengthened, and they are therefore in tension.

The shear for L_0U_1, more commonly known as the end post, is equal to 61,750 pounds; the stress is, therefore, $61,750 \times 1.34 = 82,000$ pounds. It is evident that the stress for this member is compressive.

The secant of inclination for the vertical members is 1. Therefore the numerical values of the stresses are equal to the shears. The shear for U_2L_2 is $61,750 - 42,800 = 19,000$ pounds. This shear is obtained by passing a section through U_1U_2, U_2L_2, and L_2L_3; the loads subtracted from the reaction are then those placed at L_1, L_2, and U_1.

The tensile stress in the member U_1L_1 is simply the load hanging at its foot, L_1. It is clear that this member may be removed from the truss without in any way affecting the stability of the structure, but it would then be necessary to make single stringers of double length for the end panels. In a similar manner the stress in the member U_3L_3 is 6,600 pounds compression, for its function is simply to carry the load applied at U_3.

Stresses in Chord Members.

The stresses in the chord members will first be found by the increment method and then some of the values so found will be checked by the method of moments. By the former method, the stress in L_0L_1 is equal to the shear in the member L_0U_1 multiplied by the tangent of the angle which L_0U_1 makes with the vertical, and that tangent is 0.893. Since the shear is 61,750 pounds, the stress in L_0L_1 is $61,750 \times 0.893 = 55,000$ pounds, and it is tension.

It will not be necessary to consider the signs of any of the chord stresses of simply supported trusses, as the lower-chord members are always in tension, and the upper-chord members, including the end post, are always in compression.

The stress in L_1L_2 is the same as that in L_0L_1, for no horizontal force is applied at the point L_1. The stress in L_2L_3 is equal to the stress in L_1L_2 plus the shears in U_1L_2 and U_2L_2 multiplied by their respective tangents, i.e., $55{,}000 + (37{,}050 \times 0.893) = 88{,}000$ pounds tension.

By passing a section through U_1U_2, U_2L_2, and L_2L_3 it is seen that the stress in U_1U_2 is equal but opposite in kind to that in L_2L_3, or 88,000 pounds compression. The stress in U_2U_3 is equal to that in U_1U_2 plus the shears in U_2L_2 and U_2L_3 times their respective tangents; that is, equal to $88{,}000 + (12{,}350 \times 0.893) = 100{,}000$ pounds.

Checking the stress in U_2U_3 by the method of sections, it will be necessary to pass a section through U_2U_3, U_2L_3, and L_2L_3. Taking the center of moments at L_3, with clockwise rotation positive, the equation of moments becomes

$$(61{,}750 \times 75) - 24{,}700[(2 \times 25) + (1 \times 25)] - U_2U_3 \times 28 = 0.$$
$$\therefore\ U_2U_3 = 100{,}000 \text{ pounds compression.}$$

In the same way, the center of moments for U_1U_2 is at L_2, and

$$[61{,}750 \times (2 \times 25)] - 24{,}700 \times (1 \times 25) - U_1U_2 \times 28 = 0.$$
$$\therefore\ U_1U_2 = 88{,}000 \text{ pounds compression.}$$

In the case of the end post, the center of moments is at L_1. The lever-arm of L_0U_1 is 26.4 feet; therefore

$$61{,}750 \times 25 - U_1L_0 \times 26.4 = 0.$$
$$\therefore\ U_1L_0 = 82{,}000 \text{ pounds compression.}$$

Art. 69. Maximum Floor-beam Reaction.

It has previously been shown that the weight of the moving wheel loads is carried to each transverse floor-beam by the adjacent stringers. Hence each floor-beam is a pier for two adjacent spans of stringers, and it becomes necessary to determine that position of the moving load on those two spans which will subject the floor-beam to its greatest load.

In Fig. 10 let a section of the beam be shown at R, while l and l_1 are the two adjacent stringer spans traversed

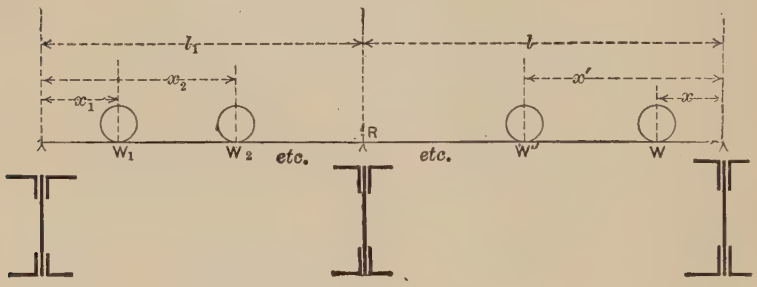


FIG. 10.

by the moving load; then let the x's be measured from the right and left ends of l and l_1, while W, W', etc., W_1, W_2, etc., represent the weights or wheel concentrations resting on the two spans. The reaction R will then have the value

$$R = \frac{W_1 x_1 + W_2 x_2 + \text{etc.}}{l_1} + \frac{W x + W' x' + \text{etc.}}{l}. \quad . \quad (1)$$

If the whole system of loading move to the left by the distance Δx, the new reaction will be

$$R' = R - \frac{(W_1 + W_2 + \text{etc.})\,\Delta x}{l_1} + \frac{(W + W' + \text{etc.})\,\Delta x}{l}.$$

In that position which gives a maximum or minimum, $R_1 - R = 0$; hence

$$(W_1 + W_2 + W_3 + \text{etc.})\frac{l}{l_1} = (W + W' + W'' + \text{etc.}). \quad (2)$$

It will seldom happen that eq. (2) will be satisfied unless a concentration rest on the point R, so that the proper portion of it may be taken for one span or the other, precisely as in the problems of maximum shear and maximum moments in the case of beams.

Ordinarily the two adjacent spans are equal, or $l = l_1$.

$$\therefore\ W_1 + W_2 + \text{etc.} = W + W' + \text{etc.} \quad . \quad . \quad . \quad (3)$$

Eq. (3) shows that when the two spans are equal, the amounts of load of each side of R must also be equal.

After the proper position of loading has been determined, eq. (1) will give the maximum reaction desired.

Art. 70. Position of Moving Load Causing Maximum Stresses in Web Members of Trusses with Parallel and Horizontal Chords.

In determining the moving-load stresses in any member of a bridge-truss, the load must evidently be so placed as to make those stresses the greatest possible. Stresses of one kind only can exist in either chord, i.e., tension or compression, but web members may be subjected to reversals of stress and the amounts of such reversals must be determined.

In trusses with parallel and horizontal chords, the web stresses vary directly as the shears. It will only be necessary, therefore, to determine the maximum shears for the various members, and the stresses may at once be obtained by multiplying by the proper secant.

Fig. 11 represents a truss in its most general form, the

web members being unequally inclined to a vertical. The position of the load causing a maximum shear in the panel

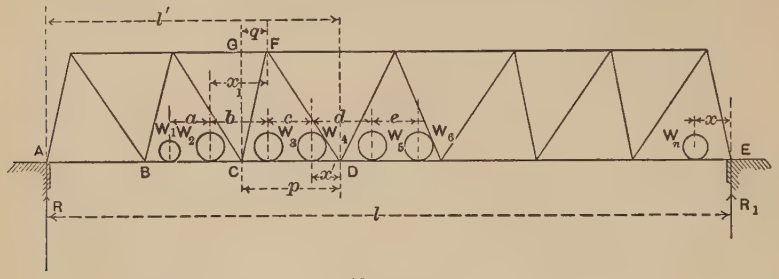


FIG. 11.

CD will be determined. The value of this shear when found will determine the stresses in the members CF and DF. The moving loads, consisting of the weights W_1, W_2, . . . W_n separated by the fixed distances a, b, c, d, . . . , are supposed to advance on the bridge from the right. The weights W_1, W_2, . . . have passed the panel in question and the weights W_3, W_4, . . . are in the panel, W_4 being distant x' from its right end, D. The last load, W_n, is at the distance x from E. The length of span AE is l, while the length of the panel DC is p. With this assumed position of loading the reaction R at A will be

$$R = \left\{ \begin{aligned} & W_1 \frac{a+b+c+\ldots+x}{l} + W_2 \frac{b+c+d+\ldots+x}{l} \\ & \quad + W_3 \frac{c+d+e+\ldots+x}{l} + \ldots + W_n \frac{x}{l}. \end{aligned} \right. \quad (1)$$

The parts of the weights W_3, W_4, . . . resting on the panel CD which pass to C are $W_3 \cdot \frac{c+x'}{p}$, $W_4 \cdot \frac{x'}{p}$, ... Hence the shear in the panel CD will be

$$S = R - (W_1 + W_2 + \ldots) - \left(W_3 \frac{c+x'}{p} + W_4 \frac{x'}{p} + \ldots \right). \quad (2)$$

The maximum value of this shear may be determined by obtaining its first derivative with respect to x and x' (x' varying at the same rate as x) and placing it equal to zero. That is,

$$\frac{dS}{dx} = \left(\frac{W_1}{l} + \frac{W_2}{l} + \frac{W_3}{l} + \ldots + \frac{W_n}{l}\right) - \left(\frac{W_3}{p} + \frac{W_4}{p} + \ldots\right) = 0, \quad (3)$$

or

$$W_1 + W_2 + W_3 + \ldots + W_n = \frac{l}{p}(W_3 + W_4 + \ldots). \quad (4)$$

The shear in the panel in question will therefore take its greatest value *when* $\frac{l}{p} = n$ *times the moving load which it contains is equal to, or most nearly equal to, the entire moving load on the bridge.*

That equality will seldom or never exist unless one of the weights W is placed on a panel-point, since $W_1 + W_2 + \ldots$ is seldom or never an exact divisor of the entire load on the bridge. If a weight rests on a panel-point, any part of such a weight may be taken as acting in one adjacent panel and the remainder in the other; the desired equality may thus be obtained.

In case eq. (4) should hold, the position of the moving load is a matter of indifference so long as the panel in question contains the same number of loads $W_1 + W_2 + \ldots$, as there is no trace of x' in that equation. A load may then always be taken as resting at the rear extremity of the panel where the greatest shear in it exists.

These considerations frequently essentially simplify computations.

The criterion, eq. (4), applies as well to a uniform and continuous loading as to a series of isolated weights.

The problem has been so treated as to include weights $W_1, W_2, \ldots$ in advance of the panel in question. It is to be

noted, however, that such weights cause a shear opposite to that of those loads located to the right of the panel. This is at once evident, since, in actually determining the shear, the full values W_1, W_2, . . . must be subtracted from the reaction, to which they have contributed but a part of themselves. The character of the loading under certain circumstances may be so peculiar, however, that the loads to the right of the panel increase the positive shear in greater amount than the loads to the left decrease it.

In general practice it will be found that no load passes the panel in question.

If each of the weights W_1, W_2, etc., is equal to any other, and if $a=b=c=d= \ldots =p$, i.e., if uniform loads separated by panel lengths are employed, eq. (4) shows that the first weight, W_1, must be taken at the first extremity of the panel in question. The same result holds if the first weight, W_1, is not exceeded in amount by any that follows it, provided that a, b, c, etc., still equal p.

Art. 71. Moving-load Web Stresses in a Six-panel Pratt Truss.

The criterion of Art. 70, deduced for the maximum web stresses in trusses having parallel and horizontal chords, will be applied to the six-panel truss treated in Art. 68.

The length of truss is 150 feet and its depth 28 feet; the live loading taken is designated by Cooper as E 50 and is given in detail on page 56. The criterion, eq. (4), Art. 70, shows that n times the weights in the panel must equal the total load on the bridge, n representing the ratio of the panel length in question to the total truss length. In this truss n is 6 for all the web members.

The moment tabulation, shown on page 94, for the concentrations of E 40 loading will be employed in this

problem. As the specifications require E 50 loading, it will be necessary to multiply all stresses found for E 40 by 1.25, the ratio of the two loadings.

Member U_1L_2.

If wheel No. 4 be taken at L_2, the total load on the bridge will be $284{,}000 + (9 \times 2{,}000) = 302{,}000$ pounds. The weight in the panel will be 50,000 or 70,000 pounds, according as wheel No. 4 is considered to the right or left of L_2. The conditions of the criterion are therefore fulfilled.

The left-hand reaction is

$$R = \frac{14{,}944{,}000 + (284{,}000 \times 14) + \frac{(9)^2 \times 2{,}000}{2}}{150}$$

$$= \frac{19{,}005{,}000}{150} = 126{,}500 \text{ pounds.}$$

The negative shear or loading carried to L_1 is found by taking moments about L_2 and dividing by the panel length; it is equal to $\frac{480{,}000}{25} = 19{,}200$ pounds. The shear in the panel itself is therefore $126{,}500 - 19{,}200 = 107{,}300$ pounds. The secant of the inclination is 1.34, and the proper loading requires a factor of 1.25; the stress in U_1L_2 is therefore

$$U_1L_2 = 107{,}300 \times 1.34 \times 1.25 = 181{,}000 \text{ pounds tension.}$$

Member U_2L_2.

It will be found that with wheel No. 3 at L_3 the total load on the bridge is 245,000 pounds and that the criterion is satisfied. The reaction is $\frac{10{,}816{,}000}{150} = 72{,}200$ pounds; the negative shear is $\frac{230{,}000}{25} = 9{,}200$; the shear in the

panel is therefore $72{,}200 - 9{,}200 = 63{,}000$ pounds. As the secant of the inclination is 1.0, the stress is

$$U_1L_2 = 63{,}000 \times 1 \times 1.25 = 79{,}000 \text{ pounds compression.}$$

Member U_2L_3 (Main Web Member, Left of the Center).

The position of the loading for greatest stress in this member is the same as for U_2L_2, for the reason that the web members meeting in the unloaded chord take their maximum stresses for the same position of loading. The secant of the inclination is, however, 1.34, so that

$$U_2L_3 = 63{,}000 \times 1.34 \times 1.25 = 106{,}000 \text{ pounds tension.}$$

Member U_2L_3 (Counter-member, Right of the Center).

The maximum stress in this member found with the load advancing from the right will be the greatest compression in it. If this compressive stress be greater than the dead-load tensile stress (always present), the member must be constructed of such form as to withstand either tension or compression, or this counter-stress, as it is called, must be provided for by another member in the same panel, such as U_3L_2, but capable of carrying tension only.

The criterion for the maximum counter-stress is satisfied with the wheel load 2 at L_4. Loads 1 to 10 are on the truss, wheel No. 10 being 2 feet distant from the right abutment. Hence the reaction is

$$R = \frac{4{,}632{,}000 + (152 \times 2{,}000)}{150} = \frac{4{,}936{,}000}{150} = 32{,}900 \text{ pounds.}$$

The shear in the panel is $32{,}900 - \left(\frac{80{,}000}{25}\right) = 29{,}700$ pounds. Hence the stress in $U_2L_3 = 29{,}700 \times 1.34 \times 1.25 =$

50,000 pounds compression. Since this compresion is greater than the dead-load tension found on page 150, a counter-brace U_3L_2 will be inserted. The maximum stress in this member will evidently be 50,000 pounds tension, for its inclination to the vertical is the same as U_2L_3, and its shear is the same. It should be observed that if any panel contains two intersecting web members of the kind here shown, the exact determination of their stresses becomes impossible by the ordinary static equations of equilibrium. It is necessary to assume that one of such members only acts at a time, choosing always that member most convenient for the purpose.

Member U_3L_3.

The position of loading just determined for U_2L_3 furnishes the maximum compressive stress in U_3L_3, the counter-member U_3L_2 being assumed to act. Its amount is

$U_3L_3 = 29{,}700 \times 1 \times 1.25 = 38{,}000$ pounds compression.

Counter-members in Panel L_1L_2.

Whether counters are necessary or not in this panel is easily determined. The position of loading, moving from left to right, causing the maximum counter-stress is found when wheel No. 2 is at L_1. The reaction then is

$$R = \frac{1{,}640{,}000 + (103{,}000 \times 1)}{150} = 11{,}600 \text{ pounds.}$$

The shear is $11{,}600 - 3{,}200 = 8{,}400$ pounds, and the stress in the counter would therefore be

$$8{,}400 \times 1.34 \times 1.25 = 14{,}000 \text{ pounds}$$

and less than the stress due to dead load. Counters are, therefore, not required in this panel.

Member L_1U_1.

This member is not a true web member; its function is simply to transfer the floor-beam load at its foot to U_1. The criterion for a maximum floor-beam reaction (Art. 69) requires for this truss that the loads in the two adjacent panels be equal to each other. If wheel No. 13 be placed directly over the floor-beam, this condition is fulfilled, as shown in Fig. 12.

The reaction at L_1 due to the loads on L_0L_1 is 30,800 pounds, and that due to the loads on L_1L_2 is 24,800 pounds.

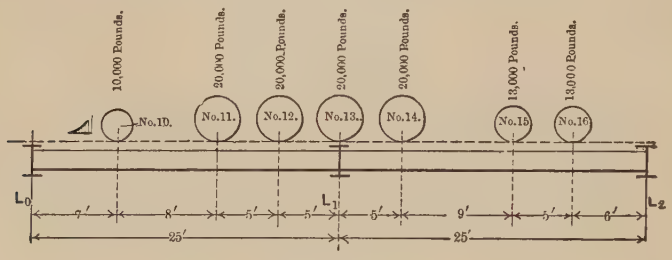


FIG. 12.

Wheel No. 13, weighing 20,000 pounds, rests directly at L_1. The maximum floor-beam reaction or stress in U_1L_1 is therefore

$$U_1L_1 = (30{,}800 + 24{,}800 + 20{,}000) \times 1.25$$
$$= 95{,}000 \text{ pounds tension.}$$

Art. 72. Position of Moving Load Causing Maximum Chord Stresses in Trusses with Parallel and Horizontal Chords.

Case I. Upper-chord Members.

The stress in an upper-chord member is found by the method of sections by dividing the moment of the external forces taken about some center of moments (a lower panel-

point) by the proper lever-arm. In order to determine the maximum stress, it is only necessary, then, to determine the maximum bending moment at the proper point, precisely as in the case of a simply supported beam (Chapter III, Art. 57). The criterion there developed, viz.,

$$\frac{l'}{l} = \frac{W_1 + W_2 + \ldots + W_n'}{W_1 + W_2 + W_3 + \ldots + W_n}, \quad \ldots \quad (1)$$

may be used for this case in exactly the same way, since the transference of the loads on the structure to the panel-points does not affect the bending moments at those points.

Example.

Let there be found the maximum stress caused by Cooper's E 50 standard concentrations in the upper-chord member U_1U_2 of the truss already treated in the preceding articles. Passing a section through U_1U_2, U_1L_2, and L_1L_2, the center of moments is found at L_2. In the criterion, therefore, $l' = 50$ feet, $l = 150$ feet, and the sum of the weights on l' must be one third of the sum of all the weights on the bridge.

This condition will be fulfilled when wheel No. 8 is at L_2, for in that case the total load on the bridge is $284{,}000 + (34 \times 2{,}000) = 352{,}000$ lbs., and the weights on l' may be taken at either 116,000 or 129,000 pounds. The product $(34 \times 2{,}000)$ indicates that there are 34 feet of uniform load on the bridge. In this analysis the table on page 94, given for Cooper's E 40, is employed. The stresses thus found must therefore be multiplied by 1.25 in order that they comply with the required load, Cooper's E 50.

The left-hand reaction is next to be found; it may be obtained by taking moments about the right end of the truss. By the aid of the moment diagram,

$$R = \frac{14{,}944{,}000 + (284 \times 39) + \frac{(34)^2 \times 2{,}000}{2}}{150}$$

$$= \frac{14{,}944{,}000 + 11{,}080{,}000 + 1{,}155{,}000}{150} = 181{,}000 \text{ pounds.}$$

In this equation 14,944,000 represents the moment of all the wheels, Nos. 1 to 18, about the last wheel, No. 18; 284,000, the sum of the wheel loads, including No. 18; 39, the distance of wheel No. 18 from the right end of the truss; and 34, the length of uniform load whose moment is expressed by $\frac{wx^2}{2}$, where x represents its length and w its intensity or amount per linear foot.

The moment about L_2 is then

$M = 181{,}000 \times 50 - 2{,}851{,}000 = 6{,}199{,}000$ foot-pounds.

The quantity 2,851,000 is obtained directly from the moment diagram; it is the moment of wheels Nos. 10 to 17 about No. 17; or of wheels Nos. 1 to 8 about No. 8.

This moment M is then to be divided by the lever-arm of U_1U_2 and the result multiplied by 1.25 to reduce the value for the loading E 50; i.e.,

$$U_1U_2 = \frac{6{,}199{,}000 \times 1.25}{28} = 278{,}000 \text{ pounds,}$$

i.e., a compression of 278,000 pounds.

Case II. Lower-chord Members.

Those trusses in which the upper panel-point taken as the center of moments is vertically above a panel-point of the lower chord require the same criterion deduced for the upper-chord members. This results from the fact that the transference of the loading to the truss by means of the floor-beams and stringers does not affect the bending mo-

ment at or vertically above a panel-point. For those trusses in which the center of moments falls within a panel, as in the case where the web members are all inclined, a different criterion for the lower chords must be developed. Such a criterion must naturally reduce to the form of Case I when the upper-chord panel-points are vertically over the lower.

Let it be required to determine the position of the moving load for the maximum stress in the lower-chord mem-

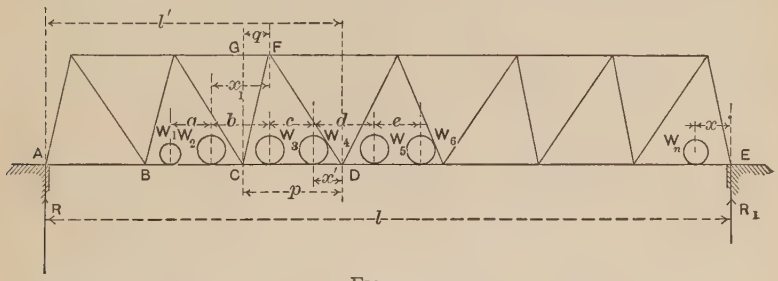


FIG. 13.

ber CD, Fig. 13, and the expression for the corresponding moment.

As before, let the train load move from E to A. Let q represent the horizontal distance of F from C, i.e., $q=GF$. Let x_1 represent the horizontal distance from F of the first load to the left of C, and let that load be represented by W_n'. That portion of the loads resting on CD which are transferred to C is

$$W_3\frac{(c+\ldots+x')}{p}+\ldots+W_4\left(\frac{x'}{p}\right).$$

l' will now represent $AC+GF=AC+q$. By taking moments about F,

$$M=Rl'-[W_1(a+\ldots+x_1)+W_2(\ldots+x_1)+\ldots+W_n'\cdot x_1]$$
$$-\frac{q}{p}[W_3(c+\ldots+x')+\ldots+W_4\cdot x']. \quad (2)$$

The second term in the right-hand member of eq. (2) is the negative moment of the loads between A and C, while the third term is the moment due to that portion of the loads resting in CD which is transferred to C.

Assuming x, x', and x_1 to vary at the same rate, the position of the loading causing the maximum moment is found by placing the first derivative of eq. (2) equal to zero; that is,

$$\frac{dM}{dx} = \frac{l'}{l}(W_1 + W_2 + W_3 + \ldots + W_n) - (W_1 + W_2 + \ldots + W_n') - \frac{q}{p}(W_3 + \ldots + W_4) = 0. \quad (3)$$

The condition for a maximum or minimum then takes the form:

$$\frac{l'}{l} = \frac{W_1 + W_2 + \ldots + W_n' + \frac{q}{p}(W_3 + \ldots + W_4)}{W_1 + W_2 + W_3 + \ldots + W_n}. \quad (4)$$

Eq. (4) is simply the general expression of which eq. (1) is a special form. The numerator of the right-hand member of eq. (4) includes not only the sum of all the weights upon AC, but a certain proportion, namely, $\frac{q}{p}$, of the weights in the panel cut by the section.

It will be found that the equation is generally satisfied by placing a wheel load either at the right or left end of the panel in question.

The preceding methods are equally applicable to a series of concentrations followed by a uniform load, or to a uniform load alone, or to the case of a series of concentrations as treated above. It is only to be noted that in all cases of moments the lever-arm of the uniform load is the distance from its center of gravity to the point taken as the center of moments.

Example.

As an example, the maximum stress caused by Cooper's E 40 will be found in the member L_2L_3 of the six-panel Warren truss shown in Fig. 14. The panel lengths, all equal, are 29 feet; the length of truss is 174 feet, and the uniform depth is 30 feet; the inclinations of the web members are all equal.

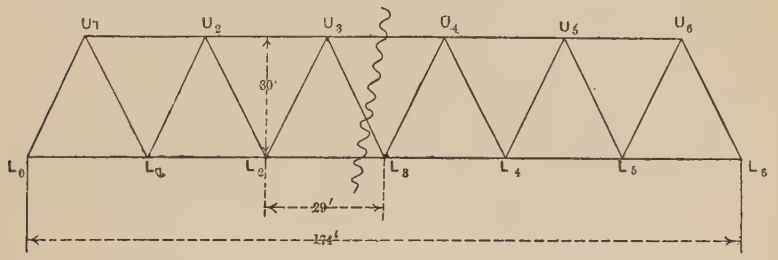


FIG. 14.

Passing a section through U_3U_4, U_3L_3, and L_2L_3, the center of moments is found at U_3. The quantities in eq. (4) have therefore the following values:

$$l' = 72.5 \text{ feet},$$
$$l = 174 \text{ feet},$$
$$\frac{q}{p} = \frac{1}{2}.$$

The moving load is assumed to pass from right to left, and the criterion will be applied with wheel No. 8 at L_2. Wheel No. 1 will then be 15 feet from L_0, and there will be 50 feet of uniform load in the truss. Wheels Nos. 9 to 12 will be in the panel L_2L_3, wheel No. 12 being 3 feet from L_3. Eq. (4) then yields results as follows:

$$\frac{72.5}{174} = \frac{129{,}000 + \frac{1}{2}(63{,}000)}{284{,}000 + (50 \times 2{,}000)} \quad \text{or} \quad \frac{116{,}000 + \frac{1}{2}(76{,}000)}{284{,}000 + (50 \times 2{,}000)}$$
$$= \frac{160{,}500}{384{,}000} \quad \text{or} \quad \frac{154{,}000}{384{,}000}.$$

The ratio $\frac{l'}{l}$ will be found to lie between these values, and the position of the loading taken will give a maximum value of the moment at U_3. In general, several positions of the loading will satisfy the criterion, and in such cases the greatest of the moments so found is the only one to be used to determine the maximum chord stress desired.

The reaction R at L_0 is found as follows, with the aid of the moment table of page 94:

$$R = \frac{14{,}944{,}000 + 284{,}000 \times 55 + \frac{2{,}000 \times 50 \times 50}{2}}{174}$$
$$= 190{,}500 \text{ pounds.}$$

The reaction at L_2 of the loads in the panel L_2L_3 is

$$R_p = \frac{503{,}000 + 63{,}000 \times 3}{29} = 23{,}800 \text{ pounds.}$$

Therefore the moment about U_3 is

$$M = 190{,}500 \times 72.5 - 2{,}851{,}000 - 129{,}000 \times 14.5 - 23{,}800 \times 14.5$$
$$= 8{,}734{,}000 \text{ foot-pounds.}$$

The lever-arm of L_2L_3 is 30 feet; therefore the stress is

$$L_2L_3 = \frac{8{,}734{,}000}{30} = 291{,}000 \text{ pounds tension.}$$

The stress in U_3U_4 would be found by taking the maximum bending moment at L_3, and the method of treatment is precisely that of Case I.

Art. 73. Moving-load Chord Stresses in a Six-panel Pratt Truss.

The moving-load chord stresses for the Pratt truss of Art. 68, reproduced as Fig. 15, will be found for Cooper's E 50 concentrations. As before, the table on page 94, constructed for E 40 loading, will be employed and the stresses will then be multiplied by 1.25. The criterion is that of Art. 71, viz.,

$$\frac{l'}{l} = \frac{W_1 + W_2 + \ldots + W_n'}{W_1 + W_2 + W_3 + \ldots + W_n}, \quad . \quad . \quad . \quad . \quad (1)$$

where $W_1 + W_2 + \ldots + W_n'$ represent the weights on the length l', and $W_1 + W_2 + W_3 + \ldots + W_n$ the weights on the entire span of length l.

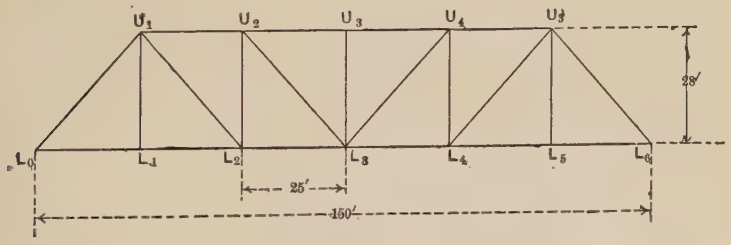


FIG. 15.

Member L_0U_1.

For this member the center of moments is at L_1. The value of l' is then 25 feet, and of l 150 feet. The ratio of the two series of weights on these lengths must then be $\frac{1}{6}$. If wheel No. 4 be placed at L_1, the weights on l' may be taken at either 50,000 or 70,000 pounds as wheel No. 4 is taken either to the right or left of L_1 respectively. The total weight on the truss is $284{,}000 + (34 \times 2{,}000) = 352{,}000$, where 34 represents the length of the uniform load. Since $\frac{1}{6}$ lies between $\frac{50{,}000}{352{,}000}$ and $\frac{70{,}000}{352{,}000}$, the position of loading taken produces a maximum.

As already explained, other positions of the loading may also fulfill the conditions of the criterion, and the absolute maximum should always be obtained. Graphical methods* greatly facilitate the determination of that maximum, since the moment or funicular polygon once drawn for any series of loads and with any desired pole distance will furnish the value of the moment at any point in a span, when the proper closing line has been drawn.

The left-hand reaction R is obtained as already explained in Art. 72:

$$R = \frac{14{,}944{,}000 + (284{,}000 \times 39) + \frac{(34)^2 \times 2{,}000}{2}}{150} = 181{,}000 \text{ lbs.}$$

The force acting opposite to R at L_0 is equal to the reaction at that point of the wheel loads in the panel L_0L_1. The moment of wheels Nos. 1 to 4 about wheel No. 4 is 480,000 foot-pounds (from moment diagram). The force desired is obtained by dividing this quantity by 25 feet, the panel length, i.e., $\frac{480{,}000}{25} = 19{,}000$ pounds.

Taking moments of these external forces about L_1 and dividing by the lever-arm of L_0U_1, 18.7 feet, there is found

$$L_0U_1 = \frac{181{,}000 \times 25 - 19{,}000 \times 25}{18.7} \times 1.25 = 272{,}000 \text{ lbs. comp.}$$

Instead of taking moments about L_1, it would prove more simple to treat L_0U_1 as a web member and multiply the shear in the panel L_0L_1 by the secant, 1.34, of the angle between L_0U_1 and a vertical. This shear has already been found to be

$$181{,}000 - 19{,}000 = 162{,}000 \text{ pounds;}$$

therefore the stress is

$$162{,}000 \times 1.34 \times 1.25 = 272{,}000 \text{ pounds compression.}$$

* See the Authors' "Influence Lines," page 77.

Member L_0L_1.

The maximum live-load stress for this member occurs with the same position of loading as for L_0L_1. Its value is obtained by multiplying the shear for that member, viz. 162,000, by the tangent, 0.893, of the angle between L_0U_1 and the vertical:

$$L_0L_1 = 162{,}000 \times .893 \times 1.25 = 181{,}000 \text{ pounds tension.}$$

Member L_1L_2.

Since the member U_1L_1 is vertical and the only one besides L_0L_1 and L_1L_2 passing through the point L_1, it follows that the maximum stress in L_1L_2 is the same as in L_0L_1:

$$L_1L_2 = 181{,}000 \text{ pounds tension.}$$

Member L_2L_3.

This member has already been fully treated in Art. 72. Its maximum stress is

$$L_2L_3 = 278{,}000 \text{ pounds tension.}$$

Member U_1U_2.

The value of the maximum stress in this member is the same as that found in L_2L_3; for if a section cut U_1U_2, U_2L_2, and L_2L_3, the member U_2L_2, being vertical, has no horizontal component of stress, and U_1U_2 and L_2L_3 being horizontal, the stresses in them must be equal and opposite in kind. Therefore

$$U_1U_2 = 278{,}000 \text{ pounds compression.}$$

Member U_2U_3.

The ratio of the lengths to be used in the criterion is 75/150, or $\frac{1}{2}$. The conditions of that equation are then fulfilled by placing wheel No. 12 at U_3, as there will be 35 feet of uniform load on the truss.

The left-hand reaction is then

$$R = \frac{14{,}944{,}000 + (284{,}000 \times 40) + \frac{(35)^2 \times 2{,}000}{2}}{150} = 183{,}500 \text{ lbs.}$$

The moment of all the external forces to the left of U_3 about U_3 is

$$M = 183{,}500 \times 75 - 6{,}708{,}000 = 7{,}042{,}000,$$

and the stress in U_2U_3 is

$$U_2U_3 = \frac{7{,}042{,}000 \times 1.25}{28} = 313{,}000 \text{ pounds compression.}$$

Art. 74. Final Stresses and Design of the Members for the Six-panel Pratt Truss of Arts. 68, 71, and 73.

The members of the six-panel Pratt truss whose fixed- and moving-load stresses have been determined in the previous articles can now be designed with given allowed intensities of stress for dead and live loads. The truss under consideration is to be designed under the specifications of the American Bridge Co., as already abstracted in Chapter II. Those specifications require the increment $I = S\left(\frac{300}{L+300}\right)$ to be added to the live-load stresses already found, in which S is the calculated maximum live-load stress, and L is the length of the loaded distance in feet which produces the maximum stress in the member.

It is sufficiently accurate in the case of chord members to assume the entire bridge as loaded, but in the case of web members the bridge is assumed to be loaded only to that panel through which the section is passed.

Table I furnishes the values of the impact stresses and

illustrates the method of obtaining them. Table II* shows the final values of the stresses for which the members must be designed together with the allowed intensities of stress for medium steel; the final areas of cross-section and their composition. The allowed intensity of tensile stress (§ 21) is 17,000 pounds per square inch, and that for compressive stress is $p = \frac{17{,}000}{1 + \frac{l^2}{11{,}000 r^2}}$, where l = the length of member in inches, center to center of connections, and r is the radius of gyration of the section in inches, taken in the direction of failure if the column were tested to failure.

TABLE I.

Member.	Loaded Length in Feet = L.	$\frac{300}{L+300}$	S	$S\left(\frac{300}{L+300}\right)$
L_0U_1	150	0.667	272,000	181,000 pounds
L_0L_1	150	0.667	181,000	121,000 "
L_1L_2	150	0.667	181,000	121,000 "
L_2L_3	150	0.667	278,000	185,000 "
U_1U_2	150	0.667	278,000	185,000 "
U_2U_3	150	0.667	313,000	210,000 "
U_1L_1	50	0.857	95,000	81,000 "
U_2L_2	75	0.800	79,000	63,000 "
U_3L_3	50	0.857	38,000	32,000 "
U_1L_2	100	0.750	181,000	136,000 "
U_2L_3	75	0.800	106,000	85,000 "
U_3L_2	50	0.857	50,000	43,000 "

The bridge has been assumed to be pin-connected; hence the tension members are eye-bars. For further details of construction, including the consideration of wind stresses, the reader is referred to Chap. VI, "The Detailed Design of a Railroad Bridge," in the Authors' "Influence Lines for Bridge and Roof Trusses."

Plate II illustrates the stress sheet of this bridge as prepared by the American Bridge Co., and in that form it is furnished to the draughting room, where detail drawings preparatory to the manufacture are made.

* See page 172 for Table II.

TABLE II.

Member.	Dead-load Stress in Thousands of Pounds.	Live-load Stress in Thousands of Pounds.	Impact Stress in Thousands of Pounds.	Total Stress in Thousands of Pounds.	Allowed Intensity of Stress in Thousands of Pounds.	$\frac{l}{r}$	Area Required in Square Inches.	Composition of Section.
L_0U_1	− 82	−272	−181	−535	11.7	$\frac{450}{6.4}$	45.8	1 Cover-plate 24″×$\frac{7}{16}$″ = 10.5 sq. ins. 2 15″ Channels @ 50 lbs. = 29.4 2 Flats, 4″×$\frac{7}{8}$″ = 7.0
L_0L_1	+ 55	+181	+121	+357	17	....	21	2 Bars, 6″×1$\frac{1}{4}$″ = 21.0
L_1L_2	+ 55	+181	+121	+357	17	....	21	2 Bars, 6″×1$\frac{1}{4}$″ = 21.0
L_2L_3	+ 88	+278	+185	+551	17	....	32.5	4 Bars, 6″×1$\frac{1}{8}$″ = 33.0
U_1U_2	− 88	−278	−185	−551	14.3	$\frac{300}{6.6}$	38.6	1 Cover-plate, 24″×$\frac{7}{16}$″ = 10.5 2 15″ Channels @ 35 lbs. = 20.6 2 Flats, 4″×$\frac{7}{8}$″ = 7.0
U_2U_3	−100	−313	−210	−623	14.2	$\frac{300}{6.4}$	43.9	Same as above, except 2 15″ channels @ 45 lbs. = 26.5 sq. ins.
U_1L_1	+ 18	+ 95	+ 81	+194	17	....	11.4	4 angles, 5″×3$\frac{1}{2}$″×$\frac{3}{8}$″ = 12.2 gross = 10.7 net 1 Web-plate, 8″×$\frac{3}{8}$″ = 3.0 gross = 2.2 net
U_2L_2	− 19	− 79	− 63	−161	11.1	$\frac{336}{4.4}$	14.5	2 12″ Channels @ 25 lbs. = 14.8 sq. ins.
U_3L_3	− 7	− 38	− 32	− 77	11.1	$\frac{336}{4.4}$	6.9	2 12″ Channels @ 25 lbs. = 14.8
U_1L_2	+ 50	+181	+136	+367	17	....	21.0	2 Bars, 6″×1$\frac{13}{16}$″ = 21.7
U_2L_3	+ 17	+106	+ 85	+208	17	....	12.3	2 Bars, 6″×1″ = 12.0
U_3L_2	− 17	+ 50	+ 43	+ 76	17	....	4.7	1 Bar, 5″×1″ = 5.0 adjustable

Art. 75. Stresses in an Eight-panel Pratt Highway Truss.

Uniform Moving Load.

The stresses in the truss shown in Fig. 16 due to a uniform moving load of 90 pounds per square foot on a roadway 20 feet wide will be determined.

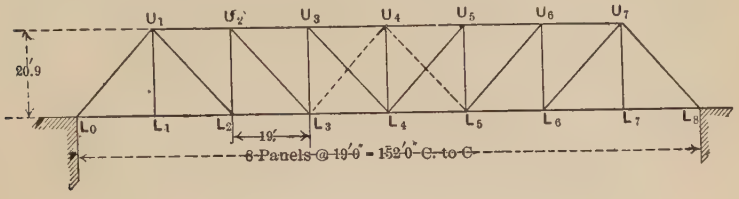


FIG. 16.

The following are the principal dimensions and fixed-load data:

Span = 152 feet. Panel length = 19 feet.
Depth = 1.1 panel lengths = 20.9 feet. Number of panels = 8.
Upper-chord fixed load = 170 lbs. per lineal foot per truss.
Lower- " " " = 340 " " " " " " "
Upper- " " " per panel per truss = 3,230 pounds.
Lower- " " " " " " " = 6,460 "

$\tan \alpha = 0.91$. $\sec \alpha = 1.35$.

The preceding loading is accounted for as follows:

The dead weight w of the steel in the trusses per foot of span may be approximately estimated by the formula $w = al$, where l equals the span length in feet and a an empirical constant, here chosen as 4.5. The weight per linear foot w then becomes 680. The weight of the floor system and roadway per linear foot may be taken at 340 pounds, making the total 1,020 pounds per linear foot of span.

Dead-load Stresses.

The dead-load reaction at L_0 is

$$3\tfrac{1}{2}(3{,}230+6{,}460)=33{,}920 \text{ pounds},$$

neglecting the panel load at the abutment as a factor in causing stresses in the truss. The following tabulation then furnishes the shears for the web members indicated. The stresses are obtained by multiplying those shears by the secants of the angle between the web members and a vertical.

Shear for $L_0U_1 = 33{,}920$,
$\therefore$ stress $= 33{,}920 \times 1.35 = -45{,}750$ pounds.

Shear for $U_1L_2 = 33{,}920 - 9{,}690 = 24{,}230$,
$\therefore$ stress $= 24{,}230 \times 1.35 = +32{,}800$ pounds.

Shear for $U_2L_2 = 33{,}920 - 12{,}920 = 21{,}000$,
$\therefore$ stress $= 21{,}000 \times 1 = -21{,}000$ pounds.

Shear for $U_2L_3 = 33{,}920 - 19{,}380 = 14{,}540$,
$\therefore$ stress $= 14{,}540 \times 1.35 = +19{,}600$ pounds.

Shear for $U_3L_3 = 33{,}920 - 22{,}610 = 11{,}310$,
$\therefore$ stress $= 11{,}310 \times 1 = -11{,}310$ pounds.

Shear for $U_3L_4 = 33{,}920 - 29{,}070 = 4{,}850$,
$\therefore$ stress $= 4{,}850 \times 1.35 = +6{,}500$ pounds.

The member U_1L_1 is simply a tie-rod and its dead-load stress is simply the lower-chord panel load of $+6{,}460$ pounds. Similarly the member U_4L_4 supports only the upper-chord panel dead load at U_4 or $-3{,}230$ pounds. The + sign indicates tension and the − sign compression.

The stress in the lower-chord member L_0L_1 is found by multiplying the end reaction by the tangent of the angle between L_0U_1 and the vertical. Therefore

$$L_0L_1 = 33{,}920 \times 0.91 = +30{,}800 \text{ pounds}.$$

The stress in L_1L_2 is equal to that in L_0L_1.

By the increment method (page 146) the stress in L_2L_3 is equal to that in L_1L_2 plus the products of the shears in the members meeting at L_2 by their tangents. That is,

$$L_2L_3 = 30{,}800 + (24{,}230 \times 0.91) = +52{,}900 \text{ pounds.}$$

The tangent of U_2L_2 is zero, and the increment of stress added by that member is also zero.

If a section be passed cutting U_1U_2, U_2L_2, and L_2L_3, it is evident that the stress in U_1U_2 is equal to that in L_2L_3 but of opposite sign:

$$U_1U_2 = -52{,}900 \text{ pounds.}$$

The stress in L_3L_4 is

$$L_3L_4 = L_2L_3 + (14{,}540 \times .91) = +66{,}100 \text{ pounds.}$$

The stress in $U_2U_3 = -L_3L_4 = -66{,}100$ pounds.

The stress in U_3U_4 is

$$U_3U_4 = U_2U_3 - (4{,}850 \times 0.91) = -70{,}500 \text{ pounds.}$$

Live-load Stresses.

Since the moving load is a uniform load of 900 pounds per linear foot per truss, the criterion of Art. 72 for greatest moments, viz.,

$$\frac{l'}{l} = \frac{W_1 + W_2 + \ldots + W_n'}{W_1 + W_2 + W_3 + \ldots + W_n},$$

will require the entire bridge to be loaded. A lower panel load will then be uniformly $900 \times 19 = 17{,}100$ pounds. The following tabulation shows the moments at the various panel-points:

At L_1 or $U_1 = 59{,}850 \times 19 = 1{,}135{,}000$ ft.-lbs.
" L_2 " $U_2 = 59{,}850 \times 38 - 17{,}100 \times 19 = 1{,}945{,}000$ ft.-lbs.
" L_3 " $U_3 = 59{,}850 \times 57 - 17{,}100(38 + 19)$
$= 2{,}431{,}000$ ft.-lbs.
" L_4 " $U_4 = 59{,}850 \times 76 - 17{,}100(57 + 38 + 19)$
$= 2{,}590{,}000$ ft.-lbs.

In order to obtain the chord stresses, each of these moments must be divided by the proper lever-arm, i.e., the depth of truss. Therefore

$$L_0L_1 = L_1L_2 = +\frac{1{,}135{,}000}{20.9} = +\ 54{,}300 \text{ pounds.}$$

$$L_2L_3 = -U_1U_2 = \frac{1{,}945{,}000}{20.9} = +\ 93{,}200 \quad "$$

$$L_3L_4 = -U_2U_3 = \frac{2{,}431{,}000}{20.9} = +116{,}000 \quad "$$

$$U_3U_4 = -\frac{2{,}590{,}000}{20.9} = -124{,}000 \quad "$$

The criterion for the web members (Art. 70) requires that n times the load in any panel must equal the total load on the bridge, $n = 8$ being the number of panels.

If the panels from right to left be numbered consecutively, beginning with zero, so that panel L_7L_8 is 0, L_6L_7 is 1, etc., the criterion will be satisfied if that portion of the structure represented by the ratio $\frac{x}{n-1}$ is covered by the uniform load, x indicating the number of the panel.

In the truss under consideration, then, $\frac{7}{7}$ths or the whole bridge must be covered for the panel L_0L_1; $\frac{6}{7}$ths for L_1L_2; $\frac{5}{7}$ths for L_2L_3, etc. The values of the shears and stresses in the web members are easily found after the position of the loading is determined.

The maximum stress in the end post L_1U_1 occurs, then,

with the entire bridge covered. The effective end reaction is 59,850 pounds, and as the secant is 1.35, the stress is

$$L_0U_1 = -59{,}850 \times 1.35 = -80{,}700 \text{ pounds.}$$

The member U_1L_1 is simply a tie-rod, and its maximum stress is caused by the maximum floor-beam reaction at its foot. If the concentrated wheel loadings specified for the floor systems be not considered, the stress would be

$$U_1L_1 = 900 \times 19 = +17{,}100 \text{ pounds.}$$

The maximum stress in U_1L_2 is found with $\frac{6}{7}$ths or 130.3 feet of the bridge covered. The reaction at L_0 is then $\frac{900 \times (130.3)^2}{2 \times 152} = 50{,}900$ pounds, and the weight acting downwards at L_1 is $\frac{900 \times (16.3)^2}{2 \times 19} = 6{,}280$ pounds. The length of uniform load on the panel L_1L_2 is 16.3 feet, as required by the criterion, since $n \times 16.3$ is then equal to 130.3.

The shear in the panel is therefore $50{,}900 - 6{,}280 = 44{,}620$ pounds, and the stress is

$$U_1L_2 = 44{,}620 \times 1.35 = +60{,}200 \text{ pounds.}$$

The members U_2L_2 and U_2L_3 take their maximum stresses or shears with the same position of loading, viz., $\frac{5}{7}$ths or 108.6 feet of the bridge covered. The reaction at L_0 is then $\frac{900 \times (108.6)^2}{2 \times 150} = 35{,}400$ pounds, and the panel reaction at L_2 is $\frac{900 \times (13.6)^2}{2 \times 19} = 4{,}380$ pounds. The shear is then $35{,}400 - 4{,}380 = 31{,}000$ pounds, and the stresses are:

$$U_2L_3 = 31{,}000 \times 1.35 = +41{,}800 \text{ pounds,}$$
$$U_2L_2 = 31{,}000 \times 1 \quad = -31{,}000 \quad \text{“}$$

Similarly, the members U_3L_3 and U_3L_4 receive their greatest stresses with $\frac{4}{7}$ths or 86.9 feet of the bridge covered with moving load. The shear in the panel L_3L_4 will be found to be $22{,}300 - 2{,}800 = 19{,}500$ pounds. Hence the stresses:

$$U_3L_3 = 19{,}500 \times 1 \quad = -19{,}500 \text{ pounds},$$
$$U_3L_4 = 19{,}500 \times 1.35 = +26{,}300 \quad ''$$

Counter-members.

If the live-load compressive stress in the member L_4U_5 be found greater than the dead-load tensile stress, that member must either be counterbraced, i.e., built of such section as will withstand either tension or compression, or a counterbrace such as U_4L_5, as in the present case, must be inserted. If the right half of the bridge only be loaded, this counterbrace will be in tension, whereas if the member L_4U_5 alone existed, it would be in compression.

The members U_4L_4 and U_4L_5 receive their greatest stresses, therefore, with $\frac{3}{7}$ths or 65.1 feet of the bridge covered with the load. The shear in the panel is $12{,}500 - 1{,}600 = 10{,}900$ pounds; therefore

$$U_4L_4 = -10{,}900 \text{ pounds},$$
$$U_4L_5 = -10{,}900 \times 1.35 = +14{,}700 \text{ pounds}.$$

If the member U_4L_5 acts for the live load, it is generally assumed to carry also the dead-load stress, in this case a compression of 6,500 pounds. The resultant stress in this member would therefore be the difference between those due to dead and live loads, i.e., $14{,}700 - 6{,}500 = 8{,}200$ pounds tension. It seems more rational and advisable, however, to design this counter-member for the entire live-load stress, since it is at best but a light rod.

No other counter-members will be required, for it will be found that the live-load compression in L_5U_6 is materially less than the dead-load tension.

If the truss be divided through U_3U_4 and L_3L_4, it is seen that more than three members must be cut, but if that number is exceeded, it is known from the first principles of statics that the stresses must become indeterminate. Hence, *when counterbraces are introduced, indetermination always results.* If provision is made for one system of legitimate stress analysis, however, the safety of the structure is assured.

One point more needs passing attention. It has been stated in the course of the demonstrations that the stress in certain members is tension, and compression in others. In web member U_2L_3, for example, let it be desired to determine the kind of stress. It has been seen that when the greatest main web stress exists in that member, the reaction at L_0 is 35,400 pounds, and it is evident that it is directed *upward.* At the same time the live load resting at L_2 is 4,380 pounds and is directed *down.* The difference of these forces is an *upward* shear of 31,000 pounds. Hence, if the truss is divided anywhere in the panel U_2U_3 and L_2L_3, this shear will tend to move the left portion (between the line of divisions and L_0) *upward* and past the right portion; i.e., it will tend to increase the distance between U_2 and L_3 and, consequently, produce tension in web member U_2L_3. The general principle, then, is to determine the effect of the resultant external forces on the distance between the extremities, or any other two points in the axis of the member. If the tendency is to increase this distance, the resulting stress will be tension, and compression if the reverse is the case. In trusses with parallel chords, after a little experience, the kind of stress in any member may readily be discovered at a glance, but in many structures with curved or polygonal outlines resort must be made to the general principles stated previously.

Table I exhibits the values of the stresses for both the moving and fixed loads.

If the moving load were to traverse the upper chords of this bridge, the same pairs of web members as before will not take their greatest shears together. The stresses in the inclined braces will not in any way be changed, but it will be necessary to advance the moving load by at least one panel beyond the positions taken in the through-bridge, in order to determine the greatest shears in the vertical braces of the deck-truss. Hence, by changing the bridge from the "through" type to the "deck," the vertical braces will carry considerably increased stresses, and as they are in compression, the weight of the trusses will be materially increased. Hence this truss is best adapted to carrying the moving load along its lower chord.

TABLE I.

Member.	Fixed-load Stress in Pounds.	Moving-load Stress in Pounds.	Member.	Fixed-load Stress in Pounds.	Moving-load Stress in Pounds.
$L_0L_1 = L_1L_2$	+30,800	+ 54,300	U_1L_1	+ 6,460	+17,100
L_2L_3	+52,900	+ 93,200	U_1L_2	+32,800	+60,200
L_3L_4	+66,100	+116,000	U_2L_2	−21,000	−31,000
L_0U_1	−45,750	− 80,700	U_2L_3	+19,600	+41,800
U_1U_2	−52,900	− 93,200	U_3L_3	−11,300	−19,500
U_2U_3	−66,100	−116,000	U_3L_4	+ 6,500	+26,300
U_3U_4	−70,500	−124,000	U_4L_3	− 6,500	+14,700
			U_4L_4	− 3,230	−10,900

Fig. 17 is part of the stress sheet for a similar truss, designed by the American Bridge Co. (1905), for the same span and loading as Fig. 16. There is shown only the composition of the cross-section of the members composing the structure.

Approximate Method Sometimes Employed for Web Members.

In order to simplify the determination of the stresses in the web members, for a structure carrying a uniform live

load, it is sometimes the practice to treat full panel loads only, and to consider that all the panel points between the panel cut by the section and the right end of the truss carry such loads.

That loading is actually impossible, for if the panel at the head of the moving load be completely covered by it so as to cause a full panel load at one end of the panel, there will be less than a full load at the other. The approximation

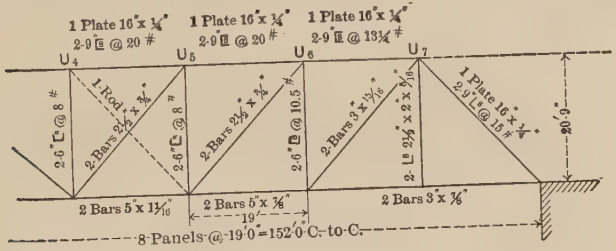


FIG. 17.

consists in considering this partial panel load a full one, so that the resulting stresses are greater than they should be, making the error on the side of safety.

In the truss of Fig. 16 the value of a panel moving load is $900 \times 19 = 17{,}100$ pounds. The following maximum web stresses result by adopting this method.

For the member U_1L_2 all panel-points (see last paragraph of Art. 71) except L_1 must sustain loads of 17,100 pounds each. The reaction at L_0 and the shear in the panel L_1L_2 have then the same values of $17{,}100\left(\frac{6+5+4+3+2+1}{8}\right) =$ 45,000 pounds, and the stress in U_1L_2 is $45{,}000 \times 1.35 =$ $+60{,}700$ pounds, or slightly greater than shown in Table I.

For the member U_2L_3 all panel-points except L_1 and L_2 must support these panel loads, and the shear is

$$17{,}100\left(\frac{5+4+3+2+1}{8}\right) = 32{,}000 \text{ pounds.}$$

The stress is then

$$32{,}000 \times 1.35 = +43{,}200 \text{ pounds.}$$

For the member U_3L_4 the stress is similarly found to be

$$17{,}100\left(\frac{4+3+2+1}{8}\right) \times 1.35 = +28{,}900 \text{ pounds.}$$

The stress for the counter U_4L_3 is

$$17{,}100\left(\frac{3+2+1}{8}\right) \times 1.35 = +17{,}300 \text{ pounds.}$$

This approximate method is thus seen to give values slightly greater than the exact method, the greatest divergence occurring in those members at the center of the span.

The exact method is so simple in its application that there is little or no reason for using any other.

Art. 76. Stresses in a Seven-panel Deck, Single-track Railroad, Howe Truss.

In this form of truss the inclined web members are in compression and the verticals are in tension. When built with timber compression members it has long been known

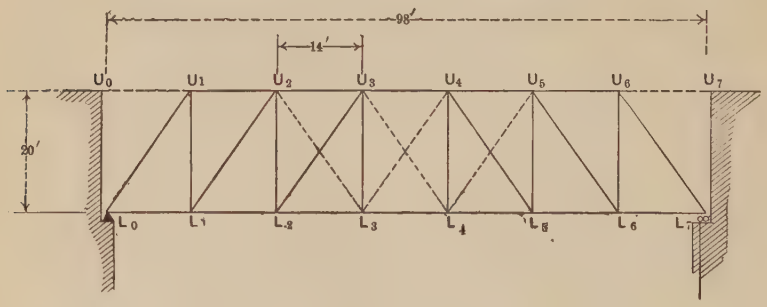


FIG. 18.

as the Howe truss. The skeleton diagram of the structure to be considered is shown in Fig. 18. The moving load

will be supposed to pass along the upper chord; hence the bridge is a "deck" structure. The following are the principal dimensions and fixed-load data:

Span = 98 feet. Panel length = 14 feet.
Depth = 20 feet. Number of panels = 7.
Upper-chord fixed load = 385 lbs per lineal foot per truss.
Lower- " " " = 215 " " " " " " "
Upper- " " " per truss panel = 5400 lbs.
Lower- " " " " " " = 3000 "
tan $L_0U_1L_1 = 0.7$. sec $L_0U_1L_1 = 1.22$.

The moving load will consist of Cooper's locomotives E 40, the weights of which are shown on p. 56; and this load will be taken as passing from right to left. The weights will be numbered W_1, W_2, etc., beginning with the pilot-wheel of the first locomotive.

The web stresses will first be determined, beginning with the first counterbrace needed. As the number of panels is seven, n in eq. (4) of Art. 70 is equal to 7. Let it be required to ascertain whether the compression counterbrace U_5L_4 is necessary. If the train is so placed that W_2 rests at U_5, the bridge will carry the weights $(W_1 \ldots W_6) = 206{,}000$ pounds, or 103,000 pounds on each truss; one truss only will be considered. Now $7W_1 = 70{,}000$ pounds and $7(W_1 + W_2) = 210{,}000$. Since the total load on the bridge is found to lie between these values, by the principles of Art. 70 it is placed to give the greatest *compressive* shear in L_4U_5, or *tensile* shear in U_4L_5. The reaction at L_0 may be found by the aid of the tabulation on p. 94, W_6 being 4 feet from U_7:

$$R = \frac{1{,}640{,}000 + (103{,}000 \times 4)}{98} = 20{,}900 \text{ pounds};$$

the downward force at U_4 is $\frac{80{,}000}{14} = 5{,}700$ pounds, and

therefore the shear is 15,200 pounds. The dead-load shear in the same panel is 8,400 pounds. As the latter is less than and opposite in kind to that of the moving load, the counter post or strut L_4U_5 must be introduced. Since the difference in these shears is very small, it is evident that no other counterbrace between U_5 and U_7 is needed, and that conclusion may be easily verified.

By proceeding in precisely the same manner for the shears in the other panels, the following quantities are found, when eq. (4) of Art. 70 is satisfied, W_n representing the last wheel on the truss, and x its distance from the right abutment:

For greatest shear in

$$\begin{aligned}
&U_4U_5 \ldots W_2 \text{ at } U_5 \ldots W_n = W_6 \ldots x = 4;\\
&U_3U_4 \ldots W_2 \text{ ``} U_4 \ldots W_n = W_9 \ldots x = 2;\\
&U_2U_3 \ldots W_2 \text{ ``} U_3 \ldots W_n = W_{11} \ldots x = 0;\\
&U_1U_2 \ldots W_3 \text{ ``} U_2 \ldots W_n = W_{14} \ldots x = 4;\\
&U_0U_1 \ldots W_3 \text{ ``} U_1 \ldots W_n = W_{16} \ldots x = 4.
\end{aligned}$$

The shears are then as follows:

Shear in $U_4U_5 = \frac{1}{98}[\ 1{,}640{,}000 + 103{,}000 \times 4] - \frac{1}{14} \times\ 80{,}000 =\ 15{,}200$ lbs.

" " $U_3U_4 = \frac{1}{98}[\ 3{,}496{,}000 + 142{,}000 \times 2] - \frac{1}{14} \times\ 80{,}000 =\ 32{,}700$ "

" " $U_2U_3 = \frac{1}{98}[\ 5{,}848{,}000 + \quad\ ] - \frac{1}{14} \times\ 80{,}000 =\ 53{,}900$ "

" " $U_1U_2 = \frac{1}{98}[\ 8{,}728{,}000 + 232{,}000 \times 4] - \frac{1}{14} \times 230{,}000 =\ 82{,}200$ "

" " $U_0U_1 = \frac{1}{98}[12{,}041{,}000 + 258{,}000 \times 4] - \frac{1}{14} \times 230{,}000 = 116{,}600$ "

The shears in the inclined web members only have been given, because each pair of braces that intersect in the chord *not* traversed by the moving load take their greatest stresses with the same position of moving load. Braces U_1L_1 and L_1U_2, U_2L_2 and L_2U_3, etc., thus go together in pairs.

The moving-load stresses, by the aid of the preceding results, will then take the following values:

$L_0U_1 = 116{,}600 \times 1.22 = -142{,}000$ lbs.	
$L_1U_2 = 82{,}200 \times 1.22 = -100{,}000$ "	$L_1U_1 = +82{,}200$ lbs.
$L_2U_3 = 53{,}900 \times 1.22 = -65{,}600$ "	$L_2U_2 = +53{,}900$ "
$L_3U_4 = 32{,}700 \times 1.22 = -39{,}100$ "	$L_3U_3 = +32{,}700$ "
$L_4U_5 = 15{,}200 \times 1.22 = -18{,}500$ "	$L_4U_4 = +15{,}200$ "

The dead-load stresses in these web members are as follows:

$$L_0U_1 = (4 \times 3{,}000 + 3 \times 5{,}400)\,1.22 = -30{,}700 \text{ lbs.}$$
$$U_1L_1 = (3 \times 3{,}000 + 2 \times 5{,}400)\,1 = +19{,}800 \text{ "}$$
$$L_1U_2 = (2 \times 3{,}000 + 2 \times 5{,}400)\,1.22 = -20{,}500 \text{ "}$$
$$U_2L_2 = (2 \times 3{,}000 + 1 \times 5{,}400)\,1 = +11{,}400 \text{ "}$$
$$L_2U_3 = (1 \times 3{,}000 + 1 \times 5{,}400)\,1.22 = -10{,}300 \text{ "}$$
$$U_3L_3 = (1 \times 3{,}000 + 0 \times 5{,}400)\,1 = +3{,}000 \text{ "}$$
$$L_3U_4 = 0$$

The positions of the moving load for the greatest chord stresses are found by the aid of eq. (1) of Art. 72, and result in the following quantities:

For upper chord:

$U_3U_4 \ldots W_7$ at $U_3 \ldots W_n = W_{16} \ldots x = 0$ feet;
$U_2U_3 \ldots W_5$ " $U_2 \ldots W_n = W_{16} \ldots x = 0$ "
$U_1U_2 \ldots W_3$ " $U_1 \ldots W_n = W_{16} \ldots x = 4$ "

The greatest stress in upper chord U_1U_2 occurs with the maximum shear in the end panel, and is found by taking the product of that shear by the tangent of its inclination to a vertical line. The shear has already been determined to be 116,600 pounds, and the tangent is 0.7. Hence

$$\text{Stress in } U_1U_2 = -116{,}600 \times 0.7 = -81{,}500 \text{ lbs.}$$

By the aid of the tabulation on page 94, the following bending moments are found:

Upp. chd. $U_3U_4 = \frac{3}{7}[12{,}041{,}000] - 2{,}155{,}000 = 3{,}005{,}000$ ft.-lbs.
" " $U_2U_3 = \frac{2}{7}[12{,}041{,}000] - 830{,}000 = 2{,}610{,}000$ "

As the depth of the truss is 20 feet, the moving-load chord stresses become

$$U_3U_4 = -3{,}005{,}000 \div 20 = -150{,}300 \text{ lbs.}$$
$$U_2U_3 = -2{,}610{,}000 \div 20 = -130{,}500 \text{ "}$$

The dead-load stresses are found most easily by the chord increment method:

$$U_3U_4 = 6(5{,}400 + 3{,}000) \times 0.7 = -35{,}300 \text{ lbs.}$$
$$U_2U_3 = 5(5{,}400 + 3{,}000) \times 0.7 = -29{,}400 \text{ "}$$
$$U_1U_2 = 3(5{,}400 + 3{,}000) \times 0.7 = -17{,}600 \text{ "}$$

It is to be observed that the upper- and lower-chord stresses are the same in pairs, i.e., in the same oblique panel. The stresses in one chord can then always be written from those in the other, as is done here:

$L_0L_1 = +\ 81{,}500$ lbs. for moving load,
$= +\ 17{,}600$ " " fixed load;
$L_1L_2 = +130{,}500$ " " moving load,
$= +\ 29{,}400$ " " for fixed load;
$L_2L_3 = +150{,}300$ " " for moving load,
$= +\ 35{,}300$ " " for fixed load;
$L_3L_4 = L_2L_3$.

The moments remain the same whether the upper or the lower extremity of any vertical brace is taken for the moment origin; hence the equality of upper- and lower-chord stresses in the same oblique panel. The member L_0U_0 is not a member of the truss proper; it is simply

a post, if used, which transfers the load brought by the floor system to its cap at U_0.

The member U_0U_1 is also not a member of the truss. It is a stringer between U_0 and U_1.

In Table I are given the values of all the stresses as determined:

Member.	Fixed-load Stresses in Pounds.	Moving-load Stresses in Pounds.	Member.	Fixed-load Stresses in Pounds.	Moving-load Stresses in Pounds.
$U_1U_2 = -L_0L_1$	−17,600	− 81,500	U_1L_1	+19,800	+ 82,200
$U_2U_3 = -L_1L_2$	−29,400	−130,500	U_2L_1	−20,500	−100,000
$U_3U_4 = -L_2L_3 = -L_3L_4$	−35,300	−150,300	U_2L_2	+11,400	+ 53,900
L_0U_1	−30,700	−142,000	U_3L_2	−10,300	− 65,600
			U_3L_3	+ 3,000	+ 32,700
			U_2L_3	.....	− 18,500
			U_3L_4	.....	− 39,100

If the truss becomes a through one (i.e., with the load on the lower chord), *the same pairs of web members do not intersect in the unloaded chord;* hence a different position of the moving load must be taken for the greatest shears in half the braces. A simple inspection of the diagram will show at once that the position of the moving load for the greatest shears in the oblique or compression braces must be the same whether that load traverses the upper or lower chord. For the vertical braces, however, the moving load must be advanced in the lower chord at least one panel (more in some cases) beyond its position on the upper. Hence the vertical-brace stresses will be greater in a through-truss than those found with the moving load on the upper chord, while the stresses in the oblique braces remain the same, showing that this type of truss is better adapted to the deck than the through form.

The following conclusions may be drawn from the preceding analyses:

The dead-load shear in any main web member is equal to the total dead load between the center of the span and the

member itself. This applies, however, only to trusses symmetrical about a vertical line at mid-span. If the number of panels in the span is odd, the shear in the inclined web members, or diagonals of the center panel, for a Howe or a Pratt truss will be zero. If the number of panels is even, the shear in each of the diagonals running from the center panel-point will be one half of an upper- and lower-chord panel load.

Art. 77. Stresses in a Six-panel, Single-track, Through Railroad Bridge, Web Members all Inclined.

The case taken will be that shown in Fig. 19. The span l is 120 feet; the depth d, 20 feet; the panel length p, 20 feet; the angle α between any web member and a vertical, 26° 40′. The moving load will be Cooper's E 40, two locomotives followed by a uniform train.

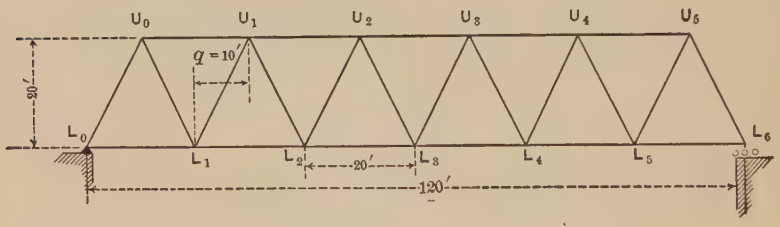


FIG. 19.

The weight per linear foot, w, of the two trusses and included bracing may be estimated by the formula $w = al$, a being an empirical constant here assumed at 6. The upper- and lower-chord panel weight per truss will then be $\frac{6 \times 120 \times 20}{2 \times 2} = 3{,}600$ pounds. The lower-chord panel-points carry also the weight of the floor system taken at 500 pounds, and the weight of ties, rails, guard-timbers, and fastenings, taken at 400 pounds per linear foot

of span. The lower-chord fixed panel weights are therefore

$$3{,}600 + \frac{900 \times 20}{2} = 12{,}600 \text{ pounds.}$$

$$\tan \alpha = 0.50,$$
$$\sec \alpha = 1.119.$$

The following tabulation will indicate sufficiently the methods of determining the dead-load shears and stresses.

Since the truss is symmetrical, the shear in any web member will be the load between that member and the center. Hence:

For Fixed Loads.

Shear in $U_2L_3 = 6{,}300$,
$\therefore$ Stress $= 6{,}300 \times 1.119 = +\ 7{,}100$ lbs.

Shear in $U_2L_2 = 6{,}300 + 3{,}600 = 9{,}900$,
$\therefore$ Stress $= 9{,}900 \times 1.119 = -11{,}000$ lbs.

Shear in $U_1L_2 = 18{,}900 + 3{,}600 = 22{,}500$,
$\therefore$ Stress $= 22{,}500 \times 1.119 = +25{,}200$ lbs.

Shear in $U_1L_1 = 18{,}900 + 7{,}200 = 26{,}100$,
$\therefore$ Stress $= 26{,}100 \times 1.119 = -\ 29{,}200$ lbs.

Shear in $U_0L_1 = 31{,}500 + 7{,}200 = 38{,}700$,
$\therefore$ Stress $= 38{,}700 \times 1.119 = +43{,}300$ lbs.

Shear in $U_0L_0 = 31{,}500 + 10{,}800 = 42{,}300$,
$\therefore$ Stress $= 42{,}300 \times 1.119 = -47{,}400$ lbs.

The stress in L_0L_1 is the product of the shear in the end panel by $\tan \alpha$;

$$\therefore\ L_0L_1 = 42{,}300 \times 0.50 = +21{,}200 \text{ pounds.}$$

By the method of chord increments the following stresses may then be found:

$$L_1L_2 = 21{,}200 + (38{,}700 + 26{,}100)0.50 = +53{,}600 \text{ lbs.},$$
$$L_2L_3 = 53{,}600 + (22{,}500 + 9{,}900)0.50 = +69{,}800 \text{ lbs.}$$

The last stress may be checked by the method of moments. The moment of the external forces acting on the left of a section passed through U_2U_3, U_2L_2, and L_2L_3, about U_2, is

$$42{,}300 \times 50 - 12{,}600(30 + 10) - 3{,}600(40 + 20)$$
$$= 1{,}395{,}000 \text{ ft.-lbs.};$$

the lever-arm of L_2L_3 is 20 feet, and therefore its stress = 69,750 pounds, which is a satisfactory check.

The stresses in the upper-chord members may also be found by the method of increments:

$$U_0U_1 = -(42{,}300 + 38{,}700)5.0 = -40{,}500 \text{ lbs.}$$
$$U_1U_2 = -[40{,}500 + (26{,}100 + 22{,}500).5] = -64{,}800 \text{ ''}$$
$$U_2U_3 = -[64{,}800 + (9{,}900 + 6{,}300).5] = -72{,}900 \text{ ''}$$

Live-load Stresses.

Since there are six panels, the criterion for the maximum web stresses requires one sixth of all the load on the bridge to be in the panel in question. The following tabulation shows the required positions of the loading on the span, W_n representing the last wheel on the bridge, and x its distance from L_6. If a uniform load should be on the bridge, its length is denoted by x. There is also given the reaction at L_0, and the downward force acting at the panel-point in advance of he panel treated. In these computations the moment tabulation given on page 94 has been employed. It is to be observed that the two braces meeting in the unloaded chord obtain their maximum

shears for the same position of loading, and since sec α is the same for all members, the stresses in each pair of those braces will be equal in amount but opposite in kind.

For Moving Load.

$L_0U_0 = -U_0L_1 \ldots\ldots W_3$ at L_1 $\quad W_n = \ldots$ $\quad x = 4$ feet;
$L_1U_1 = -U_1L_2 \ldots\ldots W_3$ " L_2 $\quad W_n = W_{16}$ $\quad x = 0$ "
$L_2U_2 = -U_2L_3 \ldots\ldots W_3$ " L_3 $\quad W_n = W_{12}$ $\quad x = 4$ "
$L_3U_3 = -U_3L_4 \ldots\ldots W_2$ " L_4 $\quad W_n = W_9$ $\quad x = 5$ "
$L_4U_4 = -U_4L_5 \ldots\ldots W_2$ " L_5 $\quad W_n = W_5$ $\quad x = 5$ "

Stress for L_0U_0

$$= \left[\tfrac{1}{120}(14{,}944{,}000 + 284{,}000 \times 9 + \tfrac{2{,}600 \times (4)^2}{2}) - \tfrac{1}{20}(230{,}000)\right] 1.119$$
$$= -151{,}000 \text{ lbs.}$$

Stress for L_1U_1

$$= \left[\tfrac{1}{120}(12{,}041{,}000) - \tfrac{1}{20}(130{,}000)\right] 1.119 = -110{,}000 \text{ lbs.}$$

Stress for L_2U_2

$$= \left[\tfrac{1}{120}(6{,}708{,}000 + 192{,}000 \times 4) - \tfrac{1}{20}(130{,}000)\right] 1.119 = -68{,}400 \text{ lbs.}$$

Stress for L_3U_3

$$= \left[\tfrac{1}{120}(3{,}496{,}000 + 142{,}000 \times 5) - \tfrac{1}{20}(80{,}000)\right] 1.119 = -34{,}800 \text{ lbs.}$$

Stress for L_4U_4

$$= \left[\tfrac{1}{120}(830{,}000 + 90{,}000 \times 5) - \tfrac{1}{20}(80{,}000)\right] 1.119 = -\ 6{,}000 \text{ lbs.}$$

The live-load stress in U_4L_4, symmetrical with U_1L_2, is -6000, while the dead-load stress is $+25{,}200$ pounds; hence this member need not be counterbraced.

The member U_3L_3 must, however, be so built as to carry a live-load stress of $-34{,}800$ and a dead-load stress of $+7{,}100$ pounds; it is therefore said to be counterbraced. If U_3L_3 is to resist tension only, a member must be introduced between the points U_2 and L_4 to take the moving-

load shear in tension. In order to provide for the movement of the load in the opposite direction, a member would also have to be introduced between U_3 and L_2. These members would be *counterbraces*.

It is now clear that the extent to which a main web member must be counterbraced is found by taking the excess of the moving-load stress over that caused by the fixed load. At first sight it would appear that the same method should hold in determining counterbrace stresses, as in the cases of the preceding articles, since it may be supposed that the main brace will carry shear until its fixed-load stress is neutralized. This presupposes, however, that there is such an exact adjustment of members that each will perform just the amount of duty assigned to it. As that is an end never completely realized, it is only prudent to suppose that *the counterbrace takes all the moving-load shear.* This procedure appears the more advisable when one reflects that counterbraces are subject to the greatest fatigue of all truss members, and that the amount of metal concerned is small.

The moving-load chord stresses remain to be found, and it will be necessary to resort to the methods of Art. 72, Cases I and II, in order to determine the proper positions of loading of the stresses in the various panels.

Since none of the web members is vertical, the positions of the moving load for the greatest stresses in the lower-chord panels will be given by eq. (4) of Art. 72. For this case q in that equation will be one half the panel length.

It will be found necessary to bring 4 and 3 feet, respectively, of the uniform load on the bridge for the stresses in the lower-chord panels L_1L_2 and L_2L_3. The condition of greatest stress in the lower-chord end panel L_0L_1 coincides with that for the end post, and it has already been found. The criterion will be satisfied in the case of L_1L_2 when wheel No. 3 rests at L_1, and for L_2L_3 when wheel No. 6

rests at L_2. The reactions at L_0 will be found to be respectively 134,100 and 130,900 pounds.

For L_1L_2 the moment at U_1 is then found to be

$$M = 134{,}100 \times 30 - 230{,}000 - 50{,}000 \times 10 - \left(\frac{460{,}000 + 53{,}000 \times 1}{20}\right) 10 = 3{,}037{,}000 \text{ ft.-lbs.}$$

For L_2L_3 the moment at U_2 is

$$M = 130{,}900 \times 50 - 1{,}640{,}000 - 103{,}000 \times 10 - \left(\frac{208{,}000 + 39{,}000 \times 4}{20}\right) 10 = 3{,}693{,}000 \text{ ft.-lbs.}$$

The negative quantities are always the negative moments caused by the downward forces acting at the panel-points between L_0 and the left end of the panel cut by the section.

The stresses are found by dividing by the depth of truss, 20 feet, i.e.,

Stress in $L_1L_2 = +151{,}800$ lbs.
" $L_2L_3 = +184{,}700$ "

The criterion for the maximum stress in the upper-chord members is eq. (1) of Art. 72, W_n' being the wheel placed at the center of moments, W_n the last wheel on the bridge, and x either the distance of W_n from the right abutment if there is no uniform load on the bridge or the length of the uniform load on the bridge.

For $U_0U_1 \ldots \frac{l'}{l} = \frac{1}{6}$; $W_n' = W_3$; $x = 4$ feet uniform load.
" $U_1U_2 \ldots \frac{l'}{l} = \frac{1}{3}$; $W_n' = W_6$; $x = 3$ " " "
" $U_2U_3 \ldots \frac{l'}{l} = \frac{1}{2}$; $W_n' = W_{10}$; $x = 7$ " " "

For these positions of loading the moments and stresses are, respectively,

M at $L_1 = 2{,}650{,}000$ ft.-lbs.; $\therefore\ U_0U_1 = -132{,}500$ lbs.
M " $L_2 = 3{,}625{,}000$ " ; $U_1U_2 = -181{,}300$ "
M " $L_3 = 3{,}834{,}000$ " ; $U_2U_3 = -191{,}700$ "

It is to be observed that the chord stresses found above may not be the absolute maxima, as, with concentrated loads, several positions may satisfy the criterion.

As a simple check on the moment at L_3, the moment at that point caused by the uniform load may be determined. This moment is expressed by $\frac{wl^2}{8}$, w being the intensity of 2000 pounds per linear foot, and l the span length in feet. In the present case its value is 3,600,000 foot-pounds. This serves to show that no large error in calculation has occurred.

The ranges of stress for which the members must be designed are shown in Table I.

TABLE I.

Member.	Fixed-load Stress in Pounds.	Moving-load Stress in Pounds.	Member.	Fixed-load Stress in Pounds.	Moving-load Stress in Pounds.
L_0L_1	+21,200		U_0L_1	+43,300	+151,000
L_1L_2	+53,600		L_1U_1	−29,200	−110,000
L_2L_3	+69,800		U_1L_2	+25,200	+110,000
L_0U_0	−47,400	−151,000	L_2U_2	−11,000	− 68,400 + 34,800
U_0U_1	−40,500		U_2L_3	+ 7,100	− 34,800 + 68,400
U_1U_2	−64,800				
U_2U_3	−72,900				

Art. 78. Stresses in a Double-intersection Pratt or Murphy-Whipple Truss.

The style of truss shown in Fig. 20, next to be treated, was at one time, in the early days of iron-bridge building, more common than any other in American practice. It

is now rarely if ever used, although many trusses of this type are still standing. Its treatment will illuminate some features of the double- or multiple-intersection systems.

It is preferable to arrange this type of truss with an even number of panels, as the stress ambiguity is then reduced to an unimportant matter. In order to show the extent to which ambiguity may arise, an odd number of panels has been selected in the present example. As is evident, the criteria of the preceding articles for web

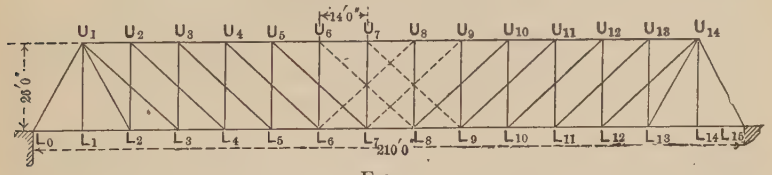


FIG. 20.

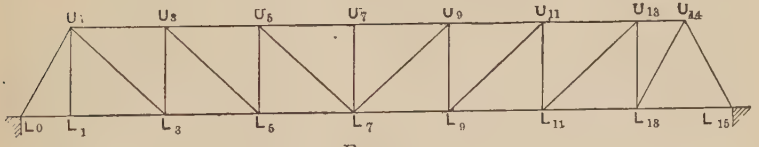


FIG. 21.

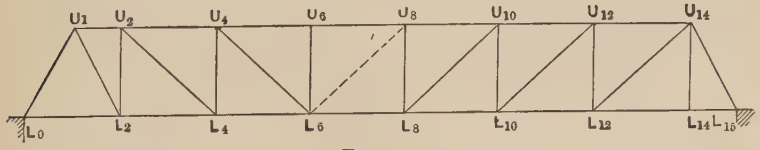


FIG. 22.

and chord stresses cannot be applied to multiple systems of triangulation. It is only possible to assume that each system acts as an independent truss, and to determine by trial the greatest possible concentrations at the head of the train in one system, and consider such concentrations the head of the moving load in that system.

In this class of structure, the use of wheel-load concentrations would complicate the problem excessively. Those concentrations may therefore be replaced by equivalent

concentrations at panel-points. In the problem under consideration the locomotives will be replaced by four panel loads, each of 40,000 pounds and represented by w'. The panel loads caused by the uniform load, represented by w, will be taken at 26,000 pounds. Only full panel loads will be considered, since the treatment to be employed is at best approximate and the use of full panel loads leads only to small errors on the side of safety. Ordinarily no two concentrations will be exactly equal, but the assumption is sufficiently accurate.*

The truss under consideration is composed of two systems of right-angled triangulation, shown in Figs. 21 and 22.

Before passing to the computations it is well to observe that although the action of the loads in one system may be considered as taking place independently of the actions of the loads in the other, at the same time equal loads symmetrically placed in reference to the center, though resting on different systems of triangulation, may be considered counterbalanced, i.e., they cause no shear in the web members between them. The web stresses due to the fixed load will be determined on the supposition that the web members shown by the dotted lines do not exist.

The data to be used are given below:

Span = 210 feet. Depth of truss = 26 feet.
Number of panels = 15. Panel length = 14 ''
Fixed load for upper-chord panel-points per truss = 9,100 lbs. = W (650 lbs. per foot).
Fixed load for lower-chord panel-points per truss = 14,000 lbs. = W' (1,000 lbs. per foot).
w = 26,000 pounds per panel per truss.

---

* The reader is referred to the Authors' ''Influence Lines,'' Chapter II, Art. 24, for the treatment involving partial panel loads.

$w' = 40{,}000$ pounds per panel per truss.

$e = w' - w = 14{,}000$ lbs.

Angle $L_{12}U_{14}L_{14} = \alpha$		Angle $L_{15}U_{14}L_{14} = \beta$	
$\tan \alpha$	$= 1.077$	$\sec \alpha$	$= 1.47$
$\tan \beta$	$= 0.538$	$\sec \beta$	$= 1.136$

The excess e will be employed in determining both the web and chord stresses.

The counterbrace L_6U_8 is the first one needed. Carrying the moving load on the bridge from left to right, panel by panel, the greatest web stresses are found to be the following:

In $L_6U_8 \ldots \frac{12}{15}w \sec \alpha + \frac{10}{15}e \sec \alpha$	$= +\ 44{,}300$ lbs.
" $L_7U_9 \ldots \frac{16}{15}w \sec \alpha + \frac{12}{15}e \sec \alpha$	$= +\ 57{,}240$ "
" $L_8U_8 \ldots \frac{12}{15}w + \frac{10}{15}e + W$	$= -\ 39{,}240$ "
" $L_8U_{10} \ldots (\frac{20}{15}w + \frac{14}{15}e)\sec \alpha + (W + W')\sec \alpha$	$= +104{,}120$ "
" $L_9U_9 \ldots \frac{16}{15}w + \frac{12}{15}e + W$	$= -\ 48{,}040$ "
" $L_9U_{11} \ldots (\frac{25}{15}w + \frac{16}{15}e + W + W')\sec \alpha$	$= +119{,}600$ "
" $L_{10}U_{10} \ldots \frac{20}{15}w + \frac{14}{15}e + 2W + W'$	$= -\ 79{,}940$ "
" $L_{10}U_{12} \ldots \{\frac{30}{15}w + \frac{18}{15}e + 2(W + W')\}\sec \alpha$	$= +169{,}060$ "
" $L_{11}U_{11} \ldots \frac{25}{15}w + \frac{16}{15}e + 2W + W'$	$= -\ 90{,}460$ "
" $L_{11}U_{13} \ldots \{\frac{36}{15}w + \frac{20}{15}e + 2(W + W')\}\sec \alpha$	$= +187{,}080$ "
" $L_{12}U_{12} \ldots \frac{30}{15}w + \frac{18}{15}e + 3W + 2W'$	$= -124{,}100$ "
" $L_{12}U_{14} \ldots \{\frac{42}{15}w + \frac{22}{15}e + 3(W + W')\}\sec \alpha$	$= +239{,}060$ "
" $L_{13}U_{13} \ldots \frac{36}{15}w + \frac{20}{15}e + 3W + 2W'$	$= -136{,}360$ "
" $L_{13}U_{14} \ldots \{\frac{49}{15}w + \frac{24}{15}e + 3(W + W')\}\sec \beta$	$= +200{,}660$ "
" $L_{14}U_{14} \ldots w' + W'$	$= +\ 54{,}000$ "
" $L_{15}U_{14} \ldots 7(w + W + W')\sec \beta + \frac{50}{15}e \sec \beta$	$= -443{,}460$ "

The stresses in each system of triangulation are found by virtually taking that system as a single truss supporting only the weights at the apices belonging to it.

The greatest chord stresses will be obtained by supposing the train to cover the entire bridge, with the four excesses e at panel-points L_1, L_2, L_3, and L_4.

Greatest stress in $U_1U_2 = 3(w+W+W')\,[2\tan\beta+(\tan\beta+\tan\alpha)]+(w+W+W')\tan\beta+e(4\tan\beta+\tan\alpha)+\frac{1}{3}e\,2\tan\beta = -473{,}000$ pounds.

Here it should be explained that since $\frac{50}{15}e = 3\frac{1}{3}e$ is found in the reaction at L_0, the three e's at panel-points L_1, L_2, and L_3, and $\frac{1}{3}$ of that at L_4 may be taken as passing directly to L_0, while $\frac{2}{3}$ of the e at L_4 passes to L_{15} through L_4U_4, U_4L_6, L_6U_8, U_8L_8, L_8U_{10}, $U_{10}L_{10}$, etc. Counterbrace L_6U_8 thus comes into action.

Greatest stress in $U_2U_3 = [2(w+W+W')+\frac{1}{3}e]\tan\alpha$
$+473{,}000 = -583{,}820$ lbs.

" " " $U_3U_4 = 2(w+W+W')\tan\alpha$
$+583{,}820 = -689{,}580$ lbs.

" " " $U_4U_5 = [(w+W+W')-\frac{2}{3}e]\tan\alpha$
$+689{,}580 = -732{,}400$ lbs.

" " " $U_5U_6 = (w+W+W')\tan\alpha$
$+732{,}400 = -785{,}280$ lbs.

The panel stresses in U_5U_6, U_6U_7, and U_7U_8 will be the same; and if the loading were uniform over the whole bridge, the panels U_5U_6, U_6U_7, U_7U_8, U_8U_9, and U_9U_{10} would all be subjected to the same stress.

Greatest stress in L_0L_1 and $L_1L_2 = [7(w+W+W')+3\frac{1}{3}e]$
$\tan\beta = +210{,}000$ lbs.

" " " $L_2L_3 = [3(w+W+W')+1\frac{1}{3}e]\tan\beta$
$+210{,}000 = 299{,}300$ lbs.

" " " $L_3L_4 = [3(w+W+W')+e]\tan\alpha$
$+299{,}300 = +473{,}020$ lbs.

" " " $L_4L_5 = [2(w+W+W')+\frac{1}{3}e]\tan\alpha$
$+473{,}020 = +583{,}820$ lbs.

Greatest stress in $L_5L_6 = 2(w+W+W')\tan\alpha$
$+583{,}820 = +689{,}580$ lbs.

" " " $L_6L_7 = (w+W+W'-\frac{4}{3}e)\tan\alpha$
$+689{,}580 = +722{,}340$ lbs.

" " " $L_7L_8 = (w+W+W')\tan\alpha$
$+722{,}340 = +775{,}220$ lbs.

It is to be noticed that diagonally opposite panels in the upper and lower chords, up to the counterbrace L_6U_8, beginning with the panels U_1U_2 and L_3L_4, are subjected to the same amounts of stress, but of opposite kinds.

If the loading were uniform over the whole bridge, this equality in pairs would continue to the center; also, the stresses in the panels U_5U_6, U_6U_7, U_7U_8, U_8U_9, U_9U_{10}, and L_7L_8 would be equal to each other.

If the end posts were vertical, there would be obvious changes in the stresses of the panels L_0L_1, L_1L_2, and L_2L_3 (that in L_0L_1 would be zero). The upper-chord panel stresses would not be changed.

The whole truss in Fig. 20 is composed of the two systems of triangulation shown in Figs. 21 and 22, and each of these is to be considered separately in checking the chord stresses by the method of moments. Denote by (R') and (R'') the reactions at the points L_0 indicated by the same letters in Figs. 21 and 22, then divide the total load supported by each system into two parts, *according to the principle of the lever*, and there will result:

$$(R') = \tfrac{16}{30}[7(w+W+W')] + \tfrac{13}{15}\cdot 2e = 207{,}570 \text{ lbs.}$$
$$(R'') = \tfrac{14}{30}[7(w+W+W')] + \tfrac{12}{15}\cdot 2e = 182{,}790 \text{ "}$$

If the diagonals are in tension, according to this value of (R''), U_6L_8 should be drawn, and not L_6U_8. The latter is taken, however, for a reason that will appear presently.

The sum of (R') and (R'') is just equal to the total reaction at L_0 in Fig. 20, as it ought to be.

If the stresses in the panels are represented by their letters, and if moments are taken about L_5 and L_3 of Fig. 21 respectively, there result

$$U_3U_5 = -L_5L_7$$
$$= [(R' \times L_0L_5) - 2(w' + W + W')(L_3L_5 + \tfrac{1}{2}L_1L_3)] \div d.$$
$$U_1U_3 = -L_3L_5 = [(R' \times L_0L_3) - (w' + W + W')L_1L_3] \div d.$$

Also taking moments about L_6 and L_4 (Fig. 22):

$$U_4U_6 = [(R'' \times L_0L_6) - 2(w' + W + W')(L_4L_6 + \tfrac{1}{2}L_2L_4)] \div d.$$
$$U_2U_4 = -L_4L_6 = [(R'' \times L_0L_4) - (w' + W + W')L_2L_4)] \div d.$$

Similar expressions will give the chord stress in every panel of Figs. 21 and 22, and having found these, the resultant stresses in Fig. 20 are simply the sums of the proper pairs taken from Figs. 21 and 22.

Thus,
$$U_4U_5 = U_3U_5 + U_4U_6,$$
$$U_5U_6 = U_5U_7 + U_4U_6,$$
$$L_3L_4 = L_3L_5 + L_2L_4,$$
etc.

This system of determination by moments may be applied to any truss with parallel chords, however many systems of triangulation there may be.

The method also applies to any irregular loading, for the stresses due to each panel load may be found separately, and the sum caused by all taken.

Web stresses may also be checked by the same method, since the increment of chord stress at any panel-point is equal to the sum of the horizontal components of the stresses in the web members intersecting at the panel-point in question. Such a check, however, is a tedious one.

Applying the above equations to U_4U_6 in Fig. 22:

$$U_4U_6 = (182{,}790 \times 84 - 2 \times 63{,}100 \times 42) \div 26 = 386{,}700 \text{ lbs.}$$

Also, to U_5U_7, in Fig. 21:

$$(U_5U_7) = (207{,}570 \times 98 - 2 \times 63{,}100 \times 70 - 49{,}100 \times 28) \div 26$$
$$= 389{,}900 \text{ lbs.}$$

But the sum of these two is 776,600 pounds, whereas $U_5U_6 = 785{,}280$ pounds. This discrepancy, not very great, is easily explained. The loading $(w+W+W')$ is counterbalanced in Fig. 20, but is not in Figs. 21 and 22.

In Fig. 20 all the load on the left of the center of the span, except $\frac{2}{3}e$ at L_4, is assumed to pass directly to L_0. Hence in Figs. 21 and 22, to be consistent with Fig. 20, there should be taken:

$$(R') = 4(w+W+W') + 2e = 224{,}400 \text{ lbs.}$$
$$(R'') = 3(w+W+W') + \tfrac{4}{3}e = 165{,}970 \text{ "}$$

Introducing these in the general formula,

$$U_5U_6 = U_4U_6 + U_5U_7 = [224{,}400 \times 98 + 165{,}970 \times 84$$
$$- 63{,}100 \times 84 - 63{,}100 \times 140 - 49{,}100 \times 28] \div 26 = 785{,}500 \text{ lbs.}$$

This result agrees sufficiently well with that obtained by the trigonometrical method.

With the last value of (R''), L_6U_8 will be in tension.

It is thus seen that with an uneven number of panels a little ambiguity exists both in reference to the greatest chord stresses and the greatest web stresses, when there are two systems of triangulation. *This ambiguity always exists, whatever the number of systems, if the component systems are not symmetrical in reference to the center line of the span, and it always disappears if they are symmetrical in reference to that line.*

With an even number of panels in the span and two systems of triangulation no ambiguity exists.

These observations in reference to ambiguity apply as well to isosceles bracing as to vertical and diagonal.

In the example taken there are only two systems of triangulation, but precisely the same method is to be followed whatever the number. In determining the web stresses, each system is supposed to carry those moving weights only which rest at its apices, and the same is true

in reference to chord stresses for unsymmetrical loading, uniform loading being supposed counterbalanced for either stresses.

The slight changes to be made for an overhead bridge, or for verticals in tension and diagonals in compression, are evident from what has already been given in preceding articles.

It is seen that any two web members intersecting in the chord not traversed by the moving load receive their greatest stresses at the same time. The principle is a general one.

When built in iron, this truss is frequently called the Linville truss.

Art. 79. Stresses in a Double-intersection Truss with Uniform Diagonal Bracing.

This truss is shown in Fig. 23, and, although taken here as an ordinary pin-connection bridge, precisely the same method of calculation is to be used for a "lattice" truss with riveted connections. The usual numbering of the panel-points will be replaced by that shown.

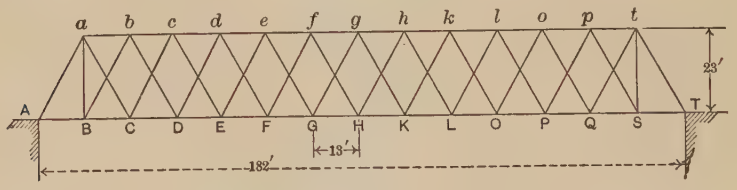


FIG. 23.

No locomotive excess will be taken, but a heavy moving load of uniform density will be assumed. The following are the data:

Span	= 182 feet.	Depth	= 23 feet.
Panel length	= 13 "	Number of panels	= 14.

Fixed load:

W (upper) = 450 lbs. per ft. = 5,850 lbs. per panel per truss.
W' (lower) = 800 " " " = 10,400 " " " " " "

Moving load:

w = 2800 lbs. per foot = 36,400 lbs. per panel per truss.
Angle $AaB = \alpha$.

$\tan \alpha$	= 0.565.	$\sec \alpha$	= 1.15.
$W \sec \alpha$	= 6,730 lbs.	$W' \sec \alpha$	= 11,960 lbs.
$\frac{w}{14} \sec \alpha$	= 3,000 "	$W \tan \alpha$	= 3,310 "
$W' \tan \alpha$	= 5,880 "	$w \tan \alpha$	= 20,560 "

The vertical members aB and tS are for tension only.

The moving load will be taken as passing from A to T, and its head will be supposed to rest at the various panel-points in succession, in the determination of the web stresses.

The stress in any member is indicated by inclosing in a parenthesis the letters which belong to it in the figure.

Head of Moving Load at D.

$$(dF) = \{\tfrac{3}{2}W' + W - \tfrac{4}{14}w\} \sec \alpha = +(11{,}000 \times \sec \alpha) \text{ lbs.}$$

Hence the stress in dF will always be tension, and this member need not be counterbraced.

Head of Moving Load at E.

$$(Ee) = \{\tfrac{6}{14}w - W' - \tfrac{3}{2}W\} \sec \alpha = -(3580 \text{ lbs.} \times \sec \alpha) \text{ lbs.}$$

Hence the stress in Ee will always be compression, and this member need not be counterbraced.

The web stresses desired are, then, the following:

$$(Ff) = -(\tfrac{1}{2}W' + W - \tfrac{9}{14}w) \sec \alpha = +\ 14{,}300 \text{ lbs.}$$
$$(eG) = (W' + \tfrac{1}{2}W - \tfrac{6}{14}w) \sec \alpha = -\ 2{,}680 \text{ "}$$

$(Gg) = -(\quad \tfrac{1}{2}W - \tfrac{12}{14}w) \sec\alpha = +\ 32{,}640$ lbs.

$(fH) = (\ \tfrac{1}{2}W' \quad - \tfrac{9}{14}w) \sec\alpha = -\ 21{,}020$ "

$(Hh) = (\ \tfrac{1}{2}W' \quad + \tfrac{16}{14}w) \sec\alpha = +\ 53{,}980$ "

$(gK) = -(\quad \tfrac{1}{2}W + \tfrac{12}{14}w) \sec\alpha = -\ 39{,}360$ "

$(Kk) = (\ W' + \tfrac{1}{2}W + \tfrac{20}{14}w) \sec\alpha = +\ 75{,}320$ "

$(hL) = -(\ \tfrac{1}{2}W' + W + \tfrac{16}{14}w) \sec\alpha = -\ 60{,}700$ "

$(Ll) = (1\tfrac{1}{2}W' + W + \tfrac{25}{14}w) \sec\alpha = +\ 99{,}660$ "

$(kO) = -(\ W' + 1\tfrac{1}{2}W + \tfrac{20}{14}w) \sec\alpha = -\ 82{,}040$ "

$(Oo) = (\ 2W' + 1\tfrac{1}{2}W + \tfrac{30}{14}w) \sec\alpha = +124{,}000$ "

$(lP) = -(1\tfrac{1}{2}W' + 2W + \tfrac{25}{14}w) \sec\alpha = -106{,}400$ "

$(Pp) = (2\tfrac{1}{2}W' + 2W + \tfrac{36}{14}w) \sec\alpha = +151{,}360$ "

$(oQ) = -(\ 2W' + 2\tfrac{1}{2}W + \tfrac{30}{14}w) \sec\alpha = -130{,}740$ "

$(Qt) = (\ 3W' + 2\tfrac{1}{2}W + \tfrac{42}{14}w) \sec\alpha = +178{,}700$ "

$(pS) = -(2\tfrac{1}{2}W' + 3W + \tfrac{36}{14}w) \sec\alpha = -158{,}080$ "

$(tT) = -\tfrac{13}{2}(W' + W + w) \sec\alpha = -394{,}480$ "

$(tS) = 3\tfrac{1}{2}W' + 3W + \tfrac{49}{14}w = +181{,}360$ "

With the moving load covering the whole bridge, the following chord stresses are found:

$(ab) = -(9\tfrac{1}{2}W' + 9W + 9\tfrac{1}{2}w) \tan\alpha = -282{,}920$ lbs.

$(bc) = (ab) - 5(W' + W + w) \tan\alpha = -431{,}660$ "

$(cd) = (bc) - 4(W' + W + w) \tan\alpha = -550{,}660$ "

$(de) = (cd) - 3(W' + W + w) \tan\alpha = -639{,}990$ "

$(ef) = (de) - 2(W' + W + w) \tan\alpha = -699{,}400$ "

$(fg) = (ef) - (W' + W + w) \tan\alpha = -729{,}140$ "

$(AB) = 6\tfrac{1}{2}(W' + W + w) \tan\alpha = +193{,}360$ "

$(BC) = (AB) + (2\tfrac{1}{2}W' + 3W + 2\tfrac{1}{2}w) \tan\alpha = +269{,}380$ "

$(CD) = (BC) + 5(W' + W + w) \tan\alpha = +418{,}120$ "

$(DE) = (CD) + 4(W' + W + w) \tan\alpha = +537{,}100$ "

$(EF) = (DE) + 3(W' + W + w) \tan\alpha = +626{,}340$ "

$(FG) = (EF) + 2(W' + W + w) \tan\alpha = +685{,}840$ "

$(GH) = (FG) + (W' + W + w) \tan\alpha = +715{,}600$ "

The following operations constitute a check on the accuracy of the chord stresses.

The horizontal forces exerted at the joints g and H, respectively, are:

$$(fg) \quad -\tfrac{1}{2}W \tan \alpha \qquad = -730{,}800 \text{ lbs.}$$
$$\text{and} \quad (GH) + \tfrac{1}{2}(W' + w) \tan \alpha = +728{,}820 \text{ ''}$$

The horizontal force exerted at either one of these joints, as found by the moment method, is:

$$\frac{7(W' + W + w) \times 0.25 \times 182}{23} = 729{,}000 \text{ lbs.}$$

The agreement is close.

It is to be observed that (Ff) is the greatest tensile stress in hL, also; and, on the other hand, that (hL) is the greatest compression stress Ff. Similar observations apply to the pairs of members eG, kK; Gg, Kg; fH, Hh.

These, consequently, are the only web members which need to be counterbraced.

Precisely the same methods of calculation apply whatever may be the number of systems of triangulation or the character of the load, or whether the truss be a through or a deck one.

If Fig. 23 represented a deck truss, however, the compressive web stresses would be increased and the tensile ones diminished, while the chord stresses would remain the same. Since the increase of compression in any web member would numerically exceed the decrease of tension in the adjacent one, the truss is better adapted to a through load than a deck load.

Art. 80. Stresses in a Double-intersection Truss, with Uniform Diagonal Bracing, and with Subdivided Panels.

An economical truss, in point of quantity of material, is that shown in Fig. 24. The truss has the ordinary isosceles bracing formed of two systems of triangulation, but a half of the floor system and moving load is carried by verticals directly to the intersections E, F, etc.

Half the weight of the trusses is supported at the apices of the main systems, as H and M in the upper chord, and half at the apices, as P and R in the lower chord,

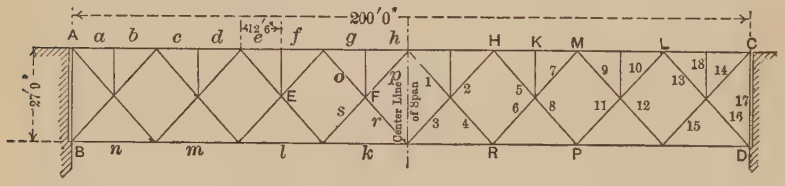


FIG. 24.

The truss chosen is a deck or overhead truss; consequently half the floor system and moving load will be supported by the verticals in compression. The weight of the floor system will be taken at 300 pounds per foot of truss and the moving load taken will be a uniform one weighing 2,700 pounds per foot of truss. Full panel live loads only will be considered. Since the live load is heavy, no excess e has been taken.

The following are the data:

Length of span = 200 ft.	Depth of truss = 27.75 ft.
Upper-panel length = 12.5 "	$\tan CDL = \tan \alpha =$ 0.9
Lower-panel length = 25.0 "	$\sec CDL = \sec \alpha =$ 1.345

Upper-chord fixed load, per main panel:

$$W = 25 \times 500 + 12\tfrac{1}{2} \times 300 = 16{,}250 \text{ lbs.}$$

Upper-chord fixed load, per intermediate panel:

$$W_1 = 12\tfrac{1}{2} \times 300 = 3{,}750 \text{ lbs.}$$

Lower-chord fixed load per panel:

$$W' = 25 \times 500 = 12{,}500 \text{ lbs.}$$
$$w = w' = 12\tfrac{1}{2} \times 2700 = 33{,}750 \text{ lbs}$$

The method of treatment is as before, that of treating each system of triangulation separately and for that load only which rests upon its own panel-points.

The intermediate loads W_1 or w may be considered applied to the trussing as follows: Fig. 25 represents a portion of the truss in question as indicated by the same letters *HMPR*. Any weight resting at K is carried down to the intersection, or two apices A and B, and the proper portion of each load is hung at each apex. In

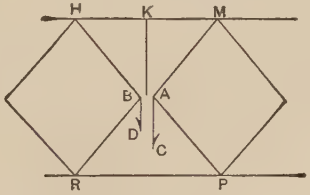


FIG. 25.

the truss in question, AC, in the adjoining figure, will represent $\frac{11}{16}$ of the weight at K, and BD $\frac{5}{16}$ of the same weight, since the point K divides the truss into two sections of $\frac{11}{16}$ and $\frac{5}{16}$ of the span respectively. Each of these parts is divided equally between the two systems of bracing. The moving load is supposed to pass on the bridge from left to right. By examination it is seen that o and s are the first members which need counterbracing. The head of the train must be at the panel-point between f and e for the greatest moving-load stress in s, and at the

panel-point between f and g for that in o, and at corresponding positions for other web members.

Fixed-load stress in $s = \frac{1}{2}(W + W_1) \sec \alpha = -12{,}460$ lbs.

Moving-load stress in $s = (1 + 3 + 8 + 5)\frac{w}{32} \sec \alpha$

$= +24{,}100$ lbs.

The composition of the numerical quantities in the brackets will at once be understood by observing that each live panel load at the upper end of a vertical strut, as between a and b, c and d, etc., is equally divided between the two systems of bracing. Each live panel load at a main panel point is evidently carried wholly by the system to which the point belongs. The moving-load shear in s is composed of $(\frac{1}{2} \times \frac{1}{16} = \frac{1}{32})w$ from panel-point a–b, $(\frac{1}{2} \times \frac{3}{16} = \frac{3}{32})w$ from panel-point c–d, $(\frac{4}{16} = \frac{8}{32})w$ from panel-point d–e, and $(\frac{1}{2} \times \frac{5}{16} = \frac{5}{32})w$ from panel-point e–f. Other shears are found in the same manner.

Fixed-load stress in $o = \frac{1}{2}(W' + W_1) \sec \alpha = +10{,}920$ lbs.

Moving-load stress in $o = (1 + 4 + 3 + 5 + 12)\frac{w}{32} \sec \alpha$

$= -35{,}460$ lbs.

Fixed-load stress in $p = \frac{1}{2}W \sec \alpha = -10{,}920$ lbs.

Moving-load stress in $p = (1 + 3 + 8 + 5 + 7)\frac{w}{32} \sec \alpha$

$= +34{,}040$ lbs.

Fixed-load stress in $r = \frac{1}{2}W' \sec \alpha = +8{,}400$ lbs.

Moving-load stress in $r = (1 + 4 + 3 + 5 + 12 + 7)\frac{w}{32} \sec \alpha$

$= -45{,}400$ lbs.

Greatest stress in $1 = (\frac{40}{32}w + \frac{1}{2}W) \sec \alpha = -37{,}640$ lbs.

" " " $2 = (\frac{41}{32}w + \frac{1}{2}W' + \frac{1}{2}W_1) \sec \alpha$

$= +69{,}060$ lbs.

" " " $3 = (\frac{32}{32}w + \frac{1}{2}W') \sec \alpha = +53{,}800$ lbs.

Greatest stress in $4 = (4\frac{9}{32}w + \frac{1}{2}W + \frac{1}{2}W_1)\sec\alpha$
$= -82{,}940$ lbs.

" " " $5 = (6\frac{1}{32}w + \frac{1}{2})W' + W_1(+W)\sec\alpha$
$= -119{,}280$ lbs.

" " " $8 = (7\frac{2}{32}w + \frac{1}{2})W' + 2W_1(+W)\sec\alpha$
$= -137{,}440$ lbs.

" " " $6 = (4\frac{9}{32}w + \frac{1}{2}W + W + \frac{1}{2}W_1)\sec\alpha$
$= +99{,}740$ lbs.

" " " $7 = (6\frac{0}{32}w + \frac{1}{2}W + W' + W_1)\sec\alpha$
$= +117{,}860$ lbs.

" " " $9 = (8\frac{4}{32}w + \frac{3}{2}W + W' + W_1)\sec\alpha$
$= -173{,}760$ lbs.

" " " $12 = (9\frac{7}{32}w + \frac{3}{2}W + W' + \frac{3}{2}W_1)\sec\alpha$
$= -194{,}760$ lbs.

" " " $11 = (7\frac{2}{32}w + \frac{3}{2}W' + W_1 + W)\sec\alpha$
$= +154{,}260$ lbs.

" " " $10 = (8\frac{5}{32}w + \frac{3}{2}W' + \frac{3}{2}W_1 + W)\sec\alpha$
$= +175{,}180$ lbs.

" " " $13 = (11\frac{3}{32}w + \frac{3}{2}W' + \frac{3}{2}W_1 + 2W)\sec\alpha$
$= -236{,}740$ lbs.

" " " $16 = (12\frac{8}{32}w + \frac{3}{2}W' + 2W_1 + 2W)\sec\alpha$
$= -260{,}600$ lbs.

" " " $15 = (9\frac{7}{32}w + \frac{3}{2}W + 2W' + \frac{3}{2}W_1)\sec\alpha$
$= +211{,}580$ lbs.

" " " $14 = (11\frac{2}{32}w + \frac{3}{2}W + 2W' + 2W_1)\sec\alpha$
$= +235{,}320$ lbs.

" " " $17 = 12\frac{8}{32}w + 2W + 2W' + 2W_1$
$= -200{,}000$ lbs.

" " " $18 = W + w = -50{,}000$ lbs.

The stress in 17 added to the vertical component of the stress in 16 is equal to $(8w + 4W + 4W_1 + 3\frac{1}{2}W')$ the weight of the truss and its load, as it should. This constitutes a check on the work if, as was done, each web stress is found by adding a proper increment to a preceding one.

For the greatest chord stresses the load will cover the whole bridge.

Stress in a or $b = (3\frac{1}{2}w + \frac{3}{2}W + 2W' + 2W_1) \tan \alpha$
$= -157{,}500$ lbs.

" " c or $d = (6w + 3W' + 3W_1 + 3W) \tan \alpha + 157{,}500$
$= -427{,}500$ lbs.

" " e or $f = 2(2w + W + W_1 + W') \tan \alpha + 427{,}500$
$= -607{,}500$ lbs.

" " g or $h = (2w + W + W_1 + W') \tan \alpha + 607{,}500$
$= -697{,}500$ lbs.

" " $n = (4w + 2W + 2W_1 + \frac{3}{2}W') \tan \alpha$
$= +174{,}380$ lbs.

" " $m = 3(2w + W + W_1 + W') \tan \alpha + 174{,}380$
$= +444{,}380$ lbs.

" " $l = 2(2w + W + W_1 + W') \tan \alpha + 444{,}380$
$= 624{,}380$ lbs.

" " $k = (2w + W + W_1 + W') \tan \alpha + 624{,}380$
$= +714{,}380$ lbs.

In determining these values, it is to be remembered that the increment of chord stress at any panel-point is equal to the sum of the horizontal components of the stresses in the web members intersecting at that point.

The results for g or h or k may be easily verified by the method of moments. Let l be the span in feet, and d the depth of a flanged beam, in feet also; then if w is the load per foot, the flange stress at the center, as is well known, will be $\frac{wl^2}{8d}$. To apply this to the present case, $(w + W + W_1 + W')$ must be written for w, and l and d have the values respectively of 200.00 and 27.75. Hence $\frac{wl^2}{8d} = 720{,}800$ lbs.

Since the resultant stress at either of the center joints is horizontal in direction for a uniform load from end to

end of the truss, the value corresponding to the above will be found by adding to the stress in h the horizontal component of the stress in brace 1, for the supposed uniform load; or by adding to that in panel k the horizontal component of that in brace 3.

$$\text{Horizontal component in } 1 = \left(\frac{w+W}{2}\right) \tan \alpha = 22{,}500 \text{ lbs.}$$

and

$$697{,}500 + 22{,}500 = 720{,}000 \text{ lbs.}$$

$$\text{Horizontal component in } 3 = \frac{W'}{2} \tan \alpha = 5{,}620 \text{ lbs.}$$

and

$$714{,}380 + 5620 = 720{,}000 \text{ lbs.}$$

Both of the above results are satisfactory verifications. They would have agreed exactly, but 0.9 is not the exact value of tan α.

If the bridge were a through one, the general method of calculation would be exactly the same; the slight changes in the details of the operations are sufficiently obvious after what has been said before.

As a through-truss there would be some saving of material, for the secondary verticals 18 would be in tension.

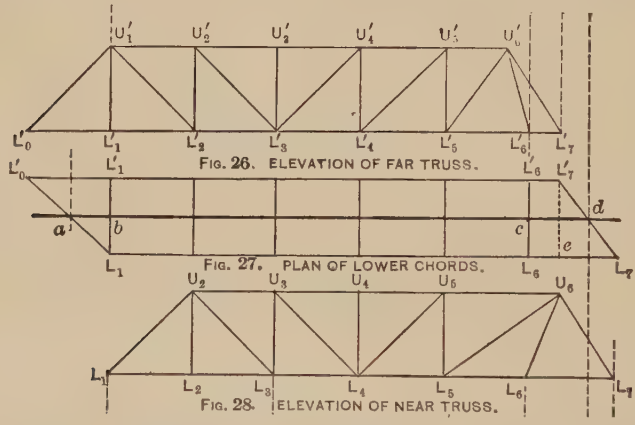
A much greater saving might be effected by using inclined end posts, in which case a short beam or girder would take the place of the end panels LC, and braces 15 would be vertical and run up to L, while braces 18, 14, and 17 would be omitted altogether.

Art. 81. Skew-bridges.

Skew-bridges are those in which the ends of the trusses do not lie in a line perpendicular to the axis of the bridge. The skew is the distance between the projections of the ends of the trusses measured on the axis of the bridge. This distance is not necessarily the same at the two ends.

In Fig. 27, which represents the plan of the lower-chord system of a skew-bridge, the skew at the left end of the bridge is a full panel length, or $L_0'L_1'$, while at the right end of the bridge it is $eL_7 = cd$. This example illustrates a skew-bridge in the most general form.

It is customary to place the floor-beams, supporting the floor system, at right angles to the trusses, as shown in the figure, and for convenience in the manufacture of the details of the portal bracing it is generally preferable to design the end posts of opposite trusses with the same



inclination. This result is attained by moving the upper end of the end post of the further truss, that is, U_6' in Fig. 26, half the skew distance back of the vertical projection of L_6', while U_6, Fig. 28, is moved half the skew distance in advance of the vertical projection of L_6. The members $U_6'L_7'$, Fig. 26, and U_6L_7, Fig. 28, will then be parallel. The upper-chord members $U_5'U_6'$ and U_5U_6 will not be equal in length, nor will $U_6'L_6'$ and U_6L_6 be either parallel or vertical; but this is usually preferable to having the end posts non-parallel.

At the left end of the bridge a simpler procedure may be followed, for the skew distance is a full panel length.

Should the skew distances at the two ends of the bridge be equal, the two trusses would be exactly alike except that they would be turned end for end.

The methods of the preceding articles apply in all respects to skew-bridges, so far as the general principles are concerned.

It will be sufficiently accurate in all cases to treat the moving load as if it were passing along the center line LL or $L'L'$ of each track, as shown in Figs. 29 and 30.

In the case of the single-track bridge, Fig. 29, if the load

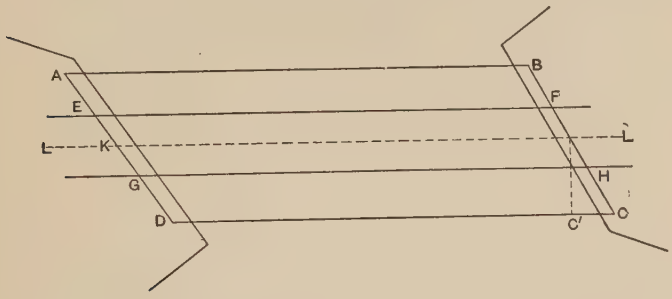


FIG. 29.

is passing from right to left, the moving load does not rest on the truss CD until it passes the point C', and continues to act on that truss *until it passes to a corresponding distance to the left of* D, it being borne in mind that all moving load is transferred to the trusses by transverse floor-beams placed normal to the axis of the bridge. It results from these considerations that if the load passes from right to left in Fig. 29 and along LL, the reactions at D will be greater than a half of those at K in the center line of the track by the amount of the half-products of the total load corresponding to those reactions multiplied by the ratio

$$\frac{CC'}{CD}.$$

Hence if R is any reaction at K for the clear span along the center line of the track and ΣW the total load, the corresponding reaction at D will be

$$\frac{R}{2}+\frac{1}{2}\frac{CC'}{CD}\Sigma W.$$

On the other hand, with the load moving in the same direction, the reaction at A will be

$$\frac{R}{2}-\frac{1}{2}\frac{CC'}{CD}\Sigma W.$$

In the equations of the preceding articles, the expression

$$(W_1+W_2+\ldots+W_n)(x\pm CC')$$

is then to be written in place of

$$(W_1+W_2+\ldots+W_n)x$$

according to the direction in which the train is moving; *but the negative portions of those equations are to remain unchanged.* It is to be remembered that the quantity x is to be measured on the center line.

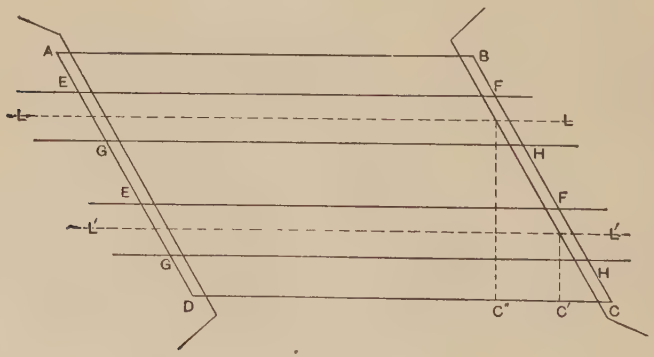


FIG. 30.

In the case of the double-track skew-bridge of Fig. 30, in which there are the two trusses AB and CD only, pre-

cisely the same observations hold. For one track, however, CC'' is to be used and CC' for the other. Separate computations are to be made for each track for each truss.

If there are three trusses in the double-track bridge in Fig. 30, each pair of trusses constitutes a single-track bridge for the track between, and is to be treated precisely as Fig. 29.

If the skew is so great that one or more floor-beams have their end or ends resting on the masonry, obvious modifications must be made according to the preceding general principles. The general method of influence lines * is, however, so much simpler than the analytical treatment, for the determination of stresses in a skew span, that no detailed illustration of the analytical method will be given.

Art. 82. Effect of Centrifugal Forces.

If the track on a railroad truss-bridge is on a curve, the center line of the track will no longer coincide with the center line between the trusses. If it be at first assumed that the outer rail is not elevated above the inner rail, and furthermore if the live load be assumed stationary, that is, with no centrifugal force acting, the moving load will no longer be equally divided between the two trusses. This is clearly shown by Fig. 31, which is the plan of the

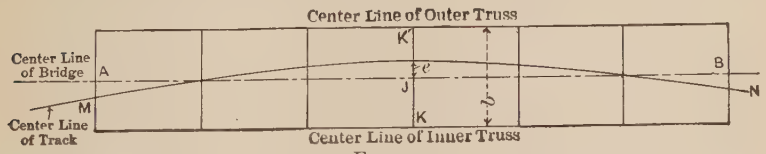


FIG. 31.

lower chord of a through-bridge. The center line of the truss is shown as AB, and the center line of the track as MN. The width between trusses is b, and at any panel

* See page 119 of the Authors' "Influence Lines."

point, such as KK, the eccentricity of the center line of the track is e. The distribution of a panel live load P, at J, to the panel-points K and K of the trusses is then found simply by the law of the lever. The outer truss will carry $\frac{P}{b}\left(\frac{b}{2}+e\right)$ and the inner truss $\frac{P}{b}\left(\frac{b}{2}-e\right)$. If, however, the outer rail be elevated above the inner by an amount

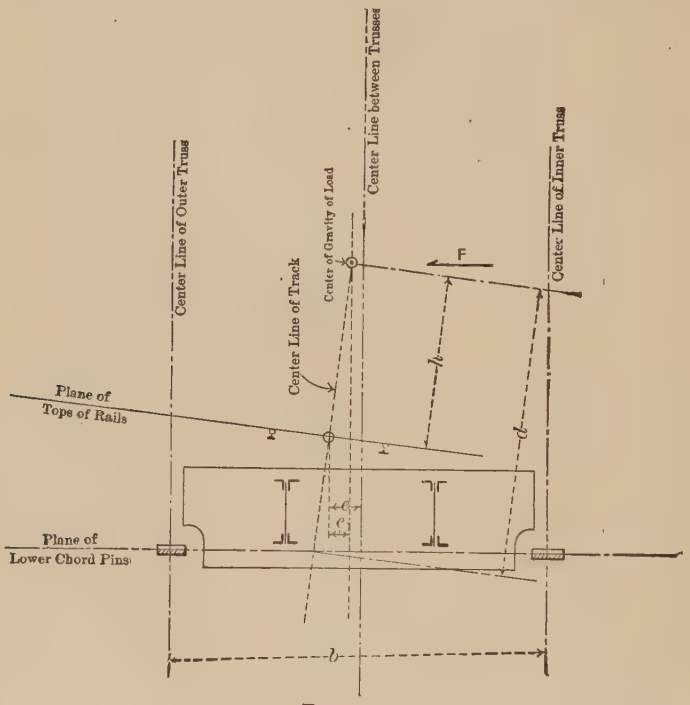


FIG. 32.

m, the center of gravity of the loading will not coincide with the center line of the track, but, as is shown in Fig. 32, it will fall toward the inner truss by an amount

$$e_1 = h \sin \alpha.$$

In this equation h is the distance of the center of gravity of the train above the plane of the tops of the rails, and α

is the angle of elevation of the rails, so that $\sin \alpha = \frac{m}{g}$, g being the gauge distance between the rails. Hence the panel load P will be divided between the two trusses in the ratios of

$$\left(\frac{b}{2} + e - h \sin \alpha\right)\frac{1}{b} \quad \text{and} \quad \left(\frac{b}{2} - e + h \sin \alpha\right)\frac{1}{b}.$$

If, finally, the train be assumed in motion so that a centrifugal force is caused to act horizontally in a radial direction at the center of gravity of the loading, an overturning effect as well as a direct thrust against the trusses will be caused. The amount of this centrifugal force, F, may be shown to be, by the laws of mechanics,

$$F = \frac{V^2}{32.2R},$$

where V = the velocity of the train in feet per second, R = the radius of curvature of the track in feet, and 32.2 is the acceleration due to gravity.

Let there be assumed two equal and opposite forces, F, acting in the plane of the pins of the lower chord and parallel to the line of action of the centrifugal force F; these forces affect in no way the equilibrium of the structure. The force, F, acting at the center of gravity may then be combined with one of the forces acting in the line of the pins so as to form a couple whose moment is $F \cdot d \cdot \cos \alpha$. The effect of this couple is to throw additional weight on the outer truss, and correspondingly to relieve the inner truss. There still remains, then, the one force, F, acting directly in the line of the lower-chord pins, to be resisted by the lower lateral bracing. Although the centrifugal force acts radially, its line of action may be taken normal to the axis of the truss without sensible error, and it will be so taken.

The couple $F \cdot d \cdot \cos \alpha$ is resisted by an equal and opposite couple caused by forces acting in the planes of the trusses with the lever-arm b, the value of the force of this latter couple being

$$\frac{F \cdot d \cdot \cos \alpha}{b},$$

since b is the distance between the trusses.

Combining finally all the effects caused by curvature, it will be found that, with a vertical panel load P, the load carried by the outer truss will be

$$\frac{P}{b}\left(\frac{b}{2} - h \sin \alpha + e + Fd \cos \alpha\right), \quad . \quad . \quad . \quad (1)$$

and that carried by the inner truss

$$\frac{P}{b}\left(\frac{b}{2} + h \sin \alpha - e - Fd \cos \alpha\right). \quad . \quad . \quad . \quad (2)$$

Eq. (2) will obtain its maximum value when the train is standing still, for then $F = 0$; the greatest load carried by the inner truss will, therefore, be

$$\frac{P}{b}\left(\frac{b}{2} + h \sin \alpha - e\right). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The greatest load carried by the outer truss is given by eq. (1).

It is to be noted that the effect of shock, in designing the members of the inner truss, is lost if the train is assumed to be without motion. The error involved in adding an impact increment to the live-load stress found with the aid of eq. (3) is, however, on the side of safety, and such allowance should be made.

The judgment of the engineer must play a prominent part in the treatment of bridge-trusses carrying a curved

railway track. If wheel concentrations are used, the positions of loading for maximum stresses may be found by the use of criteria for a straight track laid along the axis of the bridge. The corresponding panel loads may then be used in eqs. (1) and (3) for determining the actual panel loads given to the trusses by the proper eccentricities at each panel. These latter adjusted panel loads may then be used to determine the stresses in the truss members. Or, again, uniform panel loads may be used in practically the same general manner, without resorting to train concentrations. In all ordinary cases no essential error will be involved in the use of uniform panel loads.

The use of the moment table or diagram shown on page 94 is convenient for finding the values of panel loads for systems of locomotive concentrations, as the following will demonstrate.

Let $n-1$, n, and $n+1$ (Fig. 33) indicate three successive panel-points of a truss whose panel lengths are all

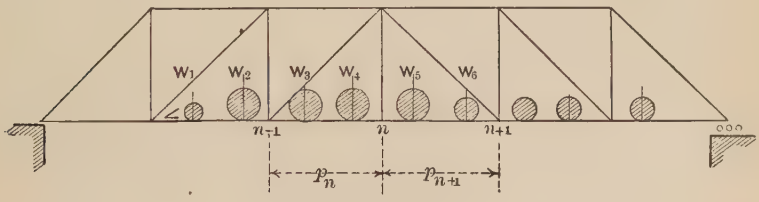


FIG. 33.

equal. The position of the wheel concentrations is assumed fixed, as shown in the figure, and it is desired to obtain the value of the panel load at n.

Let M_{n-1}, M_n, and M_{n+1} indicate the sums of the moments of all the preceding wheel loads about the $(n-1)$th, nth, and $(n+1)$th panel-points respectively, these quantities being readily obtained from the moment diagram.

That portion, P_n', of the loads $W_5+W_6+\ldots$ lying in the panel p_{n+1} and transferred to the point n may be

found in the method of moments, the center of moments being taken at $n+1$.

The moment of all wheel loads between the left end of the truss and the point $n+1$ must be decreased by the moment about the same point, $n+1$, of all wheel loads lying between the left end of the truss and the point n. That is,

$$P_n' = \frac{M_{n+1}}{p} - \frac{1}{p}[M_n + (W_1+W_2+W_3+W_4)p] \quad . \quad (1)$$

$$= \frac{M_{n+1}}{p} - \frac{M_n}{p} - W_1 - W_2 - W_3 - W_4 \quad . \quad . \quad . \quad (2)$$

Similarly, that portion P_n'' of the loads $W_3+W_4+\ldots$ lying in panel p_n which is transferred to n is found by first obtaining the portion (P_{n-1}) carried to point $n-1$ and subtracting that quantity from $W_3+W_4+\ldots$ That is,

$$P_{n-1} = \frac{M_n}{p} - \frac{M_{n-1}}{p} - W_1 - W_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\therefore P_n'' = W_3 + W_4 - \frac{M_n}{p} + \frac{M_{n-1}}{p} + W_1 + W_2 \quad . \quad . \quad (4)$$

The panel reaction P_n is then equal to $P_n' + P_n''$, i.e.,

$$P_n = \frac{M_{n-1} + M_{n+1} - 2M_n}{p} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Eq. (5) involves only quantities that may readily be found in the moment table already cited. The use of eq. (5) simplifies greatly the labor involved in obtaining panel loads.

It is obviously practicable to adjust the position of a curved track on a bridge so as to equalize to a large extent the effects of curvature on the two trusses. The most advantageous position of the track in reference to the

center line of the bridge should be sought in every case so as to make the stresses in the two trusses as nearly alike as practicable. It is also evident that the trusses of a bridge carrying a railroad track on a curve must be placed farther apart on centers in order to give suitable minimum clearance than in the case of a straight track laid on the center line of the bridge. An excellent paper showing the treatment of bridge-trusses carrying a railroad track on a curve and designed for a rapidly moving train was contributed to the American Society of Civil Engineers by Mr. Ward Baldwin, M. Am. Soc. C. E., in 1891, and it may be found in Vol. XXV of the Transactions of that Society. It is not necessary to show here a stress sheet, with the details of the computations required to meet the conditions outlined above, as such computations exhibit nothing new. If it is desired, a reference can readily be made to Mr. Baldwin's paper.

CHAPTER V.

NON-CONTINUOUS TRUSSES WITH CHORDS NOT PARALLEL.

Art. 83. General Methods.

THE determination of stresses in trusses with chords not parallel can be accomplished conveniently by the method of moments for either fixed or moving loads; or by the graphic method, especially for fixed loads.

So long as three members at most are cut by any section whatever dividing the truss into two parts, the problem of the determination of the stresses in those members is determinate. In such a case the problem pertains to the equilibrium of any system of three forces parallel to a given plane, for the solution of which there are three equations of condition.

Let Fig. 1 represent a portion of any truss divided into two parts by the section AB; and let F, G, and H be the points of intersection with the three members U_7U_8, U_7L_7, and L_6L_7, and let ΣP be the resultant of all the external forces acting on that portion of the truss lying on the left of AB. The external forces are known and the stresses in the members cut are required.

Any one of those stresses may be determined by the method of moments if the origin of moments be properly located. If the origin be taken at the point of intersection of the lines of action of any two of the stresses, the moments of those stresses will be zero. Hence as a general principle, in order to determine any one of those unknown stresses

by the method of moments, the origin of moments is to be taken at the intersection of the lines of action of the other two. The moment of the third unknown stress can then be placed equal to the resultant moment of the external forces, giving one equation with one unknown quantity.

Suppose in Fig. 1 that the stress in U_7U_8 is to be found by moments. L_7 is the point of intersection of U_7L_7 and L_6L_7; consequently it is the origin of moments. The lever-arm of that stress is the normal distance from L_7

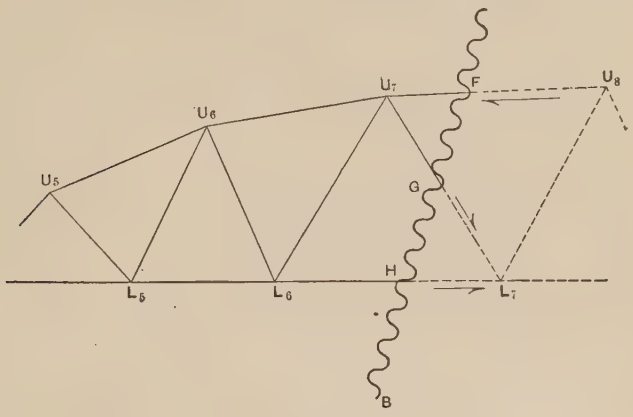


FIG. 1.

to U_7U_8. The kind of stress in U_7U_8 can always be determined by the method treated fully in Art. 65.

If any one of the three stresses is known, either of the others may be found by taking the origin of moments *anywhere* on the line of action of the third force.

This method of moments or sections is perfectly general and is independent of the character of the curve of the upper chord. It may equally well be applied to trusses with both chords curved or to trusses with parallel chords.

The graphic methods of stress solution may be found completely treated in the Authors' "Influence Lines," in

which the first chapter is devoted to the determination of the stresses in any simply supported truss. In this chapter analytical methods only will be employed.

Art. 84. Fixed-load Stresses—Example.

The preceding principles will be applied in obtaining the stresses due to the fixed or dead loads for the truss shown in Fig. 2. The following data are required:

Length of span = 200 ft.
Number of panels = 8.
Height at center = 36 ft.
Height of $U_2L_2 = 33$ ft.
Height of $U_1L_1 = 28$ ft.

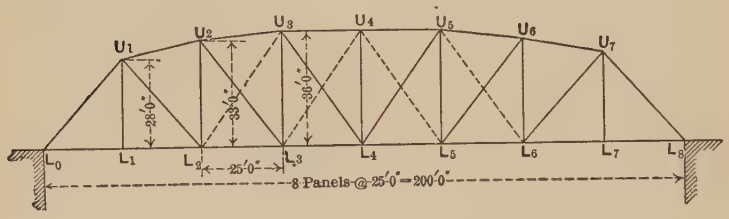


FIG. 2.

The truss is designed for a single-track through-bridge. The stresses due to the fixed and moving loads will be found separately, and those due to moving loads will be given in a succeeding article The members shown by dotted lines are the counterbraces and are not supposed to be subjected to dead-load stresses. In this problem, however, it will be supposed that when the live load brings these counterbraces into action, they will carry in compression their proper proportion of the dead load. In that case the particular main web members affected must be assumed to carry no fixed-load stresses.

The track, including rails, ties, etc., is taken to weigh 400 pounds per linear foot of bridge, while the weight of

the floor system (stringers and floor-beams) is assumed at 460 pounds per linear foot. The trusses themselves weigh 1,200 pounds per linear foot. This last value may be found by means of the usual formula, $w = al$, a in this case being taken equal to 6. The weight of the track and floor system is carried at the lower panel-points only, but the weight of the trusses is equally divided between the upper and lower chords. Each upper-chord panel weight per truss will then be $25 \times 1,200 \times \frac{1}{2} \times \frac{1}{2} = 7,500$ pounds. Each lower-chord panel weight per truss will be $7,500 + 5,000 + 5,750 = 18,250$ pounds. The reactions at the two ends of the truss will be equal with a value $3.5 \times 25,750 = 90,125$ pounds. The stresses in the various members will all be found by the method of moments.

The stress in the end post L_0U_1 is found by taking moments about L_1 of all the external forces situated to the left of a section passed through L_0U_1 and L_0L_1. In this case these forces reduce simply to the end reaction of 90,125 pounds. The lever-arm of L_0U_1 is 18.6 feet, therefore the stress

$$L_0U_1 = \frac{90,125 \times 25}{18.6} = -121,000 \text{ lbs.}$$

The stresses in the lower chords L_0L_1 and L_1L_2 are equal. Their moment is to be taken about U_1 with the lever-arm of 28 feet:

$$L_0L_1 = L_1L_2 = \frac{90,125 \times 25}{28} = +80,000 \text{ lbs.}$$

The center of moments for L_2L_3 is at U_2, and the lever-arm is 33 feet:

$$L_2L_3 = (90,125 \times 50 - 25,750 \times 25)\frac{1}{33} = +116,000 \text{ lbs.}$$

Similarly,

$$L_3L_4 = [90{,}125 \times 75 - 25{,}750(50+25)]\frac{1}{36} = +134{,}000 \text{ lbs.}$$

$$U_1U_2 = [90{,}125 \times 50 - 25{,}750 \times 25]\frac{1}{32.5} = -119{,}000 \text{ lbs.}$$

$$U_2U_3 = [90{,}125 \times 75 - 25{,}750(25+50)]\frac{1}{35.8} = -135{,}000 \text{ lbs.}$$

$$U_3U_4 = [90{,}125 \times 100 - 25{,}750(25+50+75)]\frac{1}{36}$$
$$= -144{,}000 \text{ lbs.}$$

The fixed-load stress in the hanger U_1L_1 is the load hanging at its foot, or a lower-chord panel weight, i.e.,

$$U_1L_1 = +18{,}000 \text{ lbs.}$$

The center of moments for U_1L_2 is at the intersection of U_1U_2 and L_1L_2, which is 115 feet to the left of L_0. The lever-arm of U_1L_2 about that point is 123 feet. Taking moments, there is found

$$U_1L_2 = (90{,}125 \times 115 - 25{,}750 \times 140)\frac{1}{123} = +55{,}000 \text{ lbs.}$$

The center of moments for U_2L_3 is 225 feet from L_0, and its lever-arm is 239 feet; therefore

$$U_2L_3 = [90{,}125 \times 225 - 25{,}750(250+275)]\frac{1}{239}$$
$$= +29{,}000 \text{ lbs. nearly.}$$

Since the chords U_3U_4 and L_3L_4 cut by the section in determining the stress in U_3L_4 or U_4L_3 are parallel, the stresses in the latter members may be found by multiplying the shears in the panels by the secants of the angles. In this case sec $\alpha = 1.22$. The shear is 12,875 pounds; therefore

$$U_3L_4 = -U_4L_3 = 12{,}875 \times 1.22 = +16{,}000 \text{ lbs.}$$

It is to be remembered that only one of these members acts at one time, but either may be chosen to act with the live load in order to furnish rational results.

The center of moments for U_3L_2 is the same as for U_2L_3, but the lever-arm is 226 feet; the stress is therefore

$$U_3L_2 = [90{,}125 \times 225 - 25{,}750(250 + 275)]\frac{1}{226} = -30{,}000 \text{ lbs.}$$

The fixed-load stress in the center post U_4L_4 is simply the compression caused by the upper-chord panel weight of 7,500 pounds:

$$U_4L_4 = -7{,}500 \text{ lbs.}$$

The center of moments for U_2L_2 is 115 feet from L_0, and its lever-arm is 165 feet; therefore

$$U_2L_2 = [90{,}125 \times 115 - 25{,}750 \times 140 - 18{,}250 \times 165]\frac{1}{165}$$
$$= -23{,}000 \text{ lbs.}$$

The member U_2L_3 is assumed to act in determining the above stress. If the counterbrace L_2U_3 be assumed to act, the stress in the member U_2L_2 is found by including in the equation of moments the moment of the stress in U_2L_2, since that member is also cut by the section dividing the truss.

In that case

$$U_2L_2 = [90{,}125 \times 115 - 25{,}750 \times 140 - 18{,}250 \times 165 - 30{,}000 \times 135]\frac{1}{165} = -100 \text{ lbs.}$$

Similarly for U_3L_3, if U_3L_4 acts,

$$U_3L_3 = [90{,}125 \times 225 - 25{,}750(250 + 275) - 18{,}250 \times 300]\frac{1}{300}$$
$$= -5000 \text{ lbs.}$$

and if U_4L_3 acts,

$$U_3L_3 = -5000 + \frac{16{,}000 \times 246}{300} = +8{,}400 \text{ lbs.}$$

The dead-load stresses are all shown, together with the live-load and impact stresses, in Plate III.

Art. 85. Position of Moving Load for Greatest Stress in any Web Member.

Two principal cases occur in connection with types of structures ordinarily used in engineering practice. That one to be treated first is the case arising when the intersection of the chord sections, in any panel, lies below the inclined web member in the same panel. The other arises when the intersection lies above the inclined web member. Applications of these principal cases to special features can easily be made after the general results are obtained.

The Intersection of Chord Sections below the Inclined Web Member.

Maximum Main Stress.

Let the moving load pass from right to left, as in Fig. 3; let l be the length of span; i the distance from end of span to the point of intersection, H, of the chord sections in the panel in question, m, the distance from the end of the span to the same panel, whose length is p; S the stress in the web member under consideration, and h its lever-arm about H; a, b, c, etc., the distances separating W_1 from W_2, W_2 from W_3, etc.; W_1, W_2, etc., the weights resting between G and D, and W_3, W_4, etc., the weights resting in the panel p, while W_n, distant x from E, is the last weight resting on the span from C to toward E. b' is the distance from D to the nearest weight, W_3.

The reaction at G is

$$R = W_1\left(\frac{a+b+c+\ldots+x}{l}\right) + W_2\left(\frac{b+c+\ldots+x}{l}\right) + \text{etc.} \ldots + W_n\frac{x}{l}. \quad (1)$$

By taking moments about H,

$$S \cdot h = Ri - W_1(l+i-a-b-\ldots-x) - W_2(l+i-b-c-\ldots-x) - \text{etc.} - \left(W_3\frac{p-b'}{p} + W_4\frac{p-b'-c}{p} + \text{etc.}\right)(m+i). \quad (2)$$

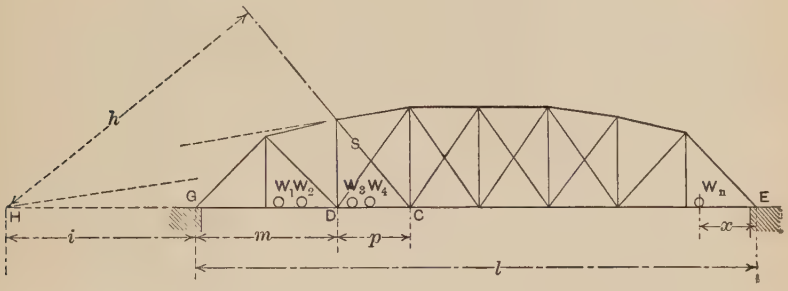


FIG. 3.

The position of the loading causing the maximum or minimum value of this stress may be found by differentiating S with respect to x, remembering that $dx = -db'$, and placing the resulting expression equal to zero:

$$dS \cdot h = (W_1 + W_2 + \ldots + Wn_n)\frac{dx}{l}i + (W_1 + W_2 + \text{etc.})dx - (W_3 + W_4 + \text{etc.})\frac{dx}{p}(m+i) = 0. \quad (3)$$

Hence

$$W_1 + W_2 + \ldots + W_n = -\frac{l}{i}(W_1 + W_2 + \text{etc.}) + (W_3 + W_4 + \text{etc.})\frac{l(m+i)}{pi}. \quad (4)$$

Eq. (4) represents the criterion to be used for determining the maximum main stress in the member S, but the following demonstration shows that it is equally applicable for maximum counter-stress if the first parenthesis in the second member represents the load between the panel p and the left end of the span, and if the second represents the load in panel p itself. For the main stress the load will extend from E to W_1, as shown, and for the

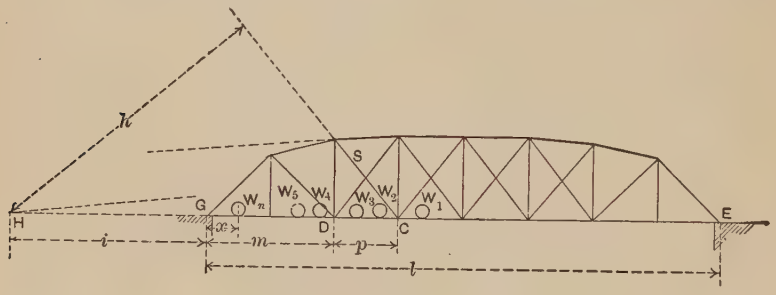


FIG. 4.

counter-stress it will extend from G to W_4 (W_4 being now the last weight toward E).

Maximum Counter-stress.

Let the moving load pass from left to right, as in Fig. 4. As before, m is the distance from the left end of the span to the left end of the panel p, but b_1 is measured from the right end of the same panel p to the nearest weight in it, W_2. The W_n weight is now the weight nearest G. The consideration of the equilibrium of the left portion of the truss requires the determination of the left reaction R'.

$$R' = W_1\left(\frac{l-a-b \ldots \text{etc.} -x}{l}\right) + W_2\left(\frac{l-b-c \ldots -x}{l}\right) + \ldots \text{etc.} \quad (5)$$

Moments about H give

$$S \cdot h = R' \cdot i - W_4(d+e \ldots \text{etc.} + x + i) - W_5(e \ldots \text{etc.} + x + i) - \ldots (m+i)\left(W_2\frac{b_1}{p} + W_3\frac{b_1+b}{p} + \text{etc.}\right). \quad (5)$$

Differentiating as before, remembering that $dx = -db_1$, and placing the resulting expression equal to zero,

$$W_1 + W_2 + W_3 + \ldots + W_n = -\frac{l}{i}(W_4 + W_5 + \ldots + W_n) + (W_2 + W_3 \ldots)\frac{l}{pi}(m+i), \quad (6)$$

which is precisely the same as eq. (4), as already noted.

Eq. (4) is perfectly general in character and covers all systems of loading whatever, but it may be put in special forms for convenient application in special cases.

Art. 86. Applications to Special Cases.

EXAMPLE I.—*Uniform Load.*

If the load is continuous, or only partially so, and w is its intensity (i.e., its amount per lineal unit) at any point distant x from E, then the various concentrations W_1, W_2, W_3, etc., will be represented by wdx. If, further, the load is uniform and continuous, w is constant, and $W_1 + W_2 + \ldots + W_n = wx_1$, x_1 representing the length of uniform load on the bridge. In the same manner, if x_2 represents the length of uniform load from D towards G, $(W_1 + W_2 + \text{etc.}) = wx_2$ and $(W_3 + W_4 + \text{etc.}) = wrp$, r being the fractional part of the panel p covered by the uniform load w. Eq. (4) then becomes

$$wx_1 = -\frac{l}{i}wx_2 + wrp\frac{l(m+i)}{pi},$$

or

$$x_1 = -\frac{l}{i}x_2 + rl\left(\frac{m}{i} + 1\right). \quad \ldots \ldots \quad (7)$$

As with the general case, eq. (7) is so written as to give both maximum main and counter stresses.

If the load is placed for the greatest main stress, for which $x_2 = 0$, while $x_1 = np + rp$, in which np is the length of loading from C towards E,

$$np + rp = rl\left(\frac{m}{i} + 1\right); \quad \therefore r = \frac{n}{\frac{l}{p}\left(\frac{m}{i} + 1\right) - 1} \quad . \quad . \quad . \quad (8)$$

If the load is placed for the greatest counter-stress,

$$x_2 = -\frac{i}{l}x_1 + ir\left(\frac{m}{i} + 1\right). \quad . \quad . \quad . \quad . \quad (9)$$

But since there must be no load between C and E, and if $np = x_2$ is the length of load from D towards G, and $x_1 = np + rp$, eq. (9) will become

$$n\left(1 + \frac{l}{i}\right) = r\frac{l}{p}\left(\frac{m}{i} + 1\right) - r; \quad \therefore r = \frac{n\left(1 + \frac{l}{i}\right)}{\frac{l}{p}\left(\frac{m}{i} + 1\right) - 1} \quad . \quad . \quad (10)$$

Eqs. (8) and (10) will enable the position of moving load to be at once computed without trial.

EXAMPLE II.—*Loads at Panel-points Only.*

If loads are located at the panel-points only, then W_1, W_2, W_3, etc., will be the panel loads, and a, b, c, etc., the panel lengths, and equal to each other in case those lengths are uniform; the parenthesis in eq. (2) multiplied by $(m + i)$ will also disappear. Substituting R from eq. (1) in eq. (2) with the last parenthesis dropped, there will result

$$S \cdot h = W_1(a+b+c+\ldots+x)\left(\frac{l}{i}+1\right)+W_2(b+c+\ldots+x)$$
$$\left(\frac{i}{l}+1\right)+\ldots \text{ etc.} + W_3(c+\ldots+x)\frac{i}{l}+\ldots+W_n\frac{xi}{l}$$
$$-(W_1+W_2+\ldots \text{ etc.})(l+i). \quad . \quad . \quad . \quad . \quad (11)$$

Only the wheels W_1 and W_2 are between G and D.

There are two groups of positive quantities in the second member of eq. (11), one multiplied by $\left(\frac{i}{l}+1\right)$ and the other by $\frac{i}{l}$.

The position of loading for a maximum of S will, in the general case, be determined by trial by ascertaining at what position this second group of positive quantities ceases to increase more rapidly (as the load progresses) than the negative difference between the first positive group and the negative last member. This can only happen if the panel weights toward or in the vicinity of W_n are very heavy relatively to those toward or in the vicinity of W_1. If the heavy panel loads are W_1 and those near it, i.e., *if the heaviest panel loads are at the head of the train*, the following analysis shows the positions for maximum stresses, in which, it is to be observed, W_1 is the rear panel load for counter-stresses.

Since

$$(a+b+c+\ldots+x)(i+l)<(i+l)l,$$

and hence

$$(a+b+c+\ldots+x)\left(\frac{i}{l}+1\right)<(i+l),$$

it is clear that *for maximum main stresses the loads must extend from the farther end of the span to the main member in question*, i.e., there are no loads W_1 and W_2.

Since the counter-shear is negative, i.e., opposed in sign to the main shear, the negative portion of the second

member of eq. (11) must be as large as possible for the maximum counter-shear, and the positive portion as small as possible. Hence the portion $W_3(c+\ldots+x)\frac{i}{l}+\ldots+W_n\frac{xi}{l}$ must be omitted and the load W_1 placed at the panel-point nearest the end G; i.e., *the load must cover that portion of the span between the counter and the nearest end of the span for the maximum counter-stresses.*

Hence for main web stresses under the assumed conditions

$$S=\frac{1}{h}\Big\{W_1(a+b+c+\ldots+x)+W_2(b+c+\ldots+x)+\ldots+W_nx\Big\}\frac{i}{l}, \quad (12)$$

and for counter web stresses

$$S=\frac{1}{h}\Big\{W_1(a+b+c+\ldots+x)+W_2(b+c+\ldots+x)+\ldots \text{etc.}\Big\}\Big(\frac{i}{l}+1\Big)-(W_1+W_2+\ldots \text{etc.})\Big(\frac{l+i}{h}\Big). \quad (13)$$

The conditions on which eqs. (12) and (13) are based are precisely the same as if the chords are parallel. In the latter case $d=\infty$, $h=\infty$ $\cos\alpha$, $\frac{i}{hl}=\frac{\sec\alpha}{l}$, $\frac{1}{h}=0$, and $\frac{l+i}{h}=\sec\alpha$.

Art. 87.—The Intersection of Chord Sections Above the Inclined Web Member.*

This case is illustrated by Fig. 5, in which the stress S in the member DC is under consideration. The moving

* The trusses treated in Art. 87 are rarely used. They are introduced only to make the treatment complete.

load is supposed to pass on the bridge from E towards G for the main web stress. H is the point of intersection of the chord sections, while GH and GD are the distances i and m respectively. All other notation remains precisely as in the previous case. The reaction, R, under G is given by eq. (1). Bearing in mind that the distance DH

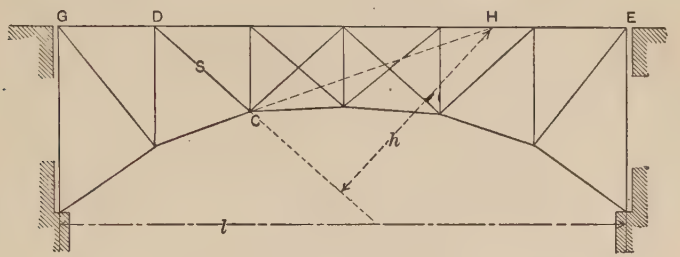


FIG. 5.

is now $(i-m)$, and taking moments about H, there will result

$$Sh = Ri - W_1(i-l+a+b+\ldots+x) - W_2(i-l+b+c+\ldots+x) - \text{etc.} - \left(W_3\frac{p-b'}{p} + W_4\frac{p-b'-c}{p} + \text{etc.}\right)(i-m). \quad (14)$$

Precisely the same operation which follows eq. (2) shows that the desired condition for a maximum is given by the following equation:

$$W_1+W_2+W_3+\ldots+W_n = \frac{l}{i}(W_1+W_2+\text{etc.}) + (W_3+W_4+\text{etc.})\frac{l(i-m)}{pi}. \quad (15)$$

The different portions of eq. (15) evidently represent exactly the same loads as the same portions of eq. (4). It is also clear, from the same considerations, that if the loads extend from E to W_1, eq. (15) gives the position

for a maximum main stress, and a maximum counter-stress if they reach from G to W_4 (i.e., to the weight farthest towards E).

Eq. (15), like eq. (4), applies to any system of loads whatever, and can be applied to special cases in the same manner.

EXAMPLE III.—*Uniform Load.*

By using the same notation and the same process of reasoning as in Ex. 1, eq. (15) takes the form for the greatest main stress:

$$r = \frac{n}{\frac{l}{p}\left(1 - \frac{m}{i}\right) - 1}. \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Or, for the greatest counter-stress:

$$r = \frac{n\left(\frac{l}{i} - 1\right)}{1 - \frac{l}{p}\left(1 - \frac{m}{i}\right)}. \quad . \quad . \quad . \quad . \quad . \quad (17)$$

EXAMPLE IV.—*Loads at Panel-points Only.*

Let it be first supposed that i is less than l; i.e., $i < l$.

The same general considerations that were given in Ex. II, applied to eq. (14), will cause it to take the form

$$Sh = -W_1(a+b+c+\ldots+x)\left(1 - \frac{i}{l}\right) - W_2(b+c+\ldots+x)$$
$$\left(1 - \frac{i}{l}\right)\ldots \text{etc.} + W_3(c+\ldots+x)\frac{i}{l} + \text{etc.}$$
$$+ (W_1 + W_2 + \text{etc.})(l-i). \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Since $(a+b+c+\ldots+x)(l-i) < l(l-i)$, hence

$$(a+b+c+\ldots+x)\left(1 - \frac{i}{l}\right) < (l-i).$$

Therefore, in order that the second member of eq. (18) shall have its greatest *positive* value, *the loads must be at all the panel-points*. Eq. (18) also shows that in the case now under consideration there can be no reversal of stress in any web member. In the inclined member, S, the stress will always be tension, and always compression in the vertical member passing through its upper extremity.

The positions for the actual maxima stresses can be found by trial only, as they will depend on the amounts of the panel loads and their locations relatively to each other.

Let it next be supposed that i is greater than l; i.e., $i > l$.

In this case $(l - i)$ becomes negative, or $(i - l)$ positive, and the conditions for maxima values are the same as those fixed for eq. (11); hence they need no further attention.

A remaining example with the intersection of chord sections *below the inclined web member*, and between it and the end of the span, can be treated in precisely the same general manner as the preceding. The moving load between G and D would lie, in the general case, partially on one side of the point of intersection and partially on the other. This form of truss, however, has little or no technical interest and needs no further attention.

The preceding treatment applies to any forms of trusses, whether deck or through, with one or the other chord horizontal. In the application of any particular formula it is only necessary that the point of intersection of the chord sections shall be located according to the conditions on which the formula is based.

The treatment also applies to any system of web members, whether they are all inclined at different angles to a vertical line or at equal angles, or, again, if, as in the figures, a part of them are vertical. It is only to be borne in mind that two web members intersecting in the unloaded chord take their greatest stresses together only *when that*

chord does not change its direction at that point of intersection. In case that direction does change, the value of i will be different for the two members, although m will remain the same.

The moment tabulation given on page 94 can be used in the application of the preceding formulæ precisely as with parallel chords. Short methods of computation, well known to every engineering office, make their practical applications easy and rapid.

In designing trusses with variable depth, special care must be taken in determining counter web stresses. It frequently happens, in cases similar to Fig. 3, that counters must be carried at least one panel nearer the end of the span than parallel chords would require. Again, the vertical web members are frequently subjected to heavy tension, with special conditions of moving load, at panel-points where the chords change direction.

Art. 88. Position of Moving Load for Greatest Chord Stresses.

Since the chord stresses under any given system of loading depend only on the truss depths at the various panel-points, measured in a direction normal to the opposite panels, the positions of loading for their maximum values with inclined chords will be identical with those determined in Art. 72 for parallel chords. No new conditions, therefore, are to be developed here. The equations of Art. 72 are to be applied exactly as they stand. It is only to be remembered that the lever-arm for any panel is the normal erected on it to the opposite panel-point.

Art. 89.—Moving-load Stresses in a Truss with Broken Upper Chord.—Example.

The truss to be taken in this example is shown in Fig. 6, and the moving load will be Cooper's E 40. The web members will be treated first.

Web Members.

U_1L_2.

Referring to the figure, it will be found that for this member $i = 115$ feet, $m = 25$ feet, $p = 25$ feet, $h = 123$ feet, and $l = 200$ feet. These quantities may be scaled with sufficient accuracy from a well-constructed drawing, or they may be

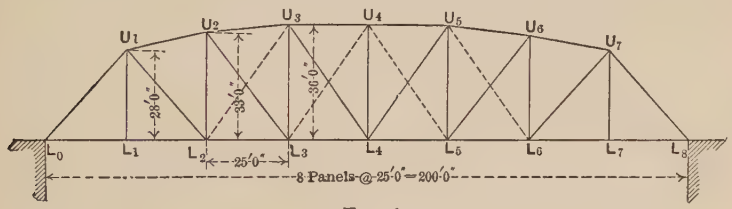


FIG. 6.

found by the usual trigonometric method. The use of the criterion, eq. (4) of Art. 85, requires the value of $\frac{l(m+i)}{pi}$, which in this case is 97.4.

Let wheel No. 3, or W_3, as the wheels may conveniently be designated, be placed at L_2. There will then be 54 feet of uniform load on the bridge, and $(W_1 + W_2 + \ldots + W_n)$ will equal $284{,}000 + (54 \times 2000) = 392{,}000$ lbs.

If $(W_3 + W_4)$ be considered either 50,000 or 70,000 lbs. (W_3 being taken either on the left or right of L_2), the criterion will be satisfied, thus indicating the position for greatest stress.

The reaction at L_0 for this loading will be found by the aid of the tabulation on page 94 to be

$$R = \left[14{,}944{,}000 + (284{,}000 \times 59) + \frac{(54)^2 \times 2{,}000}{2} \right] \frac{1}{200}$$
$$= 173{,}320 \text{ lbs.};$$

the weight or load hanging from L_1 is $\frac{230{,}000}{25} = 9{,}200$ lbs.

Taking moments about the intersection of U_1U_2 and L_1L_2, there is found

$$U_1L_2 = [173{,}320 \times 115 - 9{,}200 \times 140] \frac{1}{123} = +152{,}000 \text{ lbs.}$$

The greatest counter-stress may be obtained by the aid of the same criterion, but the quantity $\frac{l}{i} = \frac{200}{115} = 1.74$ must be included, as the following will show.

Let W_2 be placed at L_1, the train now advancing from left to right. W_6 will then be 1 foot from L_0 and $(W_1 + W_2 + \ldots + W_n)$ will be 103,000 pounds; $(W_3 + W_4)$ will be either 10,000 or 30,000 pounds, and $(W_1 + W_2)$ either 93,000 or 73,000 pounds. According to the criterion 103,000 must lie between the values of $(-1.74 \times 93{,}000) + (97.4 \times 10{,}000)$ and $(-1.74 \times 73{,}000) + (97.4 \times 30{,}000)$.

Since it is so found, this position satisfies the conditions for greatest negative stress. In determining the value of that stress, it will be more simple to treat the external forces situated to the right of the section passed through the truss.

The reaction at L_8 is then

$$R = [1{,}640{,}000 + (103{,}000 \times 1)] \frac{1}{200} = 8{,}700 \text{ lbs.},$$

and the negative weight at L_2 is 3,200 pounds. Therefore

$$U_1L_2=[8,700\times325-3,200\times165]\frac{1}{123}=-18,000 \text{ lbs.}$$

Since this compressive stress is much less than the dead-load tensile stress, this panel needs no counter-bracing.

$$U_2L_3.$$

In this member $i=225$, $m=50$, $p=25$, $h=239$, and $l=200$. Hence $\frac{l(m+i)}{pi}=11.2$, and $\frac{l}{i}=0.89$.

Placing W_3 at L_3, there will be 29 feet of uniform load on the bridge, and

$$(W_1+W_2+\ldots+W_n)=[284,000+(29,200\times2)]$$
$$=342,000 \text{ lbs.}$$

Either 30,000 or 50,000 pounds may be considered in the panel. The criterion is therefore satisfied. The reaction at L_0 becomes

$$R=\frac{1}{200}\left[14,944,000+(284,000\times34)+\frac{(29)^2\times2,000}{2}\right]$$
$$=127,120 \text{ lbs.}$$

The negative weight at L_2 is, as before, 9200 pounds. Taking moments about the intersection of the proper chord members, there is found

$$U_2L_3=[(127,120\times225)-(9,200\times275)]\frac{1}{239}=+109,000 \text{ lbs.}$$

The loads will be taken to advance from left to right for the greatest counter-stress, and W_2 will be placed at L_2. W_{10} is then 2 feet from L_0. Then

$$W_1 + W_2 + \ldots + W_n = 152{,}000,$$
$$W_1 + W_2 = 10{,}000 \text{ or } 30{,}000,$$
$$W_3 + W_4 = 142{,}000 \text{ or } 122{,}000,$$

and it will be found that the criterion is satisfied.

The reaction at L_8 becomes

$$R = [4{,}637{,}000 + (152{,}000 \times 2)]\frac{1}{200} = 24{,}700 \text{ lbs.}$$

The negative weight at L_1 will be 3,200 pounds; then

$$U_2L_3 = [24{,}700 \times 425 - 3{,}200 + 300]\frac{1}{239} = -40{,}000 \text{ lbs.}$$

Since this is greater than the dead-load stress, the counter-member L_2U_3 is brought into action. Using the same position of the loading, the same moment equation as before would result, but the lever-arm of L_2U_3 is now 226 feet instead of 239 feet. Therefore

$$L_2U_3 = \frac{9{,}540{,}000}{226} = +43{,}000 \text{ lbs.}$$

L_2U_2.

For the member L_2U_2, $i = 115$, $m = 50$, $p = 25$, $h = 165$, $l = 200$, and $\frac{l(m+i)}{pi} = 11.5$, and the same position of the loading as for U_2L_3 will satisfy the criterion. The reaction at $L_0 = 127{,}120$ pounds, and the negative weight at L_1 is 9200 pounds. Therefore

$$L_2U_2 = [127{,}120 \times 115 - 9{,}200 \times 140]\frac{1}{165} = -80{,}000 \text{ lbs.}$$

When the counter-member L_2U_3 receives its greatest tension, the post U_2L_2 will act no longer as a true web member of the truss; but the load or shear carried to

panel-point L_2 will be transferred to the abutment L_0 through the web member U_1L_2. The post L_2U_2, strictly speaking, carries, therefore, no counter-stresses, but it is subjected to some tensile stress, since it must provide for the difference in the vertical components of the upper-chord members U_1U_2 and U_2U_3 which meet at an angle at U_2. This tension has its greatest value when these upper-chord stresses have their greatest values, consistent with the action of stress in the counter L_2U_3. The proper treatment of this tensile stress is most conveniently given by the method of influence lines,* and it need not be considered by analytical methods.

The member U_3L_3 is subjected to a similar tensile stress. The values for both members are shown on the stress sheet.

U_1L_1.

This member is a hanger only, and its greatest live-load stress is caused by the maximum floor-beam reaction at its foot. The criterion to be used is eq. (3), Art. 69, Chap. IV, which is satisfied by placing W_4 at L_1. The stress caused is then 76,000 pounds tension.

U_3L_4.

The maximum stress in this member is obtained by the use of the criterion for trusses with parallel chords, viz., that n times the load in the panel should equal the total load on the bridge. In this case n, the number of panels in the bridge, is 8, and W_3 at L_4 will satisfy the criterion, there being 4 feet of uniform load on the bridge. The reaction at L_0 then equals

$$R = \left[14{,}944{,}000 + (9 \times 284{,}000) + \frac{(4)^2 \times 2{,}000}{2}\right]\frac{1}{200}$$
$$= 87{,}580 \text{ lbs.}$$

* For the detailed analysis, and the solution in detail of a problem, see the authors' "Influence Lines," page 110.

The negative weight at L_3 is 9200 lbs., and the shear is $87,500 - 9,200 = 78,400$ lbs. The secant between U_3L_4 and the vertical is 1.22, and therefore

$$U_3L_4 = 78,400 \times 1.22 = +96,000 \text{ lbs.}$$

$$L_0U_1.$$

The same criterion for parallel chord trusses is to be employed for this member as in the preceding case. It is satisfied with W_4 at L_1, thus bringing 84 feet of uniform load on the bridge. The reaction at L_0 becomes 236,400 lbs., and the load to be subtracted from it because carried by the end stringers to their end seats (if end struts be used and not an end floor-beam) will be 19,200 lbs. The shear in the panel is then 218,000 lbs. The secant of the angle is 1.34, therefore the stress is

$$L_0U_1 = -292,000 \text{ lbs.}$$

The stresses in the chord members remain to be determined.

Chord Members.

L_0L_1 *and* L_1L_2.

The maximum stress in these members is found with the same position of loading as the end post and by multiplying the effective end shear by the tangent of the angle, 0.894:

$$L_0L_1 = L_1L_2 = 218,000 \times 0.894 = +194,000 \text{ lbs.}$$

L_2L_3 *and* U_1U_2.

The center of moments for the member L_2L_3 is at U_2. The criterion, eq. (1) of Art. 72, is satisfied by placing W_7 at L_2. There is a length of 78 feet of uniform load on the bridge, and the reaction at L_0 is 223,120 lbs. The moment of the external forces about U_2 is

$$223,120 \times 50 - 2,155,000 = 9,001,000 \text{ ft.-lbs.}$$

The lever-arm of L_2L_3 is 33 feet; therefore its stress is

$$L_2L_3 = +274{,}000 \text{ lbs.}$$

The position of the loading for the maximum stress in U_1U_2 is the same as that for L_2L_3, since the center of moments is in the same vertical line. The lever-arm is 32.5 feet, therefore

$$U_1U_2 = -279{,}000 \text{ lbs.}$$

$$L_3L_4.$$

It will suffice to determine the stress in one other chord member, L_3L_4. The center of moments is at U_3 and the criterion is satisfied with W_{11} at L_3, there being 80 feet of uniform load on the span. The reaction at L_0 is 227,720 lbs., and the moment at U_3 is

$$227{,}720 \times 75 - 5{,}848{,}000 = 11{,}231{,}000 \text{ ft.-lbs.}$$

Since the lever-arm is 36 feet,

$$L_3L_4 = +313{,}000 \text{ lbs.}$$

Impact Stresses.

This bridge was designed by the American Bridge Co. under its own specifications, which provide that there be added to each live-load stress a quantity called the impact stress, depending on the length of load required to produce that particular stress. The use of this impact provision has been illustrated in Art. 74, so that it will be sufficient here to determine one or two of these impact stresses only; the values of the others may all be found on the stress sheet, Plate III, which furnishes all the data needed by the draughting room for detailing the structure.

As before, the formula to be used is $I = S\left(\frac{300}{L+300}\right)$, where I is the impact stress to be added to the calculated

live-load stress S, and L is the length in feet of that part of the span covered by the moving load causing the stress in question.

In the case of chord members the entire bridge will be considered covered. For all those members the impact factor then becomes 0.6. The live-load stress for the member L_3L_4 is 313,000 pounds, and the impact stress therefore is $.6 \times 313{,}000 = 188{,}000$ pounds. In the same manner the corresponding stresses are respectively 194,000 and 117,000 pounds for L_0L_1.

For the member L_3U_4 the length of load is 113 feet. The impact factor therefore is 0.730, and as the live-load stress is 96,000 pounds, the impact stress is 70,000 pounds.

For the member U_1L_2 $L = 163$ feet and the impact factor is 0.645. Hence, as the live-load stress is 152,000 pounds, the impact stress is 98,000 pounds.

For the hanger U_1L_1, $L = 50$ feet and the impact factor is 0.857. The live-load stress being 76,000 pounds, the impact stress is 65,000 pounds.

It is to be noted that the preceding stresses are given to the nearest 1000 pounds, as that is sufficiently exact.

Art. 90.—Trusses with Parabolic Upper Chord.

Of more interest, perhaps, than real value to the engineer is the expression for the horizontal component of the greatest stress in any web member of a truss with a parabolic upper chord. As that expression may easily be written it will be given; it may be useful at times as a numerical check.

Let Fig. 7 represent a truss of one system of triangulation, subjected to the action of vertical loads passing along the lower chord AB. It is desired to find the horizontal component of the greatest tensile stress in GH. Let Hh and Gg be verticals passing through H and G.

The following notation will be used:

p = panel length (uniform) in AB;
n = number of any panel from B; for PQ, n has the value 2, and 4 for OH;
$rp = Mg$;
$d = Gg$;
$d' = Hh$;
N = number of panels in AB;
$l = AB = Np$;
w = moving panel load;
R = reaction at B;
$n_1 p = BM$.

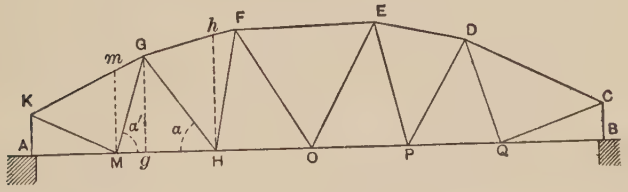


FIG. 7.

For the greatest tension in GH the panel-points covered by the moving load must extend from B to H.

The distance of the center of these loads from B is $\frac{n_1 p}{2}$. Hence the reaction at B becomes

$$R = \left(l - \frac{n_1 p}{2}\right)\frac{1}{l}(n_1 - 1)w = w\left\{(n_1 - 1) - \frac{n_1(n_1 - 1)}{2}\frac{p}{l}\right\}.$$

Taking moments about G, the stress in MH is

$$MH = \left(R(n_1 - r)p - (n_1 - 1)w\left\{(n_1 - r)p - \frac{n_1 p}{2}\right\}\right) \div d$$
$$= \frac{wp}{2d}\frac{[l - (n_1 - r)p]}{l} n_1(n_1 - 1). \quad \ldots\ldots\ldots \quad (1)$$

In order to obtain the horizontal component of the stress in GF, due to the assumed load, it is only necessary to take normals about H in precisely the same manner. The expression, however, can be derived immediately from eq. (1) by putting $r=1$, and writing d' for d.

$$\therefore \text{ Hor. Com. } GF = \frac{wp}{2d'}\frac{[l-(n_1-1)p]}{l}n_1(n_1-1). \quad . \quad (2)$$

The horizontal component of the greatest tensile stress in GH is the difference between the second members of eqs. (1) and (2); let it be called H_1.

$$\therefore H_1 = \frac{wp}{2l}\left\{\frac{l-(n_1-1)p}{d'} - \frac{l-(n_1-r)p}{d}\right\}n_1(n_1-1). \quad (3)$$

If α is the angle of inclination of GH to a horizontal line, then

$$GH = H_1 \sec \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Eqs. (3) and (4) apply to all tensile web stresses. For compressive web stresses as typified by GM there would be found the Hor. Comp. GK, instead of Hor Comp. GF, by taking moments about M; d' would then represent Mm. By making $r=0$ in eq. (1),

$$\text{Hor. Comp. } GK = \frac{wp}{2d'}\frac{(l-n_1p)}{l}n_1(n_1-1). \quad . \quad . \quad (5)$$

Hence for the horizontal component of the greatest compressive stress in GM

$$H_1' = \frac{wp}{2l}\left\{\frac{l-n_1p}{d'} - \frac{l-(n_1-r)p}{d}\right\}n_1(n_1-1) \quad . \quad (6)$$

and

$$GM = H_1' \sec \alpha'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

By means of the eqs. (3), (4), (6), and (7) every web stress in the truss may be determined by formula.

If GM is vertical, $r=0$ and $d=d'$ in eq. (6), and $H_1'=0$, as it should.

If GH is vertical, eq. (3) shows H_1 to be zero in the same manner.

It is to be borne in mind, in the application of these formulæ, that n is counted along the loaded segment; also that d', for tension, is taken at the head of the train, and one panel in front of it for compression.

If the moving load passes along the upper chord, exactly the same formulæ hold true, but d' taken at the head of the train will give compression, and tension when taken a panel length in front.

If the curve KFC is a parabola, with vertex at the center of the span, if K and C coincide with A and B respectively, and if GM and all corresponding web members are vertical, eq. (3) becomes

$$H_1=\frac{wp}{2l}\left\{\frac{l-(n_1-1)p}{d'}-\frac{l-n_1p}{d}\right\}n_1(n_1-1). \quad . \quad (8)$$

From the ordinary equation for the parabola,

$$y^2=ax$$

and

$$\frac{l^2}{4}=ad_1,$$

in which d_1 is the depth of the truss at the middle of the span. Hence

$$y^2=\frac{l^2}{4d_1}x.$$

In this equation put $y=\frac{l}{2}-n_1p$ and $x=d_1-d$, then

$y=\frac{l}{2}-(n_1-1)p$ and $x=d_1-d'$, successively. There will result

$$\frac{l^2}{4}-n_1lp+n_1^2p^2=\frac{l^2}{4d_1}(d_1-d),$$

$$\frac{l^2}{4}-(n_1-1)lp+(n_1-1)^2p^2=\frac{l^2}{4d_1}(d_1-d')$$

Remembering that $l=Np$,

$$d=d_1\frac{4(n_1N-n_1^2)}{N^2};$$

$$d'=d_1\frac{4[(n_1-1)N-(n_1-1)^2)]}{N^2}.$$

Putting these values in eq. (8), also $Np=l$, there will result

$$H_1=\frac{wp^2N^2}{8d_1l}\left\{\frac{N-(n_1-1)}{(n_1-1)N-(n_1-1)^2}-\frac{N-n_1}{n_1N-n_1^2}\right\}n_1(n_1-1).$$

$$\therefore\ H_1=\frac{wpN}{8d_1}=\frac{wl}{8d_1}=\text{constant}. \quad . \quad . \quad . \quad (9)$$

As this is the horizontal component of the greatest tension in any diagonal web member and constant, *that greatest stress itself is the hypothenuse, parallel to the brace in question, of a right-angled triangle of which the base is* $H_1=\frac{wl}{8d_1}$.

This furnishes a short method of finding the stress in any inclined web member.

The similarity between H_1 and the total stress in the horizontal chord, with the truss wholly loaded, is interesting.

If the trussing is so designed that the diagonal or

inclined braces sustain compression, eq. (6) gives precisely the same general result, but with the sign changed.

In such a case there would be substituted in the parabolic equation $y=\frac{l}{2}-n_1p$ and $x=d_1-d'$; also, $y=\frac{l}{2}-(n_1-1)p$ and $x=d_1-d$; d and d' having changed places.

If no web members are vertical, y will have for one value in the equation to the parabola $\frac{l}{2}-(n_1-r)p$, instead of $\frac{l}{2}-n_1p$, the other values to be substituted remaining the same. This new value gives

$$d=d_1\frac{4[(n_1-r)N-(n_1-r)^2]}{N^2}.$$

Making the substitutions in eq. (3) instead of eq. (8),

$$H_1=\frac{wl}{8d_1}\left(\frac{1}{n_1-1}-\frac{1}{n_1-r}\right)n_1(n_1-1).$$

$$\therefore\ H_1=\frac{wl}{8d_1}\left(\frac{1-r}{n_1-r}\right)n_1.$$

Art. 91.—The Schwedler Truss.

The general principles already applied to bowstring trusses enable the characteristics of the Schwedler truss to be easily shown, that truss being a special bowstring, having the least possible number of diagonal braces under the conditions assumed.

Fig. 8 represents the elevation of such a truss, and the problem involved is the determination of such depths, near the ends, that one diagonal only will be needed in each panel; it being premised that the inclined web members or diagonals are to sustain tension only.

Let W = total (upper and lower chord) panel fixed load;
R' = half the fixed load (or weight) of the bridge;
w = panel moving load;
l = length of span;
d = any vertical brace or truss depth, as Cc;
d_1 = vertical brace or truss depth, as Dd, adjacent to d and toward center;
p = panel length;
α = inclination of any diagonal, as Cd, to the horizontal lower chord, i.e., $Cdc = \alpha$;
x = distance from A to the intersection of the prolongation of any upper-chord panel, in the left half of the truss, with the prolongation to the left of the lower chord;
y = the normal distance from that point of intersection to the prolongation of the diagonal immediately under the upper-chord panel prolonged.

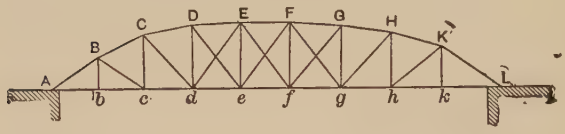


FIG. 8.

Let the moving load pass on the bridge from A toward L, and let n be the number of panel moving loads from A.

The reaction at A, for any position of the moving load, will be

$$R = R' + nw\left(1 - \frac{(n+1)p}{2l}\right). \quad . \quad . \quad . \quad . \quad (1)$$

Then let the truss be imagined divided through the panel immediately in front of the train.

If moments be taken about the point of intersection denoted by x, and if T represents the tension in the diago-

nal just in front of the train, whose lever-arm is y, there will result

$$Ty = Rx - n(W+w)\left(x + \frac{(n+1)p}{2}\right). \quad . \quad . \quad . \quad (2)$$

Eq. (2) is so written, it is important to notice, that if the second member is greater than zero, or positive, T will be tension. Hence, if T is tension,

$$Ty \geqq 0.$$

But $$y = [x + (n+1)\,p] \sin \alpha. \quad . \quad . \quad . \quad . \quad (3)$$

Also, from similar triangles,

$$\frac{d_1 - d}{p} = \frac{d_1}{x + (n+1)p}; \quad \therefore \; x + (n+1)p = \frac{pd_1}{d_1 - d}. \quad . \quad (4)$$

By the aid of eq. (3),

$$T = \frac{-R(n+1)p + n(W+w)\dfrac{(n+1)p}{2}}{[x + (n+1)p] \sin \alpha} + \frac{R - n(W+w)}{\sin \alpha} \geqq 0.$$

Hence, by the aid of eq. (4),

$$x + (n+1)p \geqq \frac{\left(R - \dfrac{n(W+w)}{2}\right)(n+1)p}{R - n(W+w)} \leqq \frac{pd_1}{d_1 - d}.$$

Using the last two members of this inequality,

$$\frac{d}{d_1} \geqq 1 - \frac{R - n(W+w)}{\left(R - \dfrac{n(W+w)}{2}\right)(n+1)}, \quad . \quad . \quad . \quad (5)$$

or

$$d \geqq d_1 \left(1 - \frac{R - n(W+w)}{\left(R - \dfrac{n(W+w)}{2}\right)(n+1)}\right) \quad . \quad . \quad . \quad (6)$$

The second member of eq. (6) is the least value of the depth d which can exist without inducing compression in the diagonal under consideration. This diagonal is the one immediately in front of the train, and the principles already given in this chapter show that if this position does not induce compression, no other will.

Inequality (6) shows that if R is greater than $n(W+w)$, or $R>n(W+w)$, d will always be less than d_1. If $R=n(W+w)$, then $d=d_1$.

Again, if $R<n(W+w)$, then will d be greater than d_1, if tension is to be found in the diagonal. But it is not admissible to make $d>d_1$; *hence, when n becomes so great that* R *is less than* $n(W+w)$, *or* $R<n(W+w)$, T *will be compression, and the diagonal must be counterbraced or else intersecting diagonals must be placed in the panels, as shown in Fig.* 8, *near the middle of the truss.*

The value of n given by

$$R<n(W+w) \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

will show the position of the head of the train when all panels between it and the center must contain intersecting diagonals. All the other panels will need but one each, sloping upward and toward the end of the truss, as shown in Fig. 8.

Since this method is independent of the direction of approach of the train, it is only necessary to consider one half of the truss.

It is seen in eq. (6) that d is given in terms of d_1; hence the latter must be known in order to find d.

The center depth is arbitrary and may be assigned at will. The depths between the center and that point indicated by n, in inequality (7), may also be assigned at will; consequently d_1, next to the first "d" to be computed, will be known. The first "d" computed will be

the "d_1" for the next "d," etc., etc., to the end of the truss.

As a margin of safety it will be well to make d a little greater than given by the second member of eq. (6).

In long spans it would be well to make the truss depth constant for a number of panels near the center, perhaps, even between the points given by eq. (7). This would make a considerable number of diagonals and panels uniform in length, which would otherwise lack uniformity. Thus the construction would be simplified and cheapened.

The loads have been taken uniformly, but precisely the same methods would hold if they were not uniform.

Example.

Let the following example (the truss shown in Fig. 8) be taken:

$$\text{Span} = l = 9p = 108 \text{ feet}; \quad \therefore \ p = 12 \text{ feet.}$$

$$\text{Center depth} = 16 \text{ feet.}$$

$$W = 16{,}000 \text{ lbs.} \qquad w = 36{,}000 \text{ lbs.}$$

$$(W + w) = 52{,}000 \text{ lbs.}$$

$$R' = 4W = 64{,}000 \text{ ``}$$

From eq. (1):

$$R = 64{,}000 + 36{,}000\left(n - \frac{n^2 + n}{18}\right). \quad . \quad . \quad . \quad (8)$$

If $n = 3$, by eqs. (8) and (7),

$$R = 64{,}000 + 36{,}000(3 - \tfrac{2}{3}) = 148{,}000 < n(W + w) = 156{,}000.$$

Hence the diagonals De and Ed, in the panel in front of the head of the train at d, must both be introduced.

If $n = 2$, by eq. (8) and eq. (7)

$$R = 124{,}000 > n(W + w) = 104{,}000.$$

Hence Cd is the only diagonal needed in the panel $CDdc$, and $d = Cc$ is to be computed from eq. (6).

The center depth $= Ee = Ff$ was taken at 16 feet; let Dd be taken at 15.5 feet.

Since $n = 2$, $R - n(W + w) = 20{,}000$,

and $$R - \frac{n(W+w)}{2} = 72{,}000.$$

Substituting these values, and $d_1 = 15.5$, in eq. (6),

$$d = 0.91 d_1 = 14.11 \text{ feet.}$$

Hence let $d = 14.5$ feet $= Cc$ (Fig. 8).

Next, let the head of the train be at b, i.e., let $n = 1$. Then, by eq. (8),

$$R = 64{,}000 + 32{,}000 = 96{,}000.$$

Also,

$$R - n(W + w) = 44{,}000 \quad \text{and} \quad R - \frac{n(W+w)}{2} = 70{,}000.$$

For this position of load, $d_1 = 14.5$ feet. Hence, by eq. (6),

$$d = 14.5\left(1 - \frac{44{,}000}{140{,}000}\right) = 9.94 \text{ feet.}$$

Hence let $d = 10.00$ feet $= Bb$ (Fig. 8).

Fig. 8 represents the truss, drawn to scale, with the various depths given or computed as above, i.e.,

$Ee = Ff = 16.0$ feet,
$Dd = Gg = 15.5$ "
$Cc = Hh = 14.5$ "
$Bb = Kk = 10.0$ "

In the three panels adjacent to each end of the truss only two main diagonals are thus seen to be necessary, and in those no compression will ever exist. In each of the three middle panels, however, two intersecting diagonals will be necessary, since no diagonal must sustain compression.

As is evident, the expression $R-n(W+w)$ is the vertical shear at the head of the train. Hence the limiting case of the inequality (7),

$$R=n(W+w),$$

gives the two points, in the two halves of the truss, at which the vertical shear at the head of the train is zero. Between these points intersecting diagonals or counter-braces are needed, and only between them.

After the truss depths are fixed by the preceding method, the stresses in the individual members are to be found in the usual manner, as in any other bowstring truss.

Art. 92.—Trusses with Subdivided Panels.

If panels are subdivided like those shown in Fig. 9, by the vertical *ab*, the load carried to the apex *a* (or *c*)

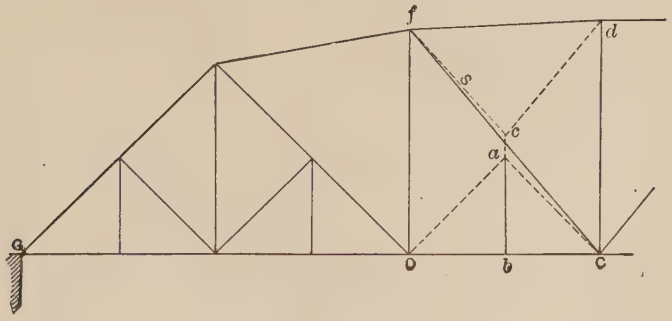


FIG. 9.

may be assumed to be divided equally between *D* and *C* by the trussed stringer *DaC*, or equally between *f* and

d by the simple truss *fcd*, or, as shown in Fig. 10 again, equally between *f* and *D* by means of the triangular bracing *Daf*, or equally between *C* and *d* by means of the triangular bracing *dcC*.

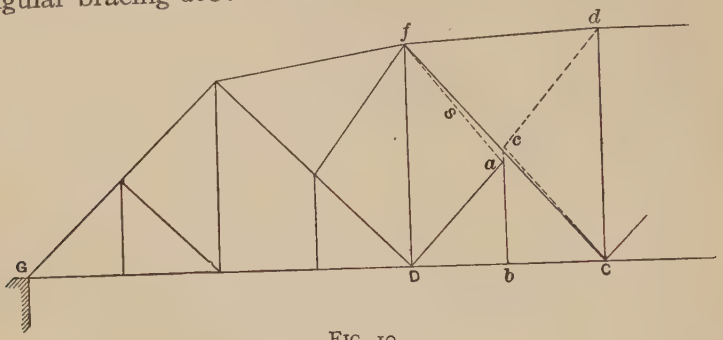


FIG. 10.

Position of Load for Maximum Web Stresses.

In the two cases presented in Fig. 9 the two simple triangular trusses *DaC* and *fcd* act as ordinary stringers in transferring load in the panel *DC* to the points *D* and *C*, or *f* and *d*. Hence the conditions assumed in Art. 85, Fig. 3, are in no way changed and eq. (4) is to be applied precisely as it stands, *but the panel length p is to be taken as DC*, i.e., with the subdivision disregarded. The greatest stress in *fC* is to be found as if *Da*, *ab*, and *dc* did not exist. If the subdivision *DaC* is employed, that greatest stress will be the actual stress in *fa*, but the actual or resultant stress in *Ca* will obviously be the stress in *fa less the compressive stress in aD* found with the position of loading unchanged.

In case the subdivision *fcd* is employed, the same greatest stress will be the actual stress in *Cc*, but the stress in *fc* will obviously be the stress in *Cc* plus that in *cd* found also with the position of loading unchanged.

If, in Fig. 10, the triangular bracing *dcC* acts to transfer

the load hung at a (or c) equally to d and C, or if the bracing faD transfers the same load equally to f and D, the conditions of application of eq. (4), Art. 85, are slightly changed. If the subdivision is made by using the bracing faD, the member cd is supposed not to exist. Then if the truss is cut anywhere through fd, cC, and bC the conditions for the greatest stress in cC will be precisely the same as those for the greatest stress in S of Fig. 3, Art. 85. The analysis given in eqs. (1), (2), (3), and (4) of that article will remain unchanged except that the panel length p will now represent the short panel bC of Fig. 10. This arises from the fact that all the load on bC is divided between the points b and C by the principle of the lever, while all the load on Db will be divided by the same law between the points D and b. That portion of the latter supported by ab will produce equal stresses of tension and compression in af and aD respectively.

Eq. (4), Art. 85, applies directly as it stands to this case, but p is to be taken as representing the short panel length bC and $(W_3+W_4+\text{etc.})$ the concentrations in it, while $(W_1+W_2+\text{etc.})$ will represent loads found on the left of b. The greatest stress in cC will at once result, but the greatest stress in fa will be equal to that in $cC+$ an amount of tension equal to the compression in aD found with the position of loading unchanged.

If the subdivision is made by using the bracing dcC (Fig. 10), the member aD will be omitted and the members fd, fc, and Db will be considered cut in order to determine the position of moving load for the greatest stress in fc. The analysis leading to eqs. (1), (2), (3), and (4), Art. 85, as well as those equations themselves, will then remain unchanged, and they will be directly applicable to the present case by taking Db as the panel length p, while $(W_3+W_4+\text{etc.})$ will represent the concentrations in it and $(W_1+W_2+\ldots)$ those found on the left of it.

These investigations cover all possible assumptions and they are applicable alike both to main and counter-stresses.

Position of Load for Maximum Chord Stresses.

In finding the position of the moving load for the greatest chord stresses the trussing shown in Fig. 11 only will be considered. The same general method of treatment is applicable to other secondary systems of trussing such as those shown in Figs. 9 and 10.

Loaded Chord.

The stresses in the two members of the lower chord lying in the same main panel, as *Db* and *bC*, Fig 11, are.

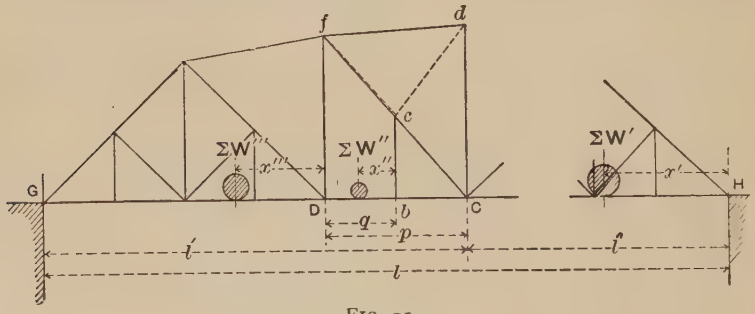


FIG. 11.

always the same for any position of the loading, for at the panel-point *b* there is no inclined member. The expression for the greatest stress desired will be obtained by the method of sections. It will always be found possible to divide the truss with any system of subdivided panels so that three members only will be cut, and so that the center of moments will always fall at one of the main panel-points. The simple criterion for the maximum bending moment at any point in a beam, namely,

$$\frac{W_1+W_2+\ldots+W_n'}{W_1+W_2+\ldots+W_n}=\frac{l'}{l}$$

of Art. 72, Chap. IV, may therefore be employed. If in Fig. 11 the supplementary trussing *fcd* be considered, the proper section to pass through the truss for the member *Db* will be *fd*, *fc*, and *Db*. The center of moments then falls at *f*, and in the above criterion the value of l' is the length *GD*, l being the total length of span. Having determined the maximum bending moment at *f*, the stress in *Db* will be found by dividing by the lever-arm *fD*. The maximum stress found for *Db* will also be the maximum stress for *bC*.

Unloaded Chord.

The same secondary system, *fcd*, shown in Fig. 11 will be considered.

The stress in a member *fd* is found by passing a section through *fd*, *fc*, and *Db*, and with the center of moments at *C*, equating the moment of the external forces situated on one side of the section with the moment due to the chord stress. Although the center of moments is at *C*, the external forces to the left of the section do not include the panel load hanging at *b*. The resulting moment equation is different, therefore, from the moment found at *C*, for a simple beam. Hence the criterion

$$\frac{W_1+W_2+\ldots+W_n'}{W_1+W_2+\ldots+W_n}=\frac{l'}{l},$$

of Art. 72, Chap. IV, cannot be used, but a similar one may easily be deduced.

The concentrations on the span will now be grouped as three different series, viz., $\Sigma W'$, $\Sigma W''$, and $\Sigma W'''$, these quantities expressing respectively the sum of the loads on the portions of the span.

$$bH=l''+p-q, \quad bD=q, \quad \text{and} \quad DG=l'-p.$$

The distances of the centers of gravity of these equivalent loads are x', x'', and x''' from the various panel-points shown in the figure.

For any position of the loading, such as shown, the reaction at G becomes

$$R = \frac{1}{l}[\Sigma W'x' + \Sigma W''(x'' + p - q + l'') + \Sigma W'''(x''' + p + l'')]. \quad (1)$$

The load in the panel Db is divided between the panel-points so that $\Sigma W'' \frac{x''}{q}$ will hang at D.

The moment at C then becomes

$$M = \frac{l'}{l}[\Sigma W'x' + \Sigma W''(x'' + p - q + l'') + \Sigma W'''(x''' + p + l'')] - \Sigma W'''(x''' + p) - \Sigma W'' \frac{x''}{q} p. \quad (2)$$

To obtain a criterion for the maximum or minimum moment, the first derivative of M is to be placed equal to zero in the usual manner, remembering that $dx' = dx'' = dx''' = dx$. Hence

$$dM = \frac{l'}{l}[\Sigma W' \cdot dx' + \Sigma W'' \cdot dx'' + \Sigma W''' \cdot dx'''] - \Sigma W'''dx''' - \Sigma W'' \frac{p}{q} dx'',$$

and

$$\frac{dM}{dx} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If ΣW represents $\Sigma W' + \Sigma W'' + \Sigma W'''$, i.e., all the weights on the bridge,

$$\Sigma W\left(\frac{l'}{l}\right) = \Sigma W'' + \Sigma W'' \frac{p}{q}. \quad . \quad . \quad . \quad . \quad (4)$$

Eq. (4) is the criterion to be employed in precisely the same manner as eq. (1) of Art. 72, Chap. IV. As is evident, however, it differs from that equation only in the right-hand member, where instead of the sum of all the weights on the length l' a portion only is taken.

There will be found in Chapter VI of the authors' "Influence Lines" the complete design, including the determination of all the fixed and moving load stresses, of a truss of this kind with subdivided panels, and it is unnecessary to repeat that matter here.

It is customary, as indicated in Fig. 12, to insert horizontal and vertical braces in deep trusses to support long

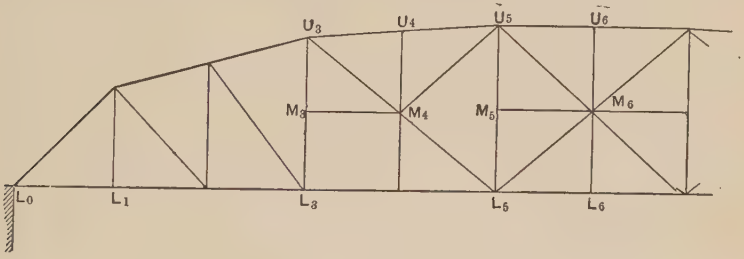


FIG. 12.

compression members either as posts or upper-chord members. In such cases horizontal struts, as M_3M_4 and M_5M_6, carry no truss stresses, their function being simply to reduce the length of compression members so as to permit higher working stresses in them. The vertical struts U_4M_4, U_6M_6 carry the panel dead loads at their upper ends, but they are not subject to live-load stresses.

Art. 93.—Stresses in Roof-trusses—Loading.

Preliminary.

Roof-trusses are designed to carry, first, their own weight; second, the weight of the roof-covering on the trusses; third, a snow load; fourth, a wind load acting in

a horizontal direction, first from the right and then from the left; and, fifth, a ceiling or other suspended weights, such as cranes, trackways, or shafting.

The weights of roof-trusses vary with the span length, with the distance between trusses, and with the loads.

Various formulas have been proposed for determining the weights of steel roof-trusses, and among them is the following, deduced by Milo S. Ketchum, Assoc. M. Am. Soc. C. E., for spans up to 150 feet:

$$W = \frac{P}{45}\left(1 + \frac{L}{5\sqrt{A}}\right),$$

where W = the weight of the truss per square foot of horizontal projection; P = the capacity of the truss in pounds per square foot of horizontal projection; L = the span of the truss in feet; and A = the distance between trusses in feet. In general, for determining the dead-load stresses it will be sufficiently accurate to use tentatively the following approximate table:

For a Span Length, in Feet, of	Weight of Truss in Pounds per Square Foot of Ground Surface Covered.
20	2
40	3
60	4
80	5
100	6

Great accuracy cannot be expected from the use of this table; but since, in any event, the greatest stresses are caused by the weight of truss-covering combined with wind and snow loads, a small error in assuming the dead weight of a roof-truss is not material. If the error is found appreciable after tentative design, the proper corrections may easily be made.

Trusses having spans up to 50 or 60 feet generally have one end supported on planed plates, but those of greater span usually have one end on rollers or rockers to provide for the effects of temperature changes. In the case of a roller or rocker end, the reaction is usually taken perpendicular to the plane on which the rollers move, although on account of friction this condition may not be rigorously exact.

The pitch of the roof is usually given as the ratio of the center height divided by the span lenth; it may vary between the limits of one half and one fifth; its more usual value is one fourth.

Snow Load.

The snow load carried by a structure depends not only on the latitude of its location, but also on the pitch of the roof. It is ordinarily specified as a load in pounds per square foot of horizontal projection. In the vicinity of New York a value of 20 pounds per square foot for roofs of small pitch or flat roofs is frequently taken, but this is decreased for roofs of greater pitch. If the pitch is 60° or more, no snow loads need be taken, although a minimum weight of 10 pounds per square foot, due to sleet, is sometimes specified. The highest value for flat roofs in cold climates is 40 pounds per square foot, while for southern latitudes the snow load disappears.

Wind Load.

In treating horizontal wind loads on roof-trusses, the component normal to the slope of the roof is usually taken, the component parallel to the slope of the roof being neglected. The intensity of horizontal wind pressure on a vertical surface is generally specified at 30 pounds per square foot.

The normal pressure on a roof due to horizontal wind force is not usually found by its simple resolution into two rectangular components, but by means of an empiric formula based upon experimental work. Two such formulæ are in common use, one by Hutton, given in eq. (1), and the other by Duchemin, given in eq. (2).

$$P_n = P \sin \alpha^{1.842 \cos \alpha - 1}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$P_n = P \frac{2 \sin \alpha}{1 + \sin^2 \alpha}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where P_n represents the normal component, P the horizontal force, and α the angle between the roof surface and a horizontal plane.

The values of the ratios of P_n to P for various slopes of roof have already been given on p. 74.

Roof-covering.

The roof-covering rests upon longitudinal purlins carried by the truss, usually at its panel points. If the purlins are not placed at the apices or panel-points, bending is caused in the chord members. Such stress conditions should ordinarily be avoided for simplicity and other reasons.

The amount of load carried to any panel-point is immediately determined by noting the area of surface and the load per square foot carried by each purlin. This area, or its horizontal and vertical projection, determines also the snow load and the wind load carried at any panel-point.

The weight of roof-covering varies with the different materials of which it is composed, but it may be closely estimated in advance.

The weights of roof-coverings may be approximately assumed as follows:

Iron Sheets.—The weight of iron or steel sheeting depends on the gauge thickness and whether the sheets are flat or corrugated. The exact weight may always be found in manufacturers' hand-books; it varies from 1 to 3 pounds per square foot for the thicknesses ordinarily employed.

Felt, Pitch, and Slag, or Gravel Roofing.—This combination of materials may weigh from 6 to 8 pounds per square foot, depending generally on the number of thicknesses of felt.

Slate.—Slate usually weighs from 7 to 9 pounds per square foot of roof.

Tile.—Terra-cotta tile 1 inch thick weighs about 6 pounds per square foot; tile roofing may vary then between 8 to 20 pounds per square foot.

Tin.—Tin without sheathing weighs from 1 to $1\frac{1}{2}$ pounds per square foot.

Shingles.—Shingles may weigh between from 2 to 3 pounds per square foot.

None of these weights includes weights of purlins or sheathing.

Wooden Coverings.—The weight of wooden roof-covering may be estimated by assuming the weight of lumber at 4 pounds per foot B.M.

The exact weight of roof-covering, including sheathing, purlins, bracing, gutters, ventilators, etc., must be calculated for each individual case. In a similar way all suspended weights or loads must be determined before the stresses can be computed.

The total loads carried by any truss having thus been estimated, the determination of the stresses in the members of the structure is the next procedure. This determination may be made by combining the various classes of loads and proceeding with resultants or by treating each class separately and subsequently combining the

stresses so found for each member. The method of procedure will be obvious in each case.

Art. 94.—Methods of Treatment.

Since the loads on a roof-truss are fixed in position, the methods of graphic statics are eminently adapted to finding by one continuous operation the stresses in all of the members, and analytical methods are usually only employed as a check on graphic computations.*

The method of moments or sections is used for that purpose, and it is generally sufficient to check the stress last found by the graphical method. Wind forces may either be resolved into their vertical and horizontal components and treated separately, or they may be treated without such resolution.

Let the roof-truss shown in Fig. 13, and known as the Fink, be treated. The data as to span, rise, and loading are shown in the figure. It will be found that a panel fixed load alone will be $1{,}175 + 400 = 1{,}575$ pounds. As in the case of bridge-trusses, the half panel load at the points of support do not affect the stresses in the members, for that load is carried directly by the points of support. Each reaction will consequently be $3\frac{1}{2} \times 1{,}575 = 5{,}510$ pounds.

If the stress in the member EM is required by the method of sections, a section must be passed through the members GH, FH, and EM, and moments taken about H, the intersection of GH and FH. The resulting moment equation will then be

$$5{,}510 \times 30 - 1{,}575(22\tfrac{1}{2} + 15 + 7\tfrac{1}{2}) = EM \times 15.$$

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* For a complete and detailed analysis of the graphical methods see the authors' "Influence Lines," pages 29–59. The flexure of supporting columns of roof-trusses is there also treated.

The solution of this equation will give the stress in the member *EM* a tension of 6,300 pounds. This should check the value of the stress found graphically.

The stress in any other member would be found in precisely the same manner, it only being necessary to pass such a section through the truss that the lever-arms of all members except the one whose stress is desired become zero. For instance, in determining the stress in

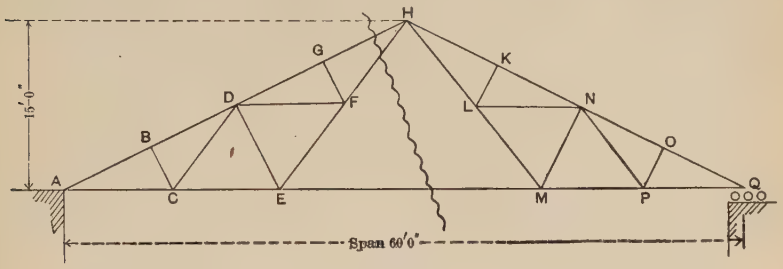


FIG. 13.

Span = 60 feet. Rise = 15 feet. Distance between trusses = 14 feet.

Dead Weights Assumed:

Roof-covering, including purlins and wind bracing, = 12 pounds per square foot of surface.

Truss = 3.8 pounds per square foot horizontal projection.
Snow load = 30 " " " " " "
Wind load = 40 " " " " vertical "

the member *CE*, a section might be passed through *DG*, *DF*, *DE*, and *CE*. The center of moments would then be found at *D*, which is the common intersection of the three members whose stress is not desired. Or *BD*, *DC*, and *CE* might be cut, making the center of moments again at *D*.

In treating wind loads on trusses it is important to include as a load on the structure the wind force acting at the abutment, for if one foot of the truss is supported on rollers, the horizontal reaction will be wholly exerted at the other foot; and if neither foot is supported on rollers, the horizontal reaction may be assumed to be equally divided between the points of support. In either case the wind load at the abutment causes stresses in

the members of the truss. For the same reason the wind stresses in a roof-truss must be found for the wind blowing from either direction, since the stresses found for these two conditions of loading are not symmetrical about the truss center line.

Art. 95. Fink Roof-truss.

The Fink truss, shown in Fig. 14, is so frequently used that formulæ for the stresses due to the *vertical* loading are given below, in which a is the length of one half the upper chord, c is half the span length, b the rise of peak, and W

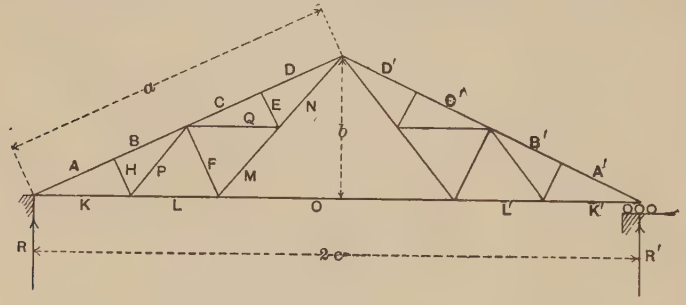


Fig. 14.

is a panel load. The following expressions may be readily derived from the figure. The primed capital letters indicate stresses.

$$A' = \frac{a}{b}R, \qquad B' = A' - W\frac{b}{a},$$

$$C' = A' - 2W\frac{b}{a}, \qquad D' = A' - 3W\frac{b}{a},$$

$$H' = E' = \frac{c}{a}W, \qquad F' = 2H' = 2\frac{c}{a}W,$$

$$P' = Q' = \tfrac{1}{2}H'\frac{a}{b} = \tfrac{1}{2}\frac{c}{b}W, \qquad K' = \frac{c}{b}R,$$

$$L' = K' - P' = \frac{c}{b}(R - \tfrac{1}{2}W), \qquad M' = \tfrac{1}{2}F'\frac{a}{b} = \frac{c}{b}W,$$

$$O' = L' - M' = \frac{c}{b}(R - \tfrac{3}{2}W), \qquad N' = M' + Q' = \tfrac{3}{2}\frac{c}{b}W.$$

Art. 96.—Other Types of Roof-trusses.

Figs. 15 to 19 illustrate other general types of roof-trusses. Those of Figs. 15 and 16 are generally used for

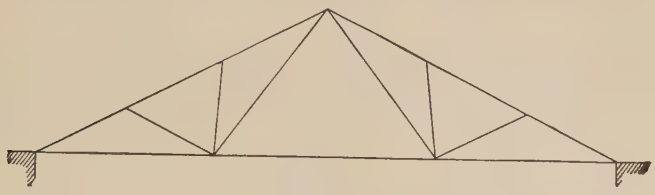


FIG. 15.

short spans up to about 50 to 60 feet, and those of Figs. 17 and 18 for span lengths up to 125 feet or more.



FIG. 16.

The saw-tooth form of truss, illustrated in Fig. 19, is used for such special purposes as roofs of exhibition halls or roofs of manufacturing establishments. Trusses are

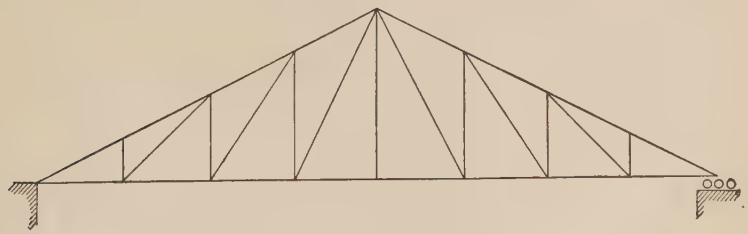


FIG. 17.

sometimes built of crescent or circular outline, and they may in addition be provided with louvres or ventilators. In many if not in all cases the sides of large buildings are

braced to the roof by oblique members,* extending from the panel-point on the lower chord adjacent to the point

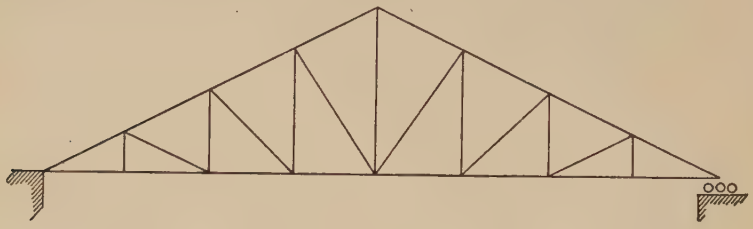


FIG. 18.

of support to some point in the side of the building. Under such conditions, the wind will cause stresses in the differ-

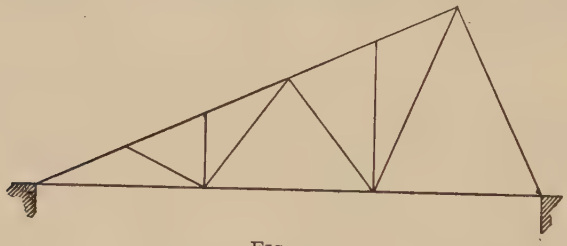


FIG. 19.

ent members of the roof-truss, to be determined independently of those already found and added algebraically to them.

* See the authors' "Influence Lines," page 48, for the detailed treatment.

CHAPTER VI.

A BRIEF STATEMENT OF INFLUENCE LINES.

Art. 97.—General.*

THE method of influence lines is the graphical method used for determining the greatest shears, bending moments, and stresses caused by a train of concentrated weights passing along a beam or bridge-truss. Obviously it must express in essence what has already been shown by the formulæ which determine the positions of moving loads for the greatest shears and bending moments and the resulting stresses.

An influence line is, therefore, a line showing the variation in any function at any section of a beam, or in any member of a truss, caused by any load moving along such a beam or truss.

Art. 98.—Influence Lines for Moments in Beams and Trusses.

It is convenient to construct these influence lines for an arbitrary load which may be considered a unit load; the effect of any other load will then be in proportion to its magnitude. The results determined from influence lines drawn for any load can, therefore, be made available for other loads by multiplying the former by the ratio

* The authors' "Graphical Method by Influence Lines" may be consulted for a complete and detailed treatment of the influence-line method.

between any desired load and that for which the influence lines are found.

AB in Fig. 1 represents a beam simply supported at each end, so that any load g resting upon it will be divided between the points of support, according to the law of the lever. Let it be desired to determine the bending moment at the section X produced by the load g in all its positions as it passes across the span from A to B. Two expressions for the bending moment must be written,

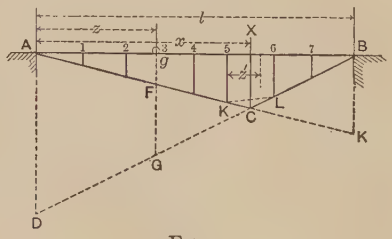


FIG. 1.

one for the load g at any point in AX, and the other for the load at any point in BX. The expression for the first bending moment is

$$M = g\frac{z}{l}(l-x), \quad \ldots\ldots\ldots \quad (1)$$

and that for the latter

$$M' = g\left(\frac{l-z}{l}\right)x. \quad \ldots\ldots\ldots \quad (3)$$

As shown in the figure, z and x, the latter locating the section at which the bending moments are to be found, are measured to the right from A. Eq. (2) shows that if the quantity $g(l-x)$ be laid off by any convenient scale, as BK at right angles to AB, XC or any similar ordinate between A and X included between AB and AK will represent the moment M by the same scale, when $z=x$,

or when z has any value between o and x. Similarly, if AD be laid off at right angles to AB by the same scale as before, to represent gx, then if $z=x$, or if it has any value between x and l, M' will be represented by XC or any similar ordinate between X and B included between BA and BD. Hence if the lines AC and CB be drawn as parts of AK and DB, any vertical intercept between AB and ACB will represent the bending moment at X produced by the load g when placed at the point from which the intercept is drawn. The lines AC and CB are the influence lines for the bending moments produced by the load g in its passage across the span AB. It is to be observed that the influence lines are continuous only when the positions of the moving load are consecutive. In case those positions are not consecutive the influence lines are polygonal in form.

If there are a number of loads g resting on the span at the same time, the total bending moments produced at X will be found by taking the sum of all the vertical intercepts between AB and ACB, drawn at the various points where those loads rest. The influence lines drawn for a single load, therefore, may be at once used for any number of loads.

The load g is considered as a unit load. If the vertical intercepts representing the bending moments by the scale used are themselves represented by y, and if W represent any load whatever, the general expression for the bending moment at X, produced by any system of loads, will be

$$\frac{1}{g}\Sigma W \cdot y. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If this expression be written as a series, the general value of the bending moment will be the following:

$$M = \frac{1}{g}(W_1 \cdot y_1 + W_2 \cdot y_2 + W_3 \cdot y_3 + \text{etc.}). \quad . \quad . \quad (4)$$

The effect of a moving train upon the bending moment at any given section is thus easily made apparent by means of influence lines. It is obvious that there will be as many influence lines to be drawn as there are sections to be considered. In the case of a truss-bridge there will be such a section at every panel-point.

A slight modification of the preceding results is to be made when the loads are applied to the beam or truss at panel-points only.

In Fig. 1, let 1, 2, 3, 4, 5, 6, and 7 be panel-points at which loads are applied, and let the load g be located at the distance z' to the right of the panel-point 5; also let the panel length be p. The reactions at 5 and 6 will then be

$$R_5 = g\frac{p-z'}{p} \quad \text{and} \quad R_6 = g\frac{z'}{p}.$$

The reaction at A will then be

$$R = g\frac{l-z}{l}.$$

Hence the moment at any section X in the panel in question will be

$$M = R\cdot x - R_5(z'-(z-x)) = g\left[\frac{l-x}{l}z - (z-z'+p-x)\frac{z'}{p}\right]. \quad (5)$$

Remembering that $(z-z')$ is a constant quantity, it is at once clear that the preceding expression is the equation of a straight line, with M and z or z' the variables. If $z'=0$, eq. (5) becomes identical with eq. (1), while if $z'=p$, it becomes identical with eq. (2). Hence the influence line for the panel in which the load is placed, as 5–6, is the straight line KL. It is manifest that when the load

g is in any other panel than that in which the section X is located, the effect of the two reactions at the extremities of that panel will be precisely the same at that section as the weight itself acting along its own line of action. Hence the two portions AK and BL of the influence line are to be constructed as if the load were applied directly to the beam or truss, and in the manner already shown. The complete influence line will then be $AKLB$, and it shows that the existence of the panel slightly reduces the bending at any section within its limits. The panel 5-6, as treated, is that of a beam in which the bending moment will, in general, vary from point to point. In a truss, however, X would always be taken at a panel-point either in the upper or lower chord.

Art. 99.—Influence Lines for Shears in Beams and Trusses.

The influence line for shears in a simple beam supported at each end can be drawn in the manner shown in Fig. 2.

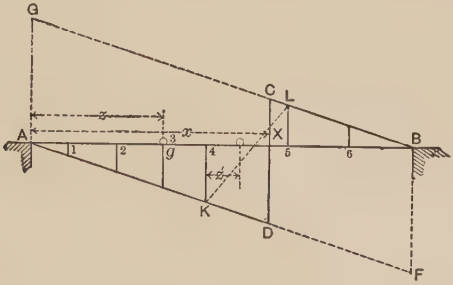


FIG. 2.

In that figure AB represents a non-continuous beam with span l supported at each end and a conventional load g at a distance z from A. The reaction at A will be

$$R = \frac{l-z}{l} \cdot g.$$

Let X be the section at which the shear for various positions of g is to be found. When g is placed at any

point between A and X the shear S at the latter point will be

$$S = R - g = -g\frac{z}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If the load is placed between B and X, the shear becomes

$$S' = R = g - g\frac{z}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Obviously these two values of the shear are equations of two parallel straight lines, that represented by eq. (6) passing through A, and that represented by eq. (7) passing through B, the constant vertical distance between them being g. Hence let BF be laid off negatively downward and AG positively upward, each being equal to g by any convenient scale. The ordinates drawn from the various positions 1, 2, 3, . . . , 6 of g on AB to AD and BC will be the shears at X produced by the load g at any point of the span, and determined by eqs. (6) and (7). The influence line, therefore, for the section X will be the broken line $ADCB$. When g is at X the sign of the shear changes, since the latter passes through the zero value.

If a train of weights W_1, W_2, W_3, etc., passes across the span, the total shear at X will be found by taking the sum of the vertical intercepts between AB and $ADCB$, drawn at the positions occupied by the various single weights of the train. If those single weights are expressed in the terms of the unit load g, the shear S will have the value

$$S = \frac{1}{g}\Sigma W \cdot y, \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

y being the general value of the intercept between AB and the influence line. The latter shows that the greatest

negative shear at X will exist when the greatest possible amount of loading is placed on AX only, while the greatest positive-shear at the same section will exist when BX only is loaded. If BX is the smaller segment of span, the latter shear is called the "counter-shear," and the former the "main shear."

If the loads are applied at panel-points of the span only, the treatment is the same in general character as that employed for bending moments. In Fig. 2 let 4 and 5 be the panel-points between which the load g is found, and let the panel length be p. Also, let z' be the distance of the weight g from panel-point 4. The reactions at A and 4 will then be

$$R = \frac{l-z}{l} \cdot g \quad \text{and} \quad R_4 = \frac{p-z'}{p} \cdot g.$$

The shear at the section X for any position of the weight g will then be

$$S = R - R_4 = g\left(\frac{z'}{p} - \frac{z}{l}\right). \quad . \quad . \quad . \quad . \quad . \quad (9)$$

As this is the equation of a straight line, with S and z or z' for the coordinates, the influence line for the panel in which the section X is located will be the straight line represented by KL in Fig. (2). If z' be placed equal to o and p successively, then will eq. (9) become identical with eqs. (6) and (7) in succession. The shears at points 4 and 5 will therefore take the same values as if the loads were applied directly to the beam. For the reasons stated in connection with the consideration of bending moments, loads in other panels than that containing the section for which the influence line is drawn will have the same effect on that section as if they were applied directly to the beam or truss. Hence $AKLB$ is the complete influence line for this case.

It is evident that there must be as many influence lines drawn as there are sections to be discussed. Also, if g is taken as some convenient unit, i.e., 1,000 or 10,000 pounds, it is clear that the labors of computation will be much reduced.

Art. 100.—Application of Influence-line Methods to Trusses.

The employment of influence lines for truss computations may be illustrated by determining the moment and shear in a single section of the truss shown in Fig. 3, when carrying the moving load (Cooper's E 40) exhibited

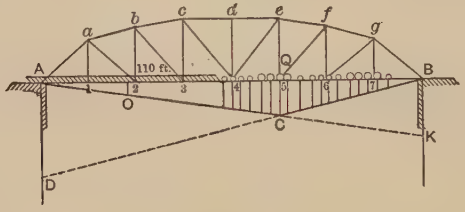


FIG. 3.

DATA.

Length of truss between centers of end pins	240 feet
Panel lengths, all equal	30 "
Height of e–5	40 "
" " f–6	36 "
" " g–7	30 "

in Fig. 4, although the use of these lines may be much extended beyond this simple procedure.

If the criterion of eq. (1), Art. 72, be applied to this moving load, passing along the truss from left to right, it will be found that the greatest bending moment is produced at the section Q when the second driving-axle of the second locomotive is placed on the truss section in question, as shown in Fig. 3.

The unit load to be used in connection with the influence lines will be taken at 10,000 pounds. Remem-

bering that the panel lengths are each 30 feet, it will be seen that the panel-point Q is 150 feet from A. Hence the product gx will be 1,500,000 foot-pounds. Similarly the product $g(l-x)$ will be 900,000 foot-pounds. Laying

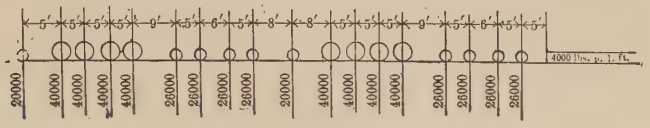


FIG. 4.

off the first of these quantities, as AD, at a scale of 1,000,000 foot-pounds per linear inch, and the second quantity, as BK, by the same scale, the influence line ACB can at once be completed. Vertical lines are next to be drawn through the positions of the various weights, including one through the center of the uniform train load 110 feet in length, resting on the truss. The vertical line through the center of the uniform train load is shown at O. By carefully scaling the vertical intercepts between AB and ACB, and remembering that each of the loads on the truss must be divided by 10,000, the following tabulated statement will be obtained, the sum of the intercepts for each set of equal weights being added into one item, and all the items of intercepts being multiplied by 1,000,000:

0.195	× 110	× 0.4	× 1,000,000	= 8,580,000	foot-pounds
1.78	× 2.6	×	"	= 4,628,000	"
2.14	× 4	×	"	= 8,560,000	"
0.485	× 2	×	"	= 970,000	"
1.525	× 2.6	×	"	= 3,965,000	"
0.9	× 4	×	"	= 3,600,000	"
0.12	× 2	×	"	= 240,000	"
				2) 30,543,000	"
			Moment for one truss	= 15,271,500	"

The lever-arm of *ef*, i.e., the normal distance from *Q* to *ef*, is 39.7 feet. Hence the stress in *ef* is

$$\frac{15,271,500}{39.7} = 384,700 \text{ pounds.}$$

All the chord stresses obviously can be found in the same manner.

In order to place the same moving load so as to produce the greatest shear at the same section *Q*, the criterion of eq. (4), Art. 85, must be employed. The dimensions of the truss give the following data to be used in that equation: $i = 210$ feet, $m = 60$ feet, and $p = 30$ feet. Hence

$$\frac{l(m+i)}{pi} = 10\tfrac{2}{7}, \quad \frac{l}{i} = 1\tfrac{1}{7}.$$

Introducing these quantities into eq. (4), and remembering that the train moves across the bridge from *A*, it would be found that the second axle of the first locomotive must be placed at the section *Q*, as shown in Fig. 5, which exhibits the lower-chord panel-points numbered from 1 to 7. The conventional unit load *g* is taken in this case at 20,000 pounds. It is represented as *AG* and *BF* (Fig. 5),

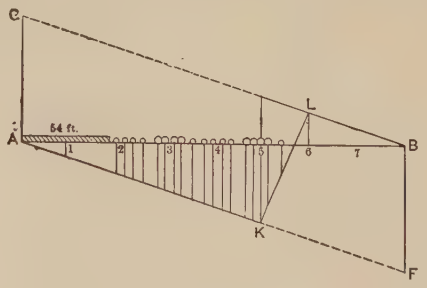


FIG. 5.

laid off at some convenient scale. *K* is immediately under panel-point 5, and *L* is immediately under panel-point 6;

hence the broken line $AKLB$ is the influence line desired. The vertical lines are then drawn from each train concentration in proper position, all as shown, including the vertical line through the center of the 54 feet of uniform train load on the left. The summation of all the vertical intercepts between AB and the influence line AKL, having regard to the scale and to the ratio between the various loads and the unit load g, will give the following tabular statement:

0.22	× 54 ×	0.2	× 10,000	= 23,760	pounds
2.2	×	1.3	× "	= 28,600	"
3.02	×	2	× "	= 60,400	"
0.9	×	1	× "	= 9,000	"
4.06	×	1.3	× "	= 53,780	"
4.53	×	2	× "	= 90,060	"
0.5	×	1	× "	= 5,000	"
				2) 270,600	"
			Shear for one truss	= 135,300	"

These simple operations illustrate the main principles of the method of influence lines. The complete development of the method is but an extension of them to meet the various conditions found in practical work.

CHAPTER VII.

SWING-BRIDGES.

Art. 101.—General Considerations.

SWING-BRIDGES may be divided into two classes: those with center-bearing turntables, Figs. 1 and 2, and those

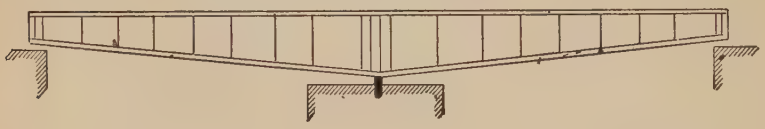


FIG. 1.

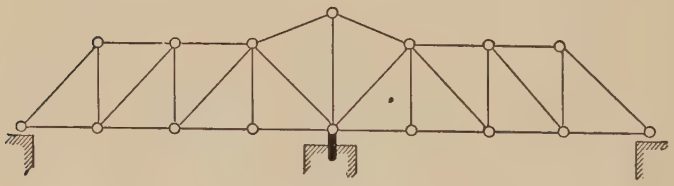


FIG 2.

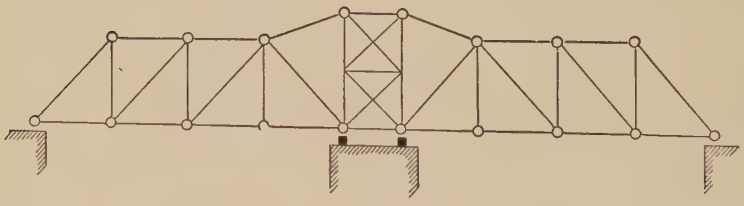


FIG. 3.

with rim-bearing turntables, Fig. 3. In the first class the entire reaction at the pivot-pier is exerted through a central

pivot, or a nest of one or more series of solid conical rollers, usually the latter. In such a case there may be, and usually is, a circular drum or other framework supported on wheels running on a circular track and used solely for the purpose of steadying the bridge while open.

The reaction at the pivot-pier of a rim-bearing turntable is exerted through the circular track on which the wheels supporting the drum or framework of the table turn. The object of the pivot in such a case is to make the bridge turn truly about a center.

There is evidently but one point of support at the pivot-pier with a center-bearing turntable, i.e., at the center.

With a rim-bearing turntable, however, there may be two or more points of support at the pivot-pier, but it will be shown hereafter that it will never be necessary to consider more than two for one system of triangulation in the bracing. Again, whether the turntable be center- or rim-bearing, there may be three different methods of supporting or securing the outer extremities of the two arms of the bridge:

I. These extremities may simply rest on supports, so that the reaction will always be zero or upward.

II. Those extremities may be fastened down or latched to the piers when the bridge is not open. The reaction may then be nothing, upward or downward.

In these two cases the reactions at the extremities of the arms will be zero when the bridge is simply closed and supporting no moving load.

In I, when the moving load is on one arm, the extremity of the other may be slightly raised from its support. In II, however, that extremity will be held down by the latching apparatus, i.e., the reaction will be downward. The object of the latching apparatus is thus seen to be the prevention of the hammering of a truss end on its support.

III. Finally, the truss ends, when the bridge is closed,

may be raised any desired amount by proper machinery. The object of this arrangement is to insure a reaction at the extremities which will always be nothing or upward, and thus obviate hammering at the truss ends.

In III the entire bridge weight does not rest at the pivot-pier, as the lifting of the ends takes up a part or even the whole of it.

Recapitulating, then, the ends of a swing-bridge may be:

(I) Simply supported;
(II) Latched down;
(III) Lifted up.

Art. 102.—General Formulæ for the Case of Ends Simply Supported—Two Points of Support at Pivot-pier—One Point of Support at Pivot-pier.

With two points of support at the pivot-pier there usually arises the case of a continuous beam resting on four points of support, as shown in Fig. 4. The notations of

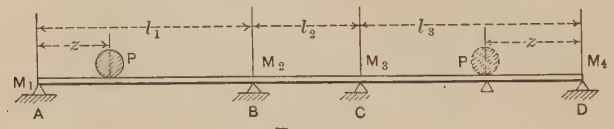


FIG. 4.

the spans and bending moments at the different points of support are sufficiently well shown in the figure. The points of support will all be taken in the same horizontal line, as the formulæ of the theorem of three moments for that condition will also apply to any configuration belonging to a state of no stress, provided the truss may be considered straight between any two points of support. Any truss may be considered straight when an equivalent solid beam has a neutral surface which is plane before flexure, and a straight solid beam is "equivalent" to a straight

truss when equal moments of inertia and resistance are found at the same section in the two structures.

The theorem of three moments in the ordinary form is not then strictly applicable to a continuous truss with one chord curved, although it is constantly employed as a sufficiently close approximation.

Two Points of Support at Pivot-pier.

Usually the load on l_2 is supported on short girders or beams resting at B and C, for there is then less complication of stresses in the trusses, and such an arrangement is probably more economical in material.

In the present case M_1 and M_4 will each be equal to zero.

Let z denote the distance of the point of application of any force, P, from the left-hand end of the left-hand span, or right-hand end of right-hand span. In l_1, z would be measured from A, while in l_3 it would be measured from D.

The formula expressing the theorem of three moments for all supporting points in the same level and for a beam straight between points of support becomes, by using the notation of Fig. 4,

$$M_1 l_1 + 2M_2(l_1 + l_2) + M_3 l_2 + \frac{1}{l_1}\overset{1}{\Sigma} P(l_1^2 - z^2)z + \frac{1}{l_2}\overset{2}{\Sigma} P(l_2^2 - z^2)z = 0.$$

The symbols $\overset{1}{\Sigma}$ and $\overset{2}{\Sigma}$ indicate summations for the spans l_1 and l_2.

Applying the above equation to spans l_1 and l_2, and then to l_2 and l_3, there will result, bearing in mind the circumstances of the present case,

$$2M_2(l_1 + l_2) + M_3 l_2 + \frac{1}{l_1}\overset{1}{\Sigma} P(l_1^2 - z^2)z = 0; \quad . \quad . \quad (1)$$

$$M_2 l_2 + 2M_3(l_2 + l_3) + \frac{1}{l_3}\overset{3}{\Sigma} P(l_3^2 - z^2)z = 0. \quad . \quad . \quad (2)$$

If eq. (1) be multiplied by l_2, and eq. (2) by $2(l_1 + l_2)$, and if the results so obtained be subtracted, and if the following notation be used,

$$c = \frac{l_1}{l_2}, \quad b = \frac{l_3}{l_2}, \quad n = \frac{z}{l_1}, \quad m = \frac{z}{l_3},$$

there will result

$$M_3 = \frac{c^2 l_2 \overset{1}{\Sigma} P(1 - n^2)n - 2b^2 l_2 (c+1)\overset{3}{\Sigma} P(1 - m^2)m}{4(b+1)(c+1) - 1}. \quad . \quad (3)$$

Eq. (1) then gives

$$M_2 = -\frac{M_3 + c^2 l_2 \overset{1}{\Sigma} P(1 - n^2)n}{2(c+1)}. \quad . \quad . \quad . \quad (4)$$

Eq. (2) will evidently give another expression for M_2, but it is not necessary to write it. These equations of moments are essential to determine the reactions.

Let R_1, R_2, R_3, and R_4 be the reactions at the points of support A, B, C, and D respectively, Fig. 4. Then, adapting the formulæ for reactions from the theorem of three moments to the notation of the present case, there may at once be written

$$R_1 = \overset{1}{\Sigma} P(1 - n) + \frac{M_2}{l_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$R_2 = \overset{1}{\Sigma} Pn - \frac{M_2}{l_1} - \frac{M_2 - M_3}{l_2}; \quad . \quad . \quad . \quad . \quad (6)$$

$$R_3 = + \frac{M_2 - M_3}{l_2} + \overset{3}{\Sigma} Pm - \frac{M_3}{l_3}; \quad . \quad . \quad . \quad (7)$$

$$R_4 = \overset{3}{\Sigma} P(1 - m) + \frac{M_3}{l_3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

As should be the case, there is found

$$R_1+R_2+R_3+R_4=\overset{1}{\Sigma}P+\overset{3}{\Sigma}P.$$

If M_2 equals M_3 (as in the case of partially continuous swing-bridges, to be explained later), R_3 in eq. (7) becomes equal to R_4 in eq. (8), but with a reversal of sign, provided there is no load on the span l_3. In eqs. (5) and (6), then,

$$R_2=\overset{1}{\Sigma}P-R_1.$$

In ordinary swing-bridges where $l_1=l_3=l$, and a single weight P rests on l_1,

$$R_1=P(1-n)\left\{1-(1+n)n\frac{2c}{4(c+1)-\frac{1}{c+1}}\right\}; \quad . \quad (5a)$$

$$R_4=P(1-n^2)n\frac{c}{4(c+1)^2-1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

$$R_3=c\{R_1-R_2-P(1-n)\}-R_4; \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

$$R_2=P-R_1-R_4+R_3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

It may sometimes be convenient to use the following equation derived from eq. (1):

$$M_3=-\{2M_2(c+1)+c^2l_2\overset{1}{\Sigma}P(1-n^2)n\}. \quad . \quad . \quad (9)$$

These are all the equations necessary for the solution of the case of two supports at the pivot-pier, existing if the turntable is rim-bearing.

One Point of Support at Pivot-pier.

If there is only one point of support at the pivot-pier, the case reduces to that of a continuous beam of two spans only, as shown in Fig. 5.

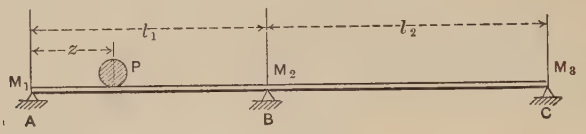


FIG. 5.

As A and C are points of support only, M_1 and M_3 are each zero; hence if m now stands for $\frac{z}{l_2}$, the equation immediately preceding eq. (1) gives

$$M_2 = -l_2 \frac{c^2 \overset{1}{\Sigma} P(1-n^2)n + \overset{2}{\Sigma} P(1-m^2)m}{2(c+1)}. \quad . . \quad (10)$$

There will also result

$$R_1 = \overset{1}{\Sigma} P(1-n) + \frac{M_2}{l_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$R_2 = \overset{1}{\Sigma} Pn - \frac{M_2}{l_1} + \overset{2}{\Sigma} Pm - \frac{M_2}{l_2}; \quad . \quad . \quad . \quad (12)$$

$$R_3 = \overset{2}{\Sigma} P(1-m) + \frac{M_2}{l_2}. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

If $l_1 = l_2 = l$, then $c = 1$ and eq. (10) becomes

$$M_2 = -\frac{l}{4}\left\{ \overset{1}{\Sigma} P(1-n^2)n + \overset{2}{\Sigma} P(1-m^2)m \right\}. \quad . \quad . \quad (13a)$$

There is again found

$$R_1 + R_2 + R_3 = \overset{1}{\Sigma} P + \overset{2}{\Sigma} P.$$

These complete the general formulæ needed for the case of ends supported.

DETERMINATION OF THE SIGNS OF THE REACTIONS.

Two Points of Support at Pivot-pier.

Some very important deductions are to be drawn from eqs. (5), (6), (7), (8), (11), (12), and (13), considering them applied to bridges with rim-bearing turntables.

Those equations are so written that a positive value of R means a reaction *upward* in direction, while a negative value indicates a *downward* reaction.

In the case of Fig. 4, let the span l_3 be supposed free of any loads P, then the term involving the summation $\overset{3}{\Sigma}$ will disappear. Eq. (3) then shows that M_3 will *always* be positive; consequently eq. (4) shows that M_2 will *always* be negative.

Using these results in connection with eqs. (5), (6), (7), and (8), it is at once seen *that R_2 and R_4 will always be positive, while R_3 will always be negative.*

It may also be shown that eq. (5) makes R_1 positive in such cases as always arise in an engineer's practice, although that equation apparently shows that R_1 may, under some circumstances, be negative, since M_2 is always negative in the case taken, while $(l_1 - z)$ is, of course, always positive.

By deducing the value of M_2 from eqs. (3) and (4), and introducing it in eq. (5), there will result

$$R_1 = \frac{1}{l_1}\left[\overset{1}{\Sigma} P(l_1 - z) - \frac{1}{l_1}\left(\frac{2(l_2 + l_3)}{4(l_1 + l_2)(l_2 + l_3) - l_2^2}\right)\overset{1}{\Sigma} P(l_1^2 - z^2)z\right].$$

If, as in Fig. 6, l_2 is very small in comparison with l_1 or l_3, and if, at the same time, z is small, there may be written for a single weight P, nearly,

$$R_1 = \frac{1}{l_1}\left[Pl_1 - \frac{1}{2l_1^2} \cdot Pl_1^2 z \right],$$

which is evidently positive.

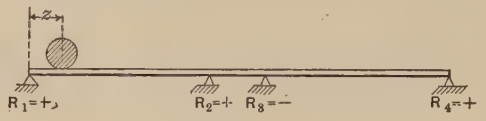


FIG. 6.

If, on the contrary, l_2 is small and z large (Fig. 7), there may be written for a single weight P, nearly:

$$R_1 = \frac{1}{l_1}\left[P(l_1 - z) - \frac{1}{2l_1^2} P(l_1 + z)(l_1 - z)z \right].$$

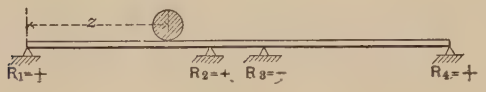


FIG. 7.

In this expression R_1 can be equal to zero only by supposing the negative quantity within the brackets to be larger, numerically, than it ought to be, i.e., by making $(l_1 + z) = 2l_1$, and $z = l_1$; hence it can never be negative.

If $l_1 = l_2 = l_3 = l$, as in Fig. 8, the general value of R_1 takes the form

$$R_1 = \frac{1}{l_1}\left[P(l - z) - \frac{4}{15 l^2} P(l^2 - z^2)z \right].$$



FIG. 8.

If z is small, there results nearly

$$R_1 = \frac{1}{l_1}\left[Pl - \frac{4}{15} Pz \right].$$

If z is large, nearly,

$$R_1 = \frac{1}{l_1}\left[P(l-z) - \frac{8}{15}P(l-z)\right].$$

In neither case, therefore, can the reaction be negative.

It may, consequently, be assumed as a principle that if l_2 is small in reference to l_1 or l_3, or if it is equal to those quantities, the reaction R_1, in the case supposed, must always be positive, and within those limits will be found all cases of swing-bridges.

Since any load on the span l_1 makes the reactions at A and D positive, considered by itself, so any load on l_3 will, of itself, make the reactions at D and A positive. Consequently, as there is never any load on l_2, the reactions at A and D *will always be positive, and the ends of the bridge will never tend to rise from their points of support.* No "hammering," therefore, can take place, in this case, at the ends.

The case of the two points of support, B and C, taken in connection with a center-bearing turntable will be considered further on.

One Point of Support at Pivot-pier.

Fig. 5 represents the case of either a center- or rim-bearing turntable with only one point of support at the pivot-pier. The two cases are coincident in all their circumstances.

Eq. (10) shows M_2 to be always negative. Consequently, if there is no load on l_2, R_1 and R_2 will always be positive, while R_3 will always be negative.

When span l_1 carries load, however, the span l_2 may at the same time support just enough load to make R_3 equal to zero; more load than that will make R_3 positive or upward in direction.

But in the present case the point C is simply *a point of support*, consequently no negative or downward reaction can exist there. It becomes necessary, therefore, to determine just how much load on l_2, combined with the full load on l_1, will make $R_3 = 0$. For this purpose, M_2 must be taken from eq. (10) and inserted in eq. (13), while in the latter R_3 must be just equal to zero. For the sake of brevity, let $\frac{1}{l_1}\overset{1}{\Sigma}P(l_1^2 - z^2)z$ be represented by A.

Then from eq. (10)

$$M_2 = -\frac{A}{2(l_1 + l_2)} - \frac{1}{2l_2(l_1 + l_2)}\overset{2}{\Sigma}P(l_2^2 - z^2)z. \quad . \quad (14)$$

A is a constant quantity so far as this operation is concerned.

Putting this value of M_2 in eq. (13) after making $R_3 = 0$,

$$2l_2(l_1 + l_2)\overset{2}{\Sigma}P(l_2 - z) - \overset{2}{\Sigma}P(l_2^2 - z^2)z = Al_2. \quad . \quad .(15)$$

Eq. (15) indicates what disposition of the load on the span l_2 will make $R_3 = 0$, and it will be seen to be of very easy application.

The determinations of R_1, R_2, and R_3, for all loading in excess of that indicated by eq. (15), will require the use of eqs. (11), (12), and (13) as they stand; for all loading less than that amount, however, $R_3 = 0$, and the reactions R_1 and R_2 are to be found by the simple principle of the lever, considering l_2 as a simple overhanging arm, or cantilever. These operations will be shown in detail hereafter.

In this case it is evident that "hammering" will take place at the ends with certain dispositions of loading.

Art. 103.—Example of Swing-bridge with Ends Simply Supported —Two Points of Support at Center—Partial Continuity.

The general formulæ of the preceding article will be applied to the truss shown by skeleton diagram in Fig. 9. The intermediate verticals U_2L_2 and U_3L_3, with U_5L_5 and the inclined web members L_0U_1 and U_4L_5, are in compression, while the remaining vertical and diagonal web members are in tension.

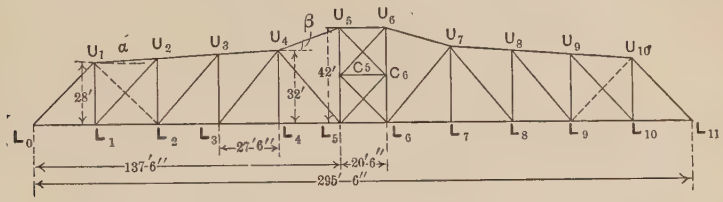


FIG. 9.

As shown in the diagram, the total length of the trusses between the centers of end pins is 295.5 feet. The following statement gives all the dimensional data required in the succeeding computations:

Center depth, $U_5L_5 = 42$ feet.
Depth, $U_4L_4 = 32$ "
Depth, $U_1L_1 = 28$ "
Length of center panel = 20.5 feet
Other panel lengths all = 27.5 feet.

The trigonometrical quantities which will be required are as follows:

$\tan L_0U_1 = 0.982$;	$\sec L_0U_1 = 1.4$;
$\tan \alpha = 0.0485$;	$\sec \alpha = 1.00121$;
$\tan \beta = 0.364$;	$\sec \beta = 1.064$;
$\tan L_1U_2 = 0.938$;	$\sec L_1U_2 = 1.371$;
$\tan L_2U_3 = 0.897$;	$\sec L_2U_3 = 1.343$;
$\tan L_3U_4 = 0.859$;	$\sec L_3U_4 = 1.319$;
$\tan U_1L_2 = 0.9762$;	$\sec U_1L_2 = 1.397$.

The upper chord $U_1 \ldots U_4$ is a straight line and it intersects the lower chord twenty panel lengths from the free extremity of the arm. The trusses are placed 16 feet apart, centers transversely. The moving load will be that specified by Cooper as E 40, shown on p. 56. The fixed load will be as follows:

Track	400	pounds per lineal foot.
Floor-beams and stringers	320	" " " "
Trusses	980	" " " "
Total	1,700	" " " "

The weight of trusses will be taken to be divided equally between the upper and lower panel-points, but the track, floor-beams, and stringers will be taken wholly at the latter. Hence the upper and lower panel fixed loads will be as follows:

Lower panel load	$\left(\frac{1,210}{2}\right) \times 27.5 =$	16,650 lbs.
Upper panel-point	$\left(\frac{490}{2}\right) \times 27.5 =$	6,750 lbs.
Total "		23,400 lbs.

In order that the weight of the locking gear at the extremity of each arm may be properly provided for, the weight concentrated at the free extremity of each truss will be taken at 10,000 pounds, which is somewhat more than one half of each of the other lower panel fixed loads.

The panel load at the top of U_5L_5 will be

$$\left(\frac{27.5+20.5}{2}\right) \times \left(\frac{490}{2}\right) = 5,880 \text{ lbs.}$$

Fixed-load Stresses.

The stresses due to the fixed loads given above will first be found. Since it is assumed that the ends of the arms are simply supported when the draw is closed, the stresses arising from the fixed or own weights are cantilever stresses identical with those of the open draw; in other

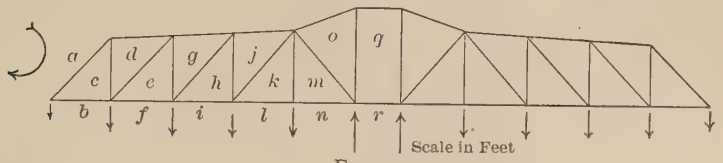


FIG. 10.

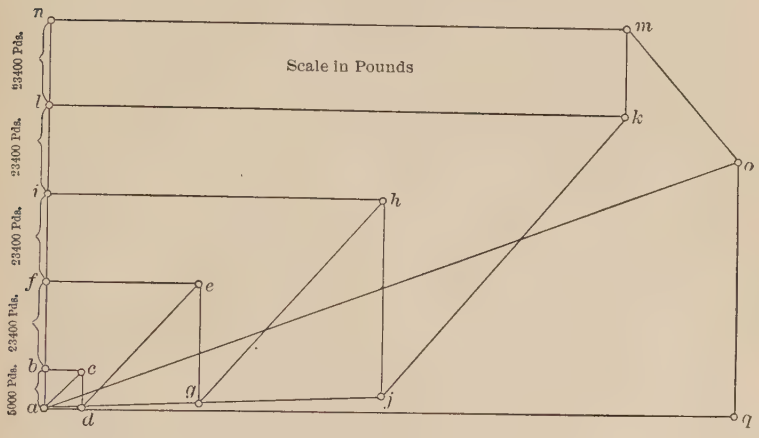


FIG. 11.

words, the entire fixed load is carried at the two points of support over the drum on the pivot-pier. These stresses are most easily found by graphical methods as shown in Fig. 11. The system of notation illustrated in Fig. 10 is employed. The diagram is started at the extremity of the left arm, at which point there are only two unknown stresses *ac* and *cb*. The final stress found is *aq*, the stress in the upper-chord member over the center span. It is

not necessary to explain in detail the construction of the figure. The accuracy of the diagram may be checked by obtaining the last stress aq by the method of sections.

Passing a section, therefore, through the members U_5U_6 and L_5L_6 and choosing as a center of moments the panel-point L_5 the following equation of moments results:

$$aq = U_5U_6 = \frac{10{,}000 \times 137.5 + 23{,}400(110 + 82.5 + 55 + 27.5)}{42}$$

$$= 186{,}000 \text{ lbs.}$$

Since this value checks that found graphically (Table I) it may be assumed that all other values are correct.

There is no dead-load stress in the counter U_1L_2. Also, since the members U_5C_6, U_6C_5, C_5L_6, and C_6L_5 are designed simply to steady the bridge when open, they sustain no dead-load stress that can be computed.

TABLE I.—STRESSES DUE TO FIXED OR DEAD LOAD.

Member.		Dead-load Stress in Pounds.	Member.		Dead-load Stress in Pounds.
L_0L_1	cb	− 9,820	U_1L_1	cd	− 16,280
L_1L_2	ej	− 40,700	L_1U_2	de	+ 45,140
L_2L_3	hi	− 89,860	U_2L_2	eg	− 38,180
L_3L_4	kl	−155,040	L_2U_3	gh	+ 73,630
L_4L_5	mn	−155,040	U_3L_3	hj	− 59,190
L_5L_6	qr	−186,000	L_3V_4	jk	+100,040
L_0U_1	ac	+ 14,000	U_4L_4	km	+ 16,700
U_1U_2	ad	+ 9,840	U_4L_5	mo	− 47,460
U_2U_3	ag	+ 40,740	L_5U_5	oq	− 73,570
U_3U_4	aj	+ 89,960			
U_4U_5	ao	+197,860			
U_5U_6	aq	+186,000			

Moving-load Stresses—Partial Continuity.

The moving-load stresses are next to be found, and in finding them the condition of "partial continuity" will be assumed, i.e., it will be assumed that no shear can pass from one arm of the bridge across the center panel

to the other. The web members in the center panel are light in section, and proportioned only to hold the bridge steady, when open, against any tendency to vertical vibration by the wind, or irregular or uneven operation of the turning machinery. These vibration stresses are indeterminate, but it is known from actual experience with long drawbridges that they are small and amply resisted by rods or braces quite inadequate to carry more than a small portion of the moving-load shear. Hence such a construction is adopted as will not afford provision for those moving-load shears at the center, leaving only the incidental stresses of turning to be cared for.

The effect of this arrangement is assumed to make either arm of the bridge a simple truss supported at each end for all moving loads on that arm, *so long as the other arm carries no moving load.*

It should be remembered, as already stated, that the span l_2 (ordinarily termed the center panel), included between the central points of support, sustains no load.

If the moving load partially covers both arms, or the whole of one arm and a part of the other, it is to be divided into three parts. Two of these parts, together with the reactions R_1 and R_4 due to them, produce equal and opposite moments at the center. For these two parts, consequently, the truss is one of *perfect continuity*, with the main diagonals at the center omitted. This last condition is admissible, because for these two parts the shear at the center will be zero.

For convenience these two parts will be called "*balanced*," while the third part will be called "*unbalanced*."

For the unbalanced part, the arm or span in which it is found will be a simple truss supported at each end.

If the panel loads are uniform in amount, balanced loads will be symmetrically placed in reference to the center.

If the panel loads are not uniform in amount, the balanced portions would be determined by equating M_2 to M_3, with the aid of eqs. (3) and (4) of Art. 102.

Coupling these statements with the principles deduced in the preceding articles, it will at once be seen that the greatest reaction R_1 at L_0, Fig. 9, will exist with the moving load over the whole of the left arm, for any moving load on the right arm will balance a part of that on the left, and relieve, to some extent, the reaction at L_0.

Although the moving load consists of various wheel concentrations followed by a uniform train, which can readily be used for the moving load on one arm only, according to the principles of Arts. 85 and 88, their use in the same general manner for balanced moving loads would lead to unnecessary complication. Hence, for the latter loads, a fixed series of panel weights, resulting from one position of the diagram concentrations, will be used in the corresponding stage of the computations. By this device the determination of the stresses will be much simplified, and the results will be essentially accurate.

Live-load Stresses in Span $L_0 \ldots L_5$ as a Simple and Non-continuous Span.

Members L_0U_1, L_1U_2, U_1L_2.

The maximum moving-load compression in the end post L_1U_1 will first be found, and then the counter-stresses and that in U_1L_1, for all of which the arm of the draw will be treated as a non-continuous span, with the moving load passing over it from the center toward the end. The positions of the moving load for the greatest value of these stresses will, hence, be found by the principles of Arts. 70, 85, and 88. Eq. (4) of Art. 70 shows that the third driver of the front locomotive must be placed at the foot of U_1L_1 in order to give the greatest shear in

L_0U_1, or, what is the same thing, the greatest reaction at its foot. There will then be 19 feet of uniform load on the span. Such a position of the moving load will make the shear in question $154{,}000 - 17{,}500 = 136{,}500$ pounds. Hence

$$L_0U_1 = -136{,}500 \times 1.4 = -191{,}000 \text{ pounds.}$$

In order to determine whether the counter U_1L_2 is necessary, let it be supposed omitted, and the resulting greatest moving-load compression found in L_1U_2. Eq. (4) of Art. 85 shows that the latter will exist with the third driver of the front locomotive at L_2. Wheel No. 17 will be $1\frac{1}{2}$ feet from L_6; whence will result a reaction at L_0, and a load at L_1 of 102,000 pounds and 17,500 pounds respectively. By taking moments about the point of intersection of the chords, and using as the lever-arm for L_1U_2 21 panel lengths divided by sec L_1U_2

$$L_1U_2 = -\frac{102{,}000 \times 20 - 17{,}500 \times 21}{21} \sec L_1U_2$$

$$= -12{,}900 \times 1.371 = -113{,}500 \text{ pounds.}$$

The fixed-load tension in L_1U_2 has already been found to be 45,100 pounds, which is much smaller than the 113,500 pounds of moving-load compression just found. Hence, as L_1U_2 can really sustain no compression, the counter U_1L_2 must be introduced, and its greatest tension found with the same position of moving load, and, also, under the assumption that L_1U_2 does not exist. Remembering that the lever-arm of U_1L_2 is 22 panel lengths divided by its secant,

$$U_1L_2 = \frac{102{,}000 \times 20 - 17{,}500 \times 21}{22} \sec U_1L_2$$

$$= +82{,}100 \times 1.4 = +115{,}000 \text{ pounds.}$$

Member L_2U_3.

By a precisely similar use of eq. (4) of Art. 85, it will be found that the greatest moving-load compression will exist in L_2U_3 with wheel No. 2 at L_3. Wheel No. 10 will then be 4 feet from L_6. This position will give a reaction of 38,100 pounds at L_0, and an advance load of 2,900 pounds at L_2. Again taking moments about the point of intersection of the chords, the greatest moving-load compression in L_2U_3 will be found to be

$$L_2U_3 = -\frac{38{,}100 \times 20 - 2{,}900 \times 22}{22} \sec L_2U_3 = -42{,}600 \text{ lbs.}$$

The fixed-load tension in L_2U_3 has already been found to be 73,630 pounds, which is almost twice the moving-load compression just found. Hence no compression can ever exist in L_2U_3, and no counter will be needed in the same panel with it, nor, indeed, any other counter than U_1L_2.

The same condition of loading will give the greatest moving-load tension in U_3L_3, but computations will show that it is only about one half the fixed-load compression (as would be inferred from the results for L_2U_3). Hence no counter-bracing of that member is required.

These operations show the method to be pursued in determining the requisite counters, and, also, that it is essentially identical with that used for non-continuous spans.

Member U_1L_1.

As U_1L_1 is a simple hanger, it will take its greatest tension with the greatest floor-beam reaction at its foot. This latter will exist with the same position of moving load as that which gives the greatest stress in L_0U_1—i.e., with the third driver of the front locomotive at L_1, and

its value is 84,200 pounds. The shear existing at the same time in L_0U_1 has been found to be 136,500 pounds, and as the coexisting vertical component of the stress in U_1U_2 is about 8,000 pounds only, the counter U_1L_2 must act in tension to carry sufficient shear over to L_0U_1 to make up its shear of 136,500 pounds. In other words, the vertical components of U_1L_2 and U_1U_2 added to the stress in U_1L_1 must equal 136,500 pounds. Hence the stress in U_1L_1 is simply the greatest possible load at its foot.

$$\therefore\ U_1L_1 = +84{,}200 \text{ pounds.}$$

Member U_4L_5.

The member U_4U_5 sustains no stress, under the condition of partial continuity, with the moving load on one arm only; hence it is to be ignored in finding the stress in U_4L_5. The greatest compression in this latter member will therefore exist with precisely the same position of moving load as was used for L_0U_1, but changed end for end on the span, so as to be headed toward the center instead of the end of the arm.

By using the same shear as for L_0U_1, therefore

$$U_4L_5 = -136{,}500 \times \sec U_4L_5 = -180{,}000 \text{ pounds.}$$

Member U_4L_4.

The tension member U_4L_4 is a simple hanger, with the maximum floor-beam reaction at its foot, as was found for U_1L_1. Hence

$$U_4L_4 = +84{,}200 \text{ pounds.}$$

Members L_3U_4 and L_2U_3.

The moving load will pass on the arm from the end toward the center, for the remaining web members. The

position of moving load for L_3U_4 in accordance with eq. (4) of Art. 85, requires the third driver to be at L_3, wheel No. 17 being $1\frac{1}{2}$ feet from L_0. This position gives a reaction of 102,000 pounds at L_5 and an advance weight of 17,500 pounds at L_5. Hence

$$L_3U_4 = \frac{102{,}000 \times 25 - 17{,}500 \times 24}{23} \quad \text{sec } L_3U_4 = +122{,}000 \text{ lbs.}$$

In the same manner it is found that the greatest tension in L_2U_3 will exist with the second driver of the first locomotive at L_2. Wheel No. 11 will then be 4 feet from L_0. The reaction at L_5 will be 47,600 pounds and the weight at L_3 8,400 pounds. Hence, by moments, as usual, about the point of intersection of the chords

$$L_2U_3 = \frac{47{,}600 \times 25 - 8{,}400 \times 23}{22} \quad \text{sec } L_2U_3 = +60{,}800 \text{ pounds.}$$

Members U_3L_3, L_1U_2, and L_2U_2.

Since U_3L_3 sustains its greatest compression for the same condition of moving load as L_2U_3,

$$U_3L_3 = -\frac{47{,}600 \times 25 - 8{,}400 \times 23}{23} = -43{,}300 \text{ pounds.}$$

Again, eq. (4) of Art. 85 shows that the first driver of the front locomotive must be at the foot of U_1L_1 in order to give L_1U_2 its greatest tension, wheel No. 6 being $3\frac{1}{2}$ feet from L_0. Hence the reaction at L_5 will be 14,500 pounds and the weight at L_2 2,900 pounds. Therefore

$$L_1U_2 = \frac{14{,}500 \times 25 - 2900 \times 22}{21} \quad \text{sec } L_1U_2 = +19{,}600 \text{ pounds.}$$

The same position of moving load gives the greatest compression in U_2L_2; hence

$$U_2L_2 = -\frac{14{,}500 \times 25 - 2{,}900 \times 22}{22} = -13{,}600 \text{ pounds.}$$

These results complete the moving-load web stresses for the moving load on one arm only, leaving the chord stresses yet to be determined.

Lower-chord Members.

The lower-chord panels L_0L_1 and L_1L_2 receive their greatest stresses with L_0U_1, and they are to be found by simply multiplying the shear in that member by the tangent for L_0U_1. Hence

$$L_0L_1 = L_1L_2 = 136{,}500 \times 0.982 = +134{,}000 \text{ pounds.}$$

Similarly, the panels L_3L_4 and L_4L_5 sustain their greatest stresses with U_4L_5. Hence

$$L_3L_4 = L_4L_5 = 136{,}500 \times 0.859 = +117{,}000 \text{ pounds.}$$

In order to determine the greatest stress in lower-chord panel L_2L_3, recourse must be had to eq. (1) of Art. 72. That equation shows that, with the moving load passing on the arm from right to left, the third driver of the second locomotive must be at L_3 with 20 feet of the uniform train load resting on the arm adjacent to the center. This position gives a bending moment about U_3 (the center of moments for L_2L_3) of 5,832,000 foot-pounds. Hence

$$L_2L_3 = \frac{5{,}832{,}000}{30.666} = +190{,}000 \text{ pounds.}$$

Upper-chord Members.

The upper-chord stresses at once result from those in the lower, but it is first to be observed that since L_1U_2 is a tension member only, it will not be stressed with the moving load on essentially the whole of the arm, since the counter U_1L_2 will then come into action. The upper-chord stresses in U_1U_2 and U_2U_3 will therefore be equal, and as L_2 will be the center of moments for those stresses, the position of the moving load for their greatest value will be precisely the same as for L_2L_3, with the exception that the direction of motion is to be reversed, thus placing the third driver of the second locomotive at the foot of U_2L_2. Hence

$$U_1U_2 = U_2U_3 = -\frac{5,832,000}{29.333} \sec \alpha = -200,000 \text{ pounds.}$$

Finally

$$U_3U_4 = -L_2L_3 \cdot \sec \alpha = -190,200 \text{ pounds.}$$

These computations determine all the greatest moving-load stresses in one arm considered as a simple non-continuous span.

Live-load Stresses in Span $L_0 \ldots L_5$ as a Continuous Span.

In finding the greatest stresses due to the moving load on both arms, the condition of partial continuity requires the use of balanced loads—i.e., loads simultaneously on each arm, which, if the trusses were continuous over the center, would produce equal and opposite center bending moments. The most obvious and simple balanced loads are those of equal magnitude placed at symmetrical panel-points in each arm, and such balanced loads will be used in the following computations.

As has already been observed, the use of the locomotive concentrations and the ordinary moment diagram for the greatest stresses in this case would lead to unnecessary complication and great labor without any corresponding advantage. Essential accuracy can be attained by computing a system of panel concentrations or weights from a position of the moving load which will give results differing in no sensible degree from those determined by more refined calculations. The desired position is largely a matter of judgment, but a heavy concentration should be found at the panel-point covered by the head of the train. In the present instance the third driving-wheel of the front locomotive will be placed over a panel-point which will be called panel 1. A small concentration will exist at the panel-point in front of panel 1; this will be called the "advance load." The desired concentrations will then be as follows:

Advance load.	17,500	pounds
At panel 1.	84,200	"
" " 2.	54,600	"
" " 3.	80,600	"
" " 4.	58,500	"

The advance load is sometimes neglected in finding the web stresses, as the resulting computations are somewhat simplified, and the small error committed is on the side of safety.

All reactions in this case must be found by the equations of Art. 102 for perfect continuity, as all loads are to be balanced. If a panel load, P, rests on the left-hand arm, or span, l_1, the reaction R_1 will be by eq. (5a) of Art. 102:

$$R_1 = P(1-n)\left\{1-(1+n)n\frac{2c}{4(c+1)-\frac{1}{c+1}}\right\} \quad . \quad . \quad (1)$$

And by eq. (6*a*) of Art. 102, the reaction R_4 will be

$$R_4 = P(1-n^2)n\frac{c}{4(c+1)^2-1}. \quad . \quad . \quad . \quad (2)$$

In this case $c = 137.5 \div 20.5 = 6.7073$.

If P be the general value of a panel load, n will have the values 0.2, 0.4, 0.6, and 0.8 in the formulæ for R_1 and R_4, and the expressions for those reactions will be

Panel load P at L_1 $R_1 = 0.716102P$; $R_4 = 0.005443P$.
" " " " L_2 $R_1 = 0.453178P$; $R_4 = 0.009526P$.
" " " " L_3 $R_1 = 0.232204P$; $R_4 = 0.010886P$.
" " " " L_4 $R_1 = 0.074153P$; $R_4 = 0.008165P$.

Since balanced loads only are to be employed in this case, it will be necessary to determine the reactions R_1 and R_4 with a panel concentration on one arm balanced by an exactly equal concentration symmetrically placed on the other. The reactions for each such concentration will be those given above, but R_1 and R_4 will be interchanged with the exchange of one arm for the other. When the two concentrations are simultaneously placed on the two arms, i.e., when they are balanced, the reaction at each end of the bridge will equal that at the other, and will be represented by the sum of R_1 and R_4, as given above for a single panel load. If R is that reaction,

For panel-points L_1 and L_{10}, $R = R_1 + R_4 = 0.7215P$.
" " L_2 " L_9, $R = R_1 + R_4 = 0.4627P$.
" " L_3 " L_8, $R = R_1 + R_4 = 0.2431P$.
" " L_4 " L_7, $R = R_1 + R_4 = 0.0823P$.

The preceding numerical work can be checked by finding M_2 for both arms fully loaded, from eq. (4) of the preceding article (having previously found M_3), and

using it in eq. (5) of the same article. The resulting value of $R_1 = R_4$ should equal the sum of the preceding values of R, because both results are for both arms fully loaded. The sum of the four values of R is $1.5096P$, while eq. (5) of Art. 102 gives $R_1 = R = 1.5097P$. Hence the check is satisfactory. In this verification all panel loads P are assumed equal, but this does not in any way affect the numerical work which it was desired to verify.

As the reactions for each of the panel loads, and the panel loads themselves, are now known, all stresses can readily be found. In all the cases except those of simple hangers, or where a trigonometric multiplier only is needed, the method of moments will be employed, precisely as with the moving load on one arm only.

Stresses in the Chord Members; Balanced Loads

In determining the stresses in the chord members for this condition of loading, influence lines * showing the moments at various points for balanced unit loads will be drawn. The variations of the stresses in the members show precisely the same variations as these moments, since the stress may always be found by dividing the moments of the external forces by the proper lever-arm.

Members L_0U_1, L_0L_1, and L_1L_2.

The center of moments for the end post L_0U_1 is at L_1, and the line *MACEGN*, of Fig. 13, is the influence line for moments at L_1. It is drawn as follows:

If loads of unity be placed at L_1 and L_{10}, the reaction R at L_0 has been found to be 0.7215. The moment of

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* An outline of the method of influence lines is furnished in Chapter VI.

the external forces about L_1 is then 0.7215×1 panel length = 0.7215. The ordinate AB directly below L_1 is therefore drawn with that value.

If loads of unity be placed at L_2 and L_9, the reaction R at L_0 is 0.4627, and the moment at L_1 is then 0.4627×1 panel length $= 0.4627$, and CD is erected immediately below L_2 with that value.

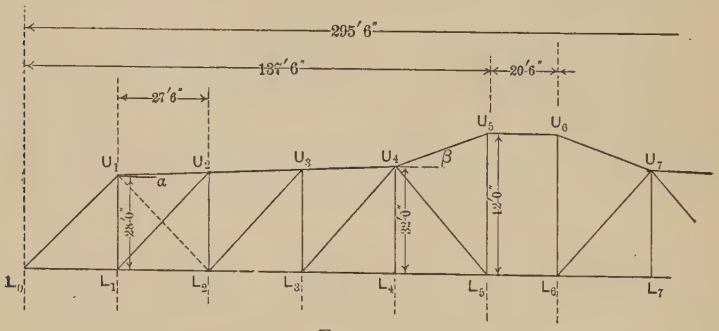


FIG. 12.

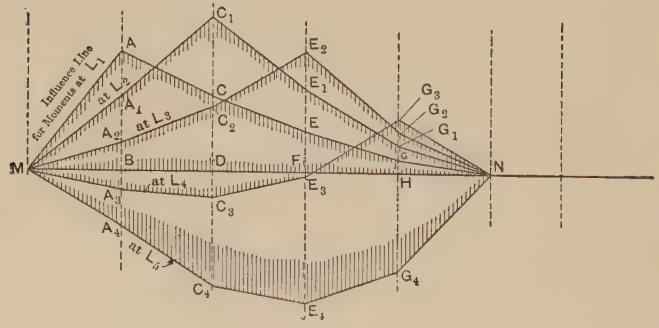


FIG. 13.

Similarly the moments at L_1 for loads at L_3 and L_8 and L_4 and L_7 are found to be 0.2431 and 0.0823, and the ordinates EF and GH are drawn correspondingly.

The completed influence line shows at once that the heaviest panel concentration, viz., 84,200 pounds, should be placed at L_1 with the advance panel concentration of

17,500 pounds at L_0. The moment at L_1 for these loads then becomes

$$\begin{aligned} 0.7215 \times 84,200 &= 60,700 \\ 0.4627 \times 54,600 &= 25,300 \\ 0.2431 \times 80,600 &= 19,600 \\ 0.0823 \times 58,500 &= 4,800 \\ \hline & 110,400 \end{aligned}$$

This moment is expressed as a product of pounds by panel length; in order to find the stress in L_0U_1, its lever-arm must also be expressed in terms of a panel length. Therefore the stress in L_0U_1 is

$$L_0U_1 = -\frac{110,400}{0.715} = -154,500 \text{ pounds.}$$

Since the center of moments for L_0L_1 and L_1L_2 is at U_1, or a point vertically above L_1, the previous influence line may be used for these members. The lever-arm is however $\frac{28}{27.5} = 1.02$ panel length.

Therefore

$$L_0L_1 = L_1L_2 = +\frac{110,400}{1.02} = +108,000 \text{ pounds.}$$

Members U_1U_2, U_2U_3.

As the counter-member U_1L_2 comes into action with the moving load over the whole of the bridge, the two upper-chord panels U_1U_2 and U_2U_3 will sustain the same stress, and panel-point L_2 will be the center of moments for their stress. The line $MA_1C_1E_1G_1N$ (Fig. 13) represents the moment influence line for this point as the center of moments and it is drawn as follows:

For loads at L_1 and L_{10},

$$M = 0.7215 \times 2 - 1 \times 1 = +0.4430 = \text{ordinate } BA_1.$$

For loads at L_2 and L_9,

$$M = 0.4627 \times 2 = +0.9254 = \text{ordinate } DC_1.$$

For loads at L_3 and L_8,

$$M = 0.2431 \times 2 = +0.4862 = \text{ordinate } FE_1.$$

For loads at L_4 and L_7,

$$M = 0.0823 \times 2 = +0.1646 = \text{ordinate } HG_1.$$

Since all these unit moments are positive, all loads will produce compression in U_1U_2 and U_2U_3. It is also clear, from the influence line, that the panel concentration of 80,600 pounds should be placed at L_2; then the weight 84,200 pounds will be at L_4, and the advance weight at L_5.

The actual moments, expressed in pounds × panel length, may now be found, as follows:

$$\begin{aligned} 58{,}500 \times 0.4430 &= 35{,}900 \\ 80{,}600 \times 0.9254 &= 74{,}500 \\ 54{,}600 \times 0.4862 &= 26{,}600 \\ 84{,}200 \times 0.1646 &= 13{,}800 \\ \text{Total} &= 150{,}800 \end{aligned}$$

The lever-arm of U_1U_2 expressed in panel lengths is

$$\frac{29.333}{27.5 \sec \alpha} = 1.06.$$

Hence $U_1U_2 = U_2U_3 = -\frac{150{,}800}{1.06} = -142{,}000$ pounds.

Members L_2L_3, U_3U_4.

In order to find the greatest stress in the lower-chord member L_2L_3, moments must be taken about U_3 vertically

above L_3. The line $MA_2C_2E_2G_2N$ is the moment influence line for unit-balanced loads for moments taken about L_3. The ordinates are found as follows:

$$\begin{aligned} A_2B &= 0.7215 \times 3 - 1 \times 2 = +0.1645 \\ C_2D &= 0.4627 \times 3 - 1 \times 1 = +0.3881 \\ E_2F &= 0.2431 \times 3 \qquad\quad = +0.7293 \\ G_2H &= 0.0823 \times 3 \qquad\quad = +0.2469 \end{aligned}$$

Since these ordinates are all positive, all loads on the bridge will produce tension in L_2L_3, or, what is the same, compression in U_3U_4. Inspection of the influence line shows that the concentration of 80,600 pounds must be placed at L_3, so that the two heaviest concentrations will be at L_3 and L_1 with the head of the train toward the right. Hence

$$\begin{aligned} 84{,}200 \times 0.1645 &= 13{,}800 \\ 54{,}600 \times 0.3881 &= 21{,}200 \\ 80{,}600 \times 0.7293 &= 58{,}700 \\ 58{,}500 \times 0.2469 &= 14{,}500 \\ \text{Total} &= 108{,}200 \end{aligned}$$

If the lever-arm of L_2L_3 in panel lengths is $\frac{30.67}{27.5} = 1.093$, and of U_3U_4 is $\frac{1.093}{\sec \alpha} = 1.090$; therefore

$$L_2L_3 = +\frac{108{,}200}{1.093} = +98{,}800 \text{ pounds}$$

$$U_3U_4 = -\frac{108{,}200}{1.090} = -99{,}000 \quad ''$$

It is evident that the stress in U_3U_4 is equal to that in L_2L_3 multiplied by the secant of the inclination of the former to a horizontal line.

Members L_3L_4 and L_4L_5.

The unit influence line for moments at U_4, for the stress in the members L_3L_4 and L_4L_5, is shown in Fig. 13 as $MA_3C_3E_3G_3N$, and the values of the ordinates are found as follows:

$$A_3B = (0.7215 \times 4) - (1 \times 3) = -0.1140$$
$$C_3D = (0.4627 \times 4) - (1 \times 2) = -0.1492$$
$$E_3F = (0.2431 \times 4) - (1 \times 1) = -0.0276$$
$$G_3H = (0.0823 \times 4) = +0.3292$$

The figure shows that the panel load at L_4 is the only one which produces tension in these members, and it is evident that one of the heaviest concentrations should be placed there. Hence, by placing the advance load of 17,500 pounds at L_3, and the heavy load of 84,200 pounds at L_4, the moment expressed in pounds $\times$ panel length is

$$17{,}500 \times (-0.0276) = - \quad 480$$
$$84{,}200 \times (+0.3292) = +27{,}700$$
$$\text{Total} = +27{,}220$$

The lever-arm of L_3L_4 is $\frac{32}{27.5} = 1.162$ panel length; hence the stress is

$$L_3L_4 = L_4L_5 = +\frac{27{,}220}{1.162} = +23{,}400 \text{ pounds.}$$

In order to find the greatest compression in these members, the advance load of 17,500 pounds is to be placed at L_3, with the head of the train toward L_4. Hence

$$54{,}600 \times (-0.1140) = - \quad 6{,}090$$
$$84{,}200 \times (-0.1492) = -12{,}600$$
$$17{,}500 \times (-0.0276) = - \quad 380$$
$$\text{Total} = -19{,}070$$

Hence

$$L_3L_4 = L_4L_5 = -\frac{19{,}070}{1.162} = -16{,}400 \text{ pounds.}$$

Members U_5U_6, L_5L_6, U_4U_5, and U_5L_5.

In order to determine the greatest stresses in the members U_5U_6 and L_5L_6, unit moments may be taken about L_5, and the influence line is shown, Fig. 13, as $MA_4C_4E_4G_4N$. The values of the various ordinates are found as follows:

$$A_4B = (0.7215 \times 5) - (1 \times 4) = -0.3925$$
$$C_4D = (0.4627 \times 5) - (1 \times 3) = -0.6865$$
$$E_4F = (0.2431 \times 5) - (1 \times 2) = -0.7845$$
$$G_4H = (0.0823 \times 5) - (1 \times 1) = -0.5885$$

The figure shows that all loads produce tension in U_5U_6 and compression in L_5L_6, and that the two heaviest concentrations should be placed at L_4 and L_2 with the head of the train toward L_6:

$$-58{,}500 \times 0.3925 = -\ 23{,}000$$
$$-80{,}600 \times 0.6865 = -\ 55{,}300$$
$$-54{,}600 \times 0.7845 = -\ 42{,}900$$
$$-84{,}200 \times 0.5885 = -\ 49{,}500$$
$$\text{Total} = -170{,}700$$

The lever-arm of U_5U_6 expressed in panel lengths is

$$\frac{42}{27.5} = 1.53.$$

Therefore

$$L_5L_6 = -U_5U_6 = -\frac{170{,}700}{1.53} = -115{,}000 \text{ pounds.}$$

This stress is also the horizontal component of U_4U_5, and the stress in the latter member is, therefore,

$$U_4U_5 = U_5U_6 \cdot \sec \beta = +122{,}000 \text{ pounds.}$$

The stress in the post U_5L_5 is the stress in U_5U_6 multiplied by tan β; therefore

$$U_5L_5 = -115{,}000 \times 0.364 = -41{,}800 \text{ pounds.}$$

Web Members.

The variations of the stresses in the web members may be illustrated by moment influence lines, as shown in Fig. 15, but the centers of moments must be taken at

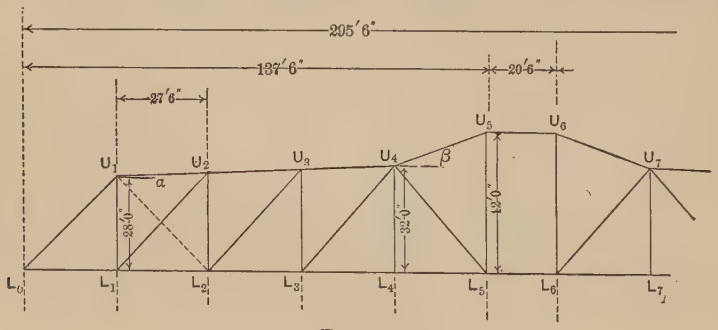


FIG. 14.

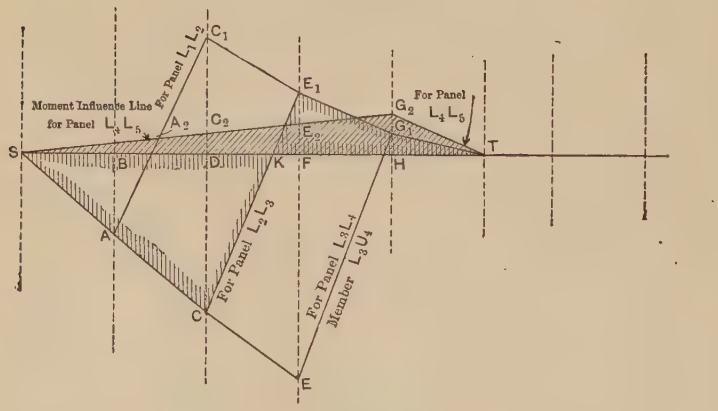


FIG. 15.

the intersections of the proper chord members. These centers of moments may fall outside of the limits of the span, and they may not be at panel-points, but in all

other respects the method of treatment is precisely as for chord members.

Members U_3L_3 and L_2U_3.

The center of moments for the stresses in these members is found 20 panel lengths distance from L_0 at the intersection of U_3U_4 and L_2L_3. If unit loads be placed at L_1 and L_{10}, the reaction at L_0 has been found to be 0.7215, and therefore the moment of the external forces situated to the left of the section passed through U_3U_4, U_3L_3, and L_2L_3 is

$$0.7215 \times 20 - 21 \times 1 = -6.57.$$

This moment is expressed in terms of a unit weight $\times$ panel length, and it is represented in Fig. 15 as AB, directly below L_1. The other ordinates of this influence line $SACE_1G_1T$ (marked "for panel L_2L_3," since the same lines also serves for L_2U_3) are found as follows:

$$\begin{aligned} AB &= (0.7215 \times 20) - (21 \times 1) = -\ 6.57 \\ CD &= (0.4627 \times 20) - (22 \times 1) = -12.746 \\ E_1F &= (0.2431 \times 20) \qquad\qquad = +\ 4.862 \\ G_1H &= (0.0823 \times 20) \qquad\qquad = +\ 1.646 \end{aligned}$$

Inspection of the influence line shows that the advance load must be at L_3 for the greatest compression in U_3L_3. The concentrations of 54,600 and 84,200 pounds will then be at L_1 and L_2 respectively. Taking moments about the center,

$$\begin{aligned} -54{,}600 \times\ 6.57 &= -\ 359{,}000 \\ -84{,}200 \times 12.746 &= -1{,}072{,}000 \\ +17{,}500 \times\ 4.862 &= +\ 85{,}000 \\ \text{Total moment} &= -1{,}346{,}000 \end{aligned}$$

this moment is expressed in pounds $\times$ panel lengths.

If the effect of a uniform load be tested, that load would extend from S to K (Fig. 15) and $SACK$ would indicate the moment area. If each panel load, however, is assumed at a full uniform panel loading, viz., $27.5 \times 2{,}000 = 55{,}000$ pounds, the moment for maximum compression in U_3L_3 becomes

$$\begin{aligned} -55{,}000 \times\ 6.57 &= -\ 362{,}000 \\ -55{,}000 \times 12.746 &= -\ 700{,}000 \\ \text{Total} &= -1{,}062{,}000 \end{aligned}$$

This condition of loading is thus seen to produce only about four fifths of the preceding moment.

The lever-arm of U_3L_3 is 23 panel lengths, and its stress therefore is

$$U_3L_3 = -\frac{1{,}346{,}000}{23} = -58{,}500 \text{ pounds.}$$

Since the center of moments for L_2U_3 is the same as for U_3L_3, the same influence line and the same position of loading may be used. The maximum tensile stress is therefore

$$L_2U_3 = +\frac{1{,}346{,}000}{22} \sec L_2U_3 = +82{,}300 \text{ pounds.}$$

It will presently be shown that this member need not be counterbraced.

Members U_2L_2, L_1U_2, and the Counter U_1L_2.

The center of moments for the stresses in these members is also 20 panel lengths to the left of L_1. The influence line for moments is $SAC_1E_1G_1T$ (Fig. 15) and the ordinates are found as follows:

$$\begin{aligned} AB &= (0.7215 \times 20) - (21 \times 1) = -6.57 \\ C_1D &= 0.4627 \times 20 = +9.254 \\ E_1F &= 0.2431 \times 20 = +4.862 \\ G_1H &= 0.0823 \times 20 = +1.646 \end{aligned}$$

Inspection of the influence line shows that the greatest compression is found for U_2L_2 with the advance load at L_2 and the concentration of 84,200 pounds at L_1. The moment then becomes

$$\begin{aligned} -84{,}200 \times 6.57 &= -553{,}000 \\ +17{,}500 \times 9.254 &= +162{,}000 \\ \text{Total} &= -391{,}000 \end{aligned}$$

Since the lever-arm of U_2L_2 is 22 panel lengths, the stress is

$$U_2L_2 = -\frac{391{,}000}{22} = -17{,}800 \text{ pounds.}$$

The lever-arm of L_1U_2 is $\frac{21}{\sec L_1U_2}$ panel lengths. Therefore the stress of that member is

$$L_1U_2 = +\frac{391{,}000}{21} \sec L_1U_2 = +25{,}600 \text{ pounds.}$$

The greatest tension in the counter U_1L_2 is found with the advance weight of 17,500 pounds at L_1, the train heading towards L_0. If the advance weight were very large, it would have to be placed at L_2. Trial only can determine which position furnishes the greater stress. The moment for this position of the loading is

$$\begin{aligned} -17{,}500 \times 6.57 &= - \quad 115{,}000 \\ +84{,}200 \times 9.254 &= + \quad 780{,}000 \\ +54{,}600 \times 4.862 &= + \quad 266{,}000 \\ +80{,}600 \times 1.646 &= + \quad 132{,}000 \\ \text{Total moment} &= +1{,}063{,}000 \end{aligned}$$

The lever-arm of U_1L_2 is $\frac{22}{\sec U_1L_2}$ panel lengths; hence the stress is

$$U_1L_2 = +\frac{1{,}063{,}000}{22} \times 1.397 = +67{,}500 \text{ pounds.}$$

In determining the stress in this counter, it is to be noted that L_1U_2 is to be neglected.

Since the stress in the counter is much less than was found with the moving load on one arm, as might have been anticipated, it is clear that no other counter-stresses need be considered. Indeed, it is evident from the general conditions of the two cases that the greatest counter web stresses must be found with the moving load on one arm only.

Hangers U_1L_1 and U_4L_4.

These members will be treated precisely as in the case of moving load on one arm only. Hence

$$U_1L_1 = U_4L_4 = +84{,}200 \text{ pounds.}$$

Members L_3U_4.

The center of moments for the stress in L_3U_4 is 20 panels to the left of L_0. The influence line for moments about this point is $SACEG_1T$, Fig. 15, and the ordinates are found as follows:

$$\begin{aligned} AB &= (0.7215 \times 20) - (1 \times 21) = -\ 6.57 \\ CD &= (0.4627 \times 20) - (1 \times 22) = -12.746 \\ EF &= (0.2431 \times 20) - (1 \times 23) = -18.138 \\ G_1H &= (0.0823 \times 20) \qquad\qquad = +\ 1.646 \end{aligned}$$

It is at once evident that the advance load, 17,500 pounds, should be at L_4 with the train facing toward L_6. The moment then becomes

$$\begin{aligned} +17{,}500 \times\ 1.646 &= +\ \ 28{,}800 \\ -84{,}200 \times 18.138 &= -1{,}540{,}000 \\ -54{,}600 \times 12.746 &= -\ \ 695{,}000 \\ -80{,}600 \times\ 6.57\ \ &= -\ \ 530{,}000 \\ \text{Total moment} &= -2{,}736{,}200 \end{aligned}$$

The lever-arm of L_3U_4 is $\left(\frac{23}{\sec L_3U_4}\right)$ panel lengths, and the stress is

$$L_3U_4 = +\frac{2{,}736{,}200}{23} \times 1.319 = +157{,}000 \text{ pounds.}$$

Member U_4L_5.

Since the upper chord U_4U_5 cuts the lower-chord member L_4L_5 at a point 0.8 panel length to the right of L_0, that point of intersection must be taken as the center of moments for U_4L_5. The influence line for moments is $SA_2C_2E_2G_2T$ (Fig. 15), found as follows:

$$\begin{aligned} A_2B &= (0.7215 \times 0.8) + (1 \times 0.2) = +0.7772 \\ C_2D &= (0.4627 \times 0.8) + (1 \times 1.2) = +1.5702 \\ E_2F &= (0.2431 \times 0.8) + (1 \times 2.2) = +2.3945 \\ G_2H &= (0.0823 \times 0.8) + (1 \times 3.2) = +3.2658 \end{aligned}$$

It is at once evident that every panel load causes compression in U_4L_5, so that the greatest compression occurs with the advance load of 17,500 pounds at L_5, with the train facing toward the right. The moment is, therefore,

$$+84{,}200 \times 3.2658 = +274{,}000$$
$$+54{,}600 \times 2.3945 = +131{,}000$$
$$+80{,}600 \times 1.5702 = +127{,}000$$
$$+58{,}500 \times 0.7772 = +\ 45{,}400$$

Total moment = 577,400

The lever-arm of U_4L_5 is $\left(\frac{4.2}{\sec U_4L_5}\right)$ panel lengths.

Therefore the stress is

$$U_4L_5 = -\frac{577{,}400 \times 1.319}{4.2} = -180{,}000 \text{ pounds.}$$

These complete all the moving-load stresses and, with those due to the fixed load, they enable all the resultant maximum stresses to be at once written. The following tabulation shows the results of all the computations.

TABLE II.—FIXED- AND MOVING-LOAD STRESSES.

Member.	Fixed-load Stress in Pounds.	Moving-load Stress in Pounds.		Member.	Fixed-load Stress in Pounds.	Moving-load Stress in Pounds.	
		Moving Load on One Arm.	Moving Load on Both Arms.			Moving Load on One Arm.	Moving Load on Both Arms.
L_0L_1	− 9,800	+134,000	+108,000	U_1L_1	− 16,300	+ 84,200	+ 84,200
L_1L_2	− 40,700	+134,000	+108,000	L_1U_2	+ 45,100	+ 19,600	+ 25,600
L_2L_3	− 89,900	+190,000	+ 98,800	U_2L_2	− 38,200	− 13,600	− 17,800
L_3L_4	−155,000	+117,000	+23,400 / −16,400	L_2U_3	+ 73,600	+ 60,800	+ 82,300
				U_3L_3	− 59,200	− 43,300	− 58,500
L_4L_5	−155,000	+117,000	+23,400 / −16,400	L_3U_4		+100,000	+122,000	+157,000
				U_4L_4	+ 16,700	+ 84,200	+ 84,200
L_5L_6	−186,000	.....	−115,000	U_4L_5	− 47,500	−180,000	−180,000
L_0U_1	+ 14,000	−191,000	−154,500	L_5U_5	− 73,600	.....	− 41,800
U_1U_2	+ 9,800	−200,000	−142,000	U_1L_2	.....	+115,000	+ 67,500
U_2U_3	+ 40,700	−200,000	−142,000				
U_3U_4	+ 89,960	−190,200	− 99,000				
U_4U_5	+197,900	.....	+122,000				
U_5U_6	+186,000	.....	+115,000				

The stresses in Table II show that the tension web member U_1L_1 must be counterbraced so as to sustain the

tension of 84,200 pounds due to the moving-load and the fixed-load compression of 16,300 pounds. In addition the lower chord members L_0L_1, L_1L_2, and L_2L_3 must be counterbraced, as must also the upper-chord panels U_1U_2, U_2U_3, and U_3U_4. These latter members are always built with sections suitable for compression.

The specifications for which the bridge is designed will always indicate in what manner the live- and dead-load stresses of opposite kinds must be combined.

From what has preceded, it is clear that the greatest counter web stresses are to be found by carrying the moving load forward from the center of the bridge toward the end, and on one arm only, at the same time considering it a simple non-continuous span.

The greatest main web stresses, on the other hand, are to be found by moving the system of balanced panel weights, already described, from the two extremities of the bridge toward the center, as has been illustrated.

It is to be remembered that verticals in compression will sustain their greatest stresses at the same time with the diagonal tension members which cut their upper extremities, if the moving load traverses the lower chord.

As one arm of the bridge is a simple truss supported at each end, for the unbalanced loads on it, it is evident that the greatest compression in the upper chord and tension in the lower will exist, near the extremities of the arms, for the moving load over the whole of one arm.

Since moving loads on both arms at the same time balance each other, it results that the greatest tension in the upper chord and compression in the lower, over the center pier, will exist with the moving load over at the whole of both arms.

This is true for the center panel only. For other panels adjacent to the center it will be necessary to take single-balanced panel moving weights and find for each,

all panels in which the stress is of the same kind as that caused by the fixed load alone and the amount of that stress in those panels.

Having obtained the results for each pair of balanced weights, they are to be combined in the manner already shown.

The greatest compression in the lower chord and tension in the upper, near the ends, however, will exist with the bridge open or closed and subjected to its own weight only.

The preceding operations constitute all that pertains to the computation of stresses in the trusses, but it is still necessary to compute the reactions R_1 and R_2 at the extremity of the arm and over the center pier respectively. The supports at the latching and locking points must be designed to resist the reaction R_1, and the drum must be designed to support the reaction R_2. R_1 will manifestly have its greatest value with the moving load on one arm only. The determination by trial of the maximum reaction at L_0, including the proportionate part of those wheel concentrations in the panel adjacent, is somewhat laborious, and as the equivalent uniform load for maximum shear of Cooper's E 40 locomotive for a span length of 137.5 feet is 5,760 pounds per linear foot of track it will be essentially accurate and much simpler to assume that the total maximum reaction to be supported at the locking and latching point at the foot of the end post is

$$R_1 = \frac{137.5 \times 2{,}880}{2} + 20{,}000 = 206{,}000 \text{ pounds.}$$

The concentration of 20,000 pounds is added to provide for the driving-wheel concentration that must be supported at the instant of entering on or leaving the arm while the whole of the latter is otherwise loaded.

The reaction for both arms covered with moving load is best determined in precisely the same manner. Each panel load will be $27.5 \times 2{,}880 = 79{,}200$ pounds. The reaction R_1 for four such panel loads will be

$$(0.7215 + 0.4627 + 0.2431 + 0.0823) \times 79{,}200 = 1.5096 \times 79{,}200$$

$$\therefore\ R_1 = 119{,}600 \text{ pounds.}$$

Hence

$$R_2' = (4 \times 79{,}200) - 119{,}600 + \left(\frac{24}{27.5} \times 79{,}200\right) + 20{,}000$$

$$= 292{,}000 \text{ pounds.}$$

The panel load added is for the half panel adjacent to the center and the central panel, while the 20,000 pounds is for the driving-wheel concentration just explained in connection with R_1. To R_2' must be added the total fixed load carried to the center. From the data given at the beginning of this article, that total fixed load will be

$$(4 \times 23{,}400) + 10{,}000 + \frac{24 \times 1{,}700}{2} = 124{,}000 \text{ pounds.}$$

Hence the total reaction R_2 to be supported at the drum will be

$$R_2 = 292{,}000 + 124{,}000 = 416{,}000 \text{ pounds.}$$

As the ends of the span are simply supported, no fixed-load reaction is to be added to the value of R_1 as determined by the moving load only.

The Omission of Counters.

It is to be observed that the design of the truss is such that the inclined web members are subject to pure tension, and the vertical posts U_2L_2 and U_3L_3 to pure compression, necessitating the introduction of the counter U_1L_2. If it

is desired to omit the latter member, making a more excellent design from an engineering point of view, L_1U_2, and perhaps other adjacent inclined web members, will be subjected to compression, with possibly some of the vertical posts in tension. All such members must be counterbraced. In the present instance the counter U_1L_2 will be omitted and the main members L_1U_2 and U_2L_2 will be counterbraced. The member U_1L_1 will also receive much greater tension than in the preceding case.

In considering these counter-stresses, the arm of the bridge is to be treated as a simple non-continuous span, as was done in the earlier part of the article, with the moving load passing from the center toward the end. Eq. (4) of Art. 85 shows that the third driver of the front locomotive should be at the foot of U_1L_1 in order to give that member its greatest tension. The first parenthesis of the second member of that equation disappears in this case because m is zero. The resulting reaction at the foot of L_0U_1 is 154,000 pounds, from which there must be subtracted 17,500 pounds due to the wheel concentrations between L_0 and L_1. Hence, by moments about the chord intersection,

$$U_1L_1 = \frac{(154{,}000 - 17{,}500)20}{21} = +130{,}000 \text{ pounds.}$$

The moving-load compression in L_1U_2 has already been found to be 113,500 pounds, in deciding upon the necessity for the counter U_1L_2 in the preceding case. The same position of the moving load (the third driver at L_3) will give the greatest tension in U_2L_2, and using the data already employed for L_1U_2, there will result

$$U_2L_2 = \frac{102{,}000 \times 20 - 17{,}500 \times 21}{22} = +76{,}000 \text{ pounds.}$$

Again, it was found that the greatest moving-load compression in L_2U_3 would be 42,600 pounds, while the fixed-load tension is 73,600 pounds; hence L_2U_3 can never suffer compression. The results for L_2U_3 show also that the fixed-load compression in U_2L_2 will always largely overbalance any possible moving-load tension. The only counterbracing needed, therefore, will be that for the members L_1U_2 and U_2L_2. These computations, in combination with the results given in Table II, show that U_1L_1, L_1U_2, and U_2L_2 will sustain the following stresses:

TABLE III.

Member.	Fixed Load.	Moving Load on One Arm.	Moving Load on Both Arms.
U_1L_1	−16,300 lbs.	+130,000 lbs.	+84,200 lbs.
L_1U_2	+45,100 "	−113,500 "	+25,600 "
U_2L_2	−38,200 "	+ 76,000 "	−17,800 "

When the member L_1U_2 sustains counter-stress, the center of moments for the upper-chord member U_1U_2 changes from L_2 to L_1. The stress is slightly less than that previously found. It is not necessary to make another computation for it.

All the other stresses remain unchanged.

Art. 104.—Example of Swing-bridge with Ends Simply Supported—Two Points of Support at Center—Complete Continuity.

The case of complete continuity to be considered in this article involves the existence of web members in the panel over the drum (i.e., the span l_2 of Art. 102), designed by actual computation to take the shear which may pass the center when the structure carries unbalanced loads.

It has been shown, in Art. 102, that when the condition of continuity is fulfilled, which is the only condition here contemplated, either of the reactions R_2 or R_3 may be

negative, i.e., downward, while both the reactions R_1 and R_4 are always positive, or upward. It will be found that provision of the nature of heavy anchorage must be made in order to meet the requirements of R_2 and R_3 with some conditions of loading of draw-spans. This indicates that the web members in the center panel will be heavy, and such will be found to be the case.

The moving-load stresses in all cases will be determined by means of the formulæ of Art. 102. The truss to be con-

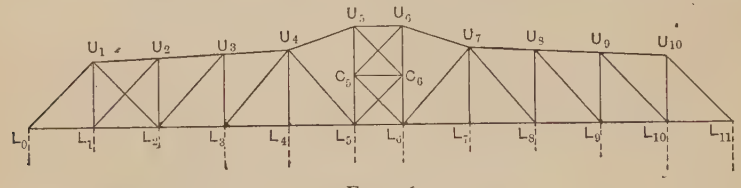


FIG. 16.

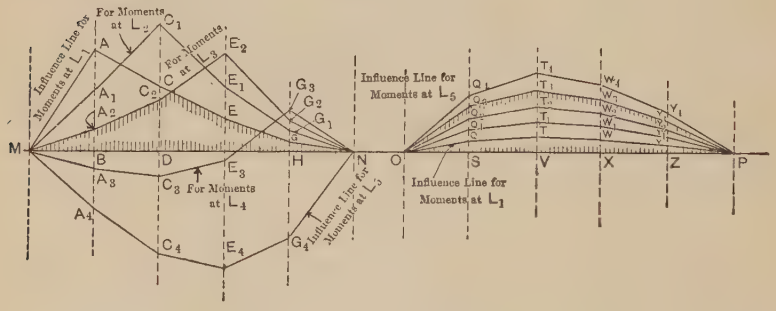


FIG. 17.

sidered is the same as treated in the preceding article, and is shown in Fig. 16. The data, reproduced from Art. 103, are as follows:

Length between centers of end pins, $L_0 \ldots L_{11} \ldots$ 295.5 feet
Panel length in arms (all equal) . . . . . 27.5 "
Length of center panel or span, $L_5 L_6$ . . . . . 20.5 "
Center depth at $L_5U_5 = 42$ feet. Depth at $U_4L_4 = 32$ feet.
Depth at $U_3L_3 = 30\frac{2}{3}$ " " " $U_2L_2 = 29\frac{1}{3}$ "
" " $U_1L_1 = 28$ " Truss centers 16 ft. apart.

Weight of rails, ties, and guards, with bolts and connections, 400 pounds per lineal foot.

Total lower panel fixed load	16,650 pounds
" upper " " "	6,750 "
Total	23,400 "
Lower panel load at end of arm	10,000 pounds
Upper " " " U_5	5,880 "

The trigonometric quantities required will not be repeated.

As the ends of the span are simply supported, the fixed-load stresses are those existing when the draw is open, and hence are identical with those found for the fixed load in Table I of the preceding article. As they are there arranged in convenient shape for combination with the moving-load stresses about to be found, they will not be reproduced here.

The moving load is identical with that used in Article 103, viz., Cooper's E 40.

For the same reasons given in that article, the system of panel concentrations which was used there will be employed here, instead of the wheel weights shown in the diagram. These panel concentrations are as follows:

Advance load	17,500 pounds
At panel 1	84,200 "
" " 2	54,600 "
" " 3	80,600 "
" " 4	58,500 "

On account of the complete continuity at the center, there will be no condition of a simple, non-continuous span for the moving load on one arm only, but all moving-load stresses will be found by the use of the preceding panel concentrations in connection with the formulæ of Art. 102.

The reactions R_1 and R_4 will be precisely the same as those found on page 308 of Art. 103. They are

Panel load P at L_1, $R_1 = 0.716102P$
" " " " L_2, $R_1 = 0.453178P$
" " " " L_3, $R_1 = 0.232204P$
" " " " L_4, $R_1 = 0.074153P$
" " " " L_7, $R_1 = 0.008165P$
" " " " L_8, $R_1 = 0.010886P$
" " " " L_9, $R_1 = 0.009526P$
" " " " L_{10}, $R_1 = 0.005443P$

As unbalanced loads will be considered in this case, these reactions will frequently be used separately in the succeeding computations; but in those special circumstances which cause the loads to be balanced, they will be united by addition, as was done in the preceding article.

Stresses in the Chord Members.

Members L_0U_1, L_0L_1, and L_1L_2.

The method of moment influence lines will be used precisely as in the case of the partially continuous swing-bridge. The center of moments for the members L_0U_1, L_0L_1, and L_1L_2 is at L_1, and the influence line for moments about this point is *MACEGN . . . OQTWYP* of Fig. 17. The values of the ordinates of this line are found by placing unit loads at the various panel-points and computing the moments for those positions. The lever-arms will be expressed in panel lengths, and if the unit load be one pound the moments will be expressed in panel lengths $\times$ pounds.

The following tabulation furnishes the ordinates of the influence line. The values of the reactions R_1 are to be taken from the tabulation shown above.

Ordinate AB	$=0.7161$	$\times 1 = 0.7161$
" CD	$=0.4532$	$\times 1 = 0.4532$
" EF	$=0.2322$	$\times 1 = 0.2322$
" GH	$=0.0741$	$\times 1 = 0.0741$
" QS	$=0.008165$	$\times 1 = 0.0082$
" TV	$=0.0109$	$\times 1 = 0.0109$
" WX	$=0.0095$	$\times 1 = 0.0095$
" YZ	$=0.0054$	$\times 1 = 0.0054$

In the figure the scale for the ordinates of the right-hand arm of the bridge has been increased ten times the natural value.

It is at once evident that a concentration should be placed at every panel-point, the heaviest of 84,200 pounds being at A, with the advance load of 17,500 pounds at L_0. The right arm of the bridge will then be covered by panel loads of 55,000 pounds each caused by the uniform train loading of 2,000 pounds per linear foot. The final moment is therefore

$0.7161 \times 84,200 =$	60,200
$0.4532 \times 54,600 =$	24,800
$0.2322 \times 80,600 =$	18,700
$0.0741 \times 58,500 =$	4,300
$0.0082 \times 55,000 =$	450
$0.0109 \times 55,000 =$	600
$0.0095 \times 55,000 =$	520
$0.0054 \times 55,000 =$	300
Total $=$	109,820

The lever-arm of L_0U_1 must also be expressed in terms of a panel length.

The stress in L_0U_1 is therefore

$$L_0U_1 = -\frac{109,820}{0.715} = -154,000 \text{ pounds.}$$

The difference between this value as compared with the stress found for L_0U_1 for the partially continuous bridge, treated with balanced loads (p. 311) is due to the fact that in the present case the loads on the right-hand arm have been taken uniformly at 55,000 pounds. The difference in favor of using exact balanced loads is slight and on the side of safety. It will be sufficiently accurate therefore in the future to use balanced loads when both arms of the bridge are covered

The lever-arm of L_0L_1 is 1.02 panel lengths; therefore

$$L_0L_1 = L_1L_2 = +\frac{109{,}820}{1.02} = +107{,}500 \text{ pounds.}$$

Members U_1U_2, U_2U_3.

The center of moments for U_2U_3 is at L_2 and the moment influence line for unit panel loads for that point is shown in Fig. 17 as $MA_1C_1E_1G_1N \ldots OQ_1T_1 \ldots P$. The various ordinates are found precisely as in the case just treated. The ordinates for the right-hand arm are exactly twice as great as those for the previous case, and they need not be recalculated. This is evident because the reactions at L_0 caused by the unit loads on $L_6 \ldots L_{11}$ are the same and, in determining the moments at L_2, the lever-arms are twice as great as those for L_1. For similar reasons the moment influence lines for L_3, L_4, and L_5, for loads on the right arm, need not be calculated in detail, as the ordinates are respectively 3, 4, and 5 times as great as for L_1.

The ordinates for the left span are calculated as follows:

Load at L_1, $A_1B = (0.7161 \times 2) - 1 \times 1 = +0.4322$
" " L_2, $C_1D = (0.4532 \times 2) = +0.9064$
" " L_3, $E_1F = (0.2322 \times 2) = +0.4644$
" " L_4, $G_1H = (0.0741 \times 2) = +0.1482$

All the ordinates are positive, showing that all loads on both arms produce compression in U_2U_3; but with the entire left arm loaded, the counter U_1L_2 comes into action and causes the stress in U_1U_2 to be the same as that in U_2U_3. Therefore for the greatest compression in these members the moving load must be so placed on the right arm as to give the reaction at L_0 its greatest value, while the heaviest panel concentrations of 84,200 and 80,600 pounds must be placed at L_2 and L_4.

This position of the loading is the same as that used for these members in the case of the partially continuous bridge (p. 312). The stresses are therefore the same, or

$$U_1U_2 = U_2U_3 = -\frac{150{,}800}{1.06} = -142{,}000 \text{ pounds.}$$

It is again to be observed that the loads on the right arm are taken equal to those on the left and that the two sets are balanced.

Members L_2L_3, U_3U_4.

The center of moments for these members is at L_3, and the moment influence line for that point is shown in Fig. 17 as $MA_2C_2E_2G_2N \ldots OT_2P$. The ordinates for the right arm require no additional explanation, while those for the left arm are found as follows:

$$\begin{aligned} A_2B &= (0.7161 \times 3) - (1 \times 2) = +0.1483 \\ C_2D &= (0.4532 \times 3) - (1 \times 1) = +0.3596 \\ E_2F &= (0.2522 \times 3) \qquad\quad = +0.6966 \\ G_2H &= (0.0741 \times 3) \qquad\quad = +0.2223 \end{aligned}$$

As each of these results is positive, it follows that all panel loads produce compression in U_3U_4 and tension in L_2L_3. It is also evident from the figure that the heavy concentrations must be placed at L_1 and L_3. This position

of the loading is therefore the same as in the case of the partially continuous trusses, and the stresses are:

$$L_2L_3 = +98{,}800 \text{ pounds}$$
$$U_3U_4 = -99{,}000 \quad \text{“}$$

Members L_3L_4, L_4L_5.

The stresses in these members are equal to each other and the unit moments are to be taken about U_4. The influence line for the left arm is $MA_3C_3E_3G_3N$, and the ordinates are found as follows:

$$A_3B = (0.7161 \times 4) - (1 \times 3) = -0.1356$$
$$C_3D = (0.4532 \times 4) - (1 \times 2) = -0.1872$$
$$E_3F = (0.2322 \times 4) - (1 \times 1) = -0.0712$$
$$G_3H = (0.0741 \times 4) = +0.2964$$

The influence line shows that all loads on the right arm, with that at L_4, produce tension in L_3L_4 and L_4L_5. Hence the heavy concentration of 84,200 pounds is to be placed at L_4, with the advance load of 17,500 pounds at L_3, while the right-arm panel-points may be assumed without essential error to carry the concentrated loads shown for L_0U_1. By the introduction of these moving loads the moment becomes

For right arm, $(450 + 600 + 520 + 300)4$	$= +\ 7{,}480$
“ panel load at L_4, $84{,}200 \times 0.2964$	$= +25{,}000$
“ “ “ “ L_3, $17{,}500 \times (-0.0712)$	$= -\ 1{,}240$
Total	$= +31{,}240$

The lever-arm of L_3L_4 is 1.162 panel lengths; therefore the stress is

$$L_3L_4 = L_4L_5 = +\frac{31{,}240}{1.162} = +26{,}900 \text{ pounds.}$$

Since the loads at the panel-points L_1, L_2, and L_3 produce compression in these members, the greatest compression in them will result from placing the heavy concentration of 84,200 pounds at panel-point L_2, with the advance load of 17,500 pounds at L_3. The actual concentrations then give

$$\begin{aligned} \text{For } L_1, &-54{,}600 \times 0.1356 = -\ 7{,}400 \\ \text{“ } L_2, &-84{,}200 \times 0.1872 = -15{,}700 \\ \text{“ } L_3, &-17{,}500 \times 0.0712 = -\ 1{,}240 \\ &\text{Total} = -24{,}340 \end{aligned}$$

Therefore

$$L_3L_4 = L_4L_5 = -\frac{24{,}340}{1.162} = -20{,}900 \text{ pounds.}$$

It is thus seen that the panels L_3L_4 and L_4L_5 are the only portions of the lower chord that can ever be subjected to compression by the moving load.

Members U_4U_5, U_5U_6, and L_5L_6.

Unit moments must be taken about L_5 in order to determine the stress in this member. The moment influence line is $MA_4C_4 \ldots NO \ldots T_4 \ldots P$, and the values of the ordinates for the left arm are found as follows:

$$\begin{aligned} A_4B &= (0.7161 \times 5) - 1 \times 4 = -0.4195 \\ C_4D &= (0.4532 \times 5) - 1 \times 3 = -0.7340 \\ E_4F &= (0.2322 \times 5) - 1 \times 2 = -0.8390 \\ G_4H &= (0.0741 \times 5) - 1 \times 1 = -0.6295 \end{aligned}$$

Hence all loads on the right arm subject U_4U_5 to compression and they must be removed when the greatest tension is sought.

The ordinates of the influence line for the right arm are the following:

$$\begin{aligned}
\text{At } L_7 &= Q_4S = 0.008165 \times 5 = 0.040825 \\
\text{“ } L_8 &= T_4V = 0.010886 \times 5 = 0.054430 \\
\text{“ } L_9 &= W_4X = 0.009526 \times 5 = 0.047630 \\
\text{“ } L_{10} &= Y_4Z = 0.005443 \times 5 = 0.027215
\end{aligned}$$

The greatest moving-load compression in U_4U_5 is therefore found with the concentrations of 84,200 and 80,600 pounds at L_7 and L_9 respectively. The moment is

$$\begin{aligned}
84{,}200 \times 0.040825 &= 3{,}440 \\
54{,}600 \times 0.054430 &= 2{,}970 \\
80{,}600 \times 0.047630 &= 3{,}840 \\
58{,}500 \times 0.027215 &= 1{,}590 \\
\text{Total} &= 11{,}840
\end{aligned}$$

As the lever-arm is 1.43 panels, the stress is

$$U_4U_5 = -\frac{11{,}840}{1.43} = -8{,}280 \text{ pounds.}$$

Since under this loading (on the right arm only) the rods C_6U_5 and L_6C_5, sloping upward to the left, do not act, the compression in U_5U_6 will be the horizontal component in U_4U_5. Hence

$$U_5U_6 = -\frac{8{,}280}{\sec \beta} = -7{,}800 \text{ pounds.}$$

If all the load on the right arm is removed and all that on the left arm is retained, the preceding results show that U_4U_5 will be subjected to its greatest tension. By placing the heavy concentrations at panel-points L_2 and L_4, and introducing all the panel concentrations on the left arm by means of the unit moments, there will result:

For panel-point L_1, $58{,}500 \times 0.4195 = 24{,}300$
" " L_2, $80{,}600 \times 0.7340 = 59{,}000$
" " L_3, $54{,}600 \times 0.8390 = 45{,}800$
" " L_4, $84{,}200 \times 0.6295 = 53{,}000$

Total $= 182{,}100$

$$\therefore\ U_4U_5 = +\frac{182{,}100}{1.43} = +127{,}500 \text{ pounds.}$$

Under this condition of loading the rods L_5C_6 and C_5U_6, sloping upward to the right, do not come into action; therefore the compression in L_5L_6 must equal the horizontal component of the tension in U_4U_5, or

$$L_5L_6 = -\frac{127{,}500}{\sec \beta} = -120{,}000 \text{ pounds.}$$

It has been shown that moving load on one arm only produces compression in U_5U_6; hence to obtain the greatest tension in that member, the whole of both arms must be loaded with balanced moving loads, which is precisely the condition used on p. 315, where there was found

$$U_5U_6 = +115{,}000 \text{ pounds.}$$

Web Members.

Hangers U_1L_1 and U_4L_4.

The member U_1L_1 may receive its live-load tension in either one of two ways: 1st, when the member L_1U_2 is acting in tension, U_1L_1 must carry, as tension, the difference of the vertical components of L_0U_1 and U_1U_2; and, 2d, when the counter-member U_1L_2 is stressed, U_1L_1 acts in tension as a single hanger. It will, however, be found that when the member L_1U_2 acts in tension there will be but

little live load on the bridge and that the stress in U_1L_1 as a simple hanger is greater than when it carries the difference of the vertical components of L_0U_1 and U_1U_2. Hence

$$U_1L_1 = +84{,}200 \text{ pounds.}$$

The member U_4L_4 is again a simple hanger, and its greatest stress is the greatest panel load at its foot. Hence

$$U_4L_4 = +84{,}200 \text{ pounds.}$$

The live-load stresses in the other web members will again be found by the method of moment influence lines,

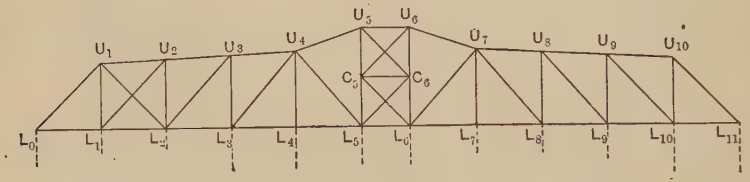


Fig. 18.

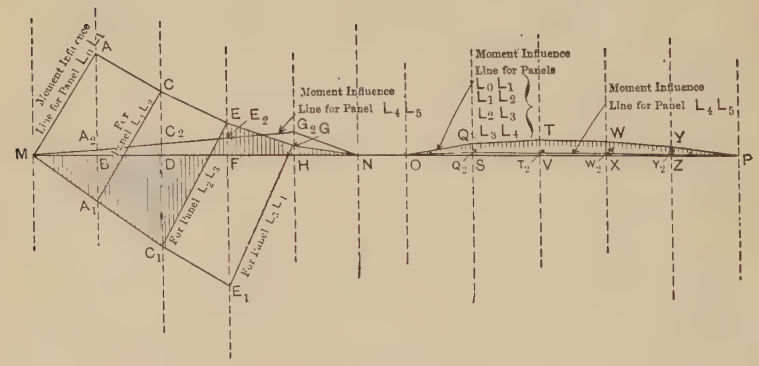


Fig. 19.

as shown in Fig. 19. The centers of moments are taken at the proper intersections of the chord members, and all lever-arms are measured in panel lengths.

Members L_2U_3 and U_3L_3.

The center of moments for these members is found twenty panels to the left of L_0, at the intersection of U_3U_4 and L_2L_3, and the moment influence line is shown (shaded) in Fig. 19 as $MA_1C_1EGN \ldots OQTWYP$. The ordinates are found as follows, those for the right arm being magnified ten times in the figure:

$$\begin{aligned}
A_1B &= (0.7161 \times 20) - (1 \times 21) = -\ 6.678 \\
CD &= (0.4532 \times 20) - (1 \times 22) = -12.936 \\
EF &= (0.2312 \times 20) = +\ 4.624 \\
GH &= (0.0742 \times 20) = +\ 1.484 \\
QS &= (0.0082 \times 20) = +\ 0.164 \\
TV &= (0.0109 \times 20) = +\ 0.218 \\
WX &= (0.0095 \times 20) = +\ 0.190 \\
YZ &= (0.0054 \times 20) = +\ 0.108
\end{aligned}$$

It is evident from the figure that the greatest tension in L_2U_3 and greatest compression in U_3L_3 will occur with heavy panel concentrations at L_1 and L_2. This will place the advance load of 17,500 pounds at L_3. The moment, expressed in pounds × panel lengths, is then

$$\begin{aligned}
54{,}600 \times\ 6.678 &= +\ 365{,}000 \\
84{,}200 \times 12.936 &= +1{,}090{,}000 \\
-17{,}500 \times\ 4.624 &= -\ 81{,}000 \\
\text{Total} &= +1{,}374{,}000
\end{aligned}$$

The lever-arm of U_3L_3 is 23 panels, and of L_2U_3, 16.4 panels. Therefore

$$U_3L_3 = -\frac{1{,}374{,}000}{23} = -59{,}800 \text{ pounds};$$

$$L_2U_3 = +\frac{1{,}374{,}000}{16.4} = +83{,}800 \text{ pounds}.$$

It will presently be shown that L_2U_3 need not be counter-braced, so that the position of the loading causing the greatest compression in that member need not be considered in detail. It is evident, however, that the right-hand arm together with panel-points L_3 and L_4 should be covered with live load.

Members U_2L_2, L_1U_2, and the Counter U_1L_2.

The center of moments for these members is at the same point as for U_3L_3. The moment influence line, marked in Fig. 19 "for panel L_1L_2," will therefore remain unchanged except for the panel-point L_2, which is now to the right of the section cutting the truss. The influence line is $MA_1CE\ldots$, the ordinate CD, expressed in the same units as before, being equal to $(+0.4532\times 20)=+9.064$.

It is at once evident that the greatest tension in L_1U_2 requires the advance load to be at L_2, with the load of 84,200 pounds at L_1. The desired moment will then be

$$+84{,}200\times 6.678 = 562{,}000$$
$$-17{,}500\times 9.064 = 159{,}000$$
$$\text{Total} = 403{,}000$$

The lever-arm of L_1U_2 is 15.3 panels; therefore

$$L_1U_2 = +\frac{403{,}000}{15.3} = +26{,}300 \text{ pounds.}$$

The greatest compression in L_1U_2 will be found with the advance weight at L_1, the head of the train facing L_0. If the advance weight were very large, it would have to be placed at L_2. Trial will quickly determine which position should be chosen. Since this position is precisely the same as for the case of the balanced loads

in the treatment of the partially continuous bridge, the stress found there may be accepted here. Since that compression is, however, greater than the tension caused by the dead load, L_1U_2 must be counterbraced, and the maximum tension in that member will be (see p. 320)

$$U_1L_2 = +67{,}500 \text{ pounds.}$$

This confirms the statement made when finding the stresses in the chord members U_1U_2 and U_2U_3 with both arms covered by the live load, the counter-member, and not L_1U_2, being assumed to act.

The member U_2L_2 takes its greatest compression with the same position of the moving load as for maximum tension in L_1U_2; therefore

$$U_2L_2 = -\frac{403{,}000}{22} = -18{,}300 \text{ pounds.}$$

If the counter-member U_1L_2 be assumed not to act, the stress in U_1L_1 may be found by the method of moments, a section being passed through U_1U_2, U_1L_1, and L_0L_1. The center of moments will be 20 panels from L_0. The corresponding moment influence line is shown in Fig. 19 as $MACE \ldots$ marked "for panel L_0L_1." The ordinate AB has the value $0.7161 \times 20 = 14.322$. It is evident that both arms of the bridge would be covered with live load for the maximum tension, but, as previously stated, this brings the counter-member U_1L_2 into action, and the stress in U_1L_1 is equal to the load hanging at its foot.

As in the case of the partially continuous swing-bridge it is clear that no other counter than U_1L_2 is required.

Member L_3U_4.

The center of moments for this member is also 20 panels from L_0, and the influence line marked in Fig. 19,

"for panel L_3L_4," is $MA_1C_1E_1HN$ . . . . The values of the ordinates are the same as for the previous cases, except that E_1F is $(0.2312 \times 20) - (1 \times 23) = -18.376$.

The advance load must be placed at L_4 for the greatest tension in U_3L_4, with the heavy concentrations at L_3 and L_1. The moment then becomes

Load at L_1,	80,600 ×	6.678 =	538,000
" " L_2,	54,600 ×	12.936 =	706,000
" " L_3,	84,200 ×	18.376 =	1,542,000
" " L_4,	−17,500 ×	1.484 =	26,000
		Total =	2,760,000

The lever-arm is 17.5 panels; therefore

$$L_3U_4 = +\frac{2,760,000}{17.5} = +158,000 \text{ pounds.}$$

Member U_4L_5.

The center of moments for this member is 0.8 panel length from the foot of the end post, and 0.2 panel length from L_1. The moment influence line is $MA_2C_2E_2G_2N$ . . . O . . . T_2 . . . P, the ordinates for which are found as follows:

$$A_2B = (0.7161 \times 0.8) + (1 \times 0.2) = +0.7729$$
$$C_2D = (0.4532 \times 0.8) + (1 \times 1.2) = +1.5626$$
$$E_2F = (0.2322 \times 0.8) + (1 \times 2.2) = +2.3858$$
$$G_2H = (0.0742 \times 0.8) + (1 \times 3.2) = +3.2594$$
$$Q_2S = (0.0082 \times 0.8) = +0.00656$$
$$T_2V = (0.0109 \times 0.8) = +0.00872$$
$$W_2X = (0.0095 \times 0.8) = +0.00760$$
$$Y_2Z = (0.0054 \times 0.8) = +0.00432$$

It is evident then that the entire bridge is to be covered with moving load for the greatest stress in U_4L_5, precisely

as was done in the case of the partially continuous bridge. Therefore, from that case,

$$U_4L_5 = -180{,}000 \text{ pounds.}$$

At the end of this article the case will be considered in which the counter U_1L_2 is omitted and the members L_1U_2 and U_2L_2 counterbraced. It has already been shown that L_2U_3 and U_3L_3 never sustain other kinds of stresses than those induced by the fixed load.

Since the function of the main web members is to carry load or shear over to the center, one condition to be fulfilled in order that they may receive their greatest stresses is, in general, to make the reaction at the end of the arm in which they are located as small as possible. For the main web stresses in the left arm, therefore, *no moving load was placed on the right arm*, because all such loads increase the upward reaction at the foot of the end post.

This condition holds in general because the chords of draw-spans are usually either parallel or so inclined to each other that their intersections lie *without* the span. If, however, their intersections lie *within* the span, as in the case of U_4L_5, the reactions at the free end of the span will have moments of the same sign as those loads between the moment origin and the web members in question. Hence in those cases the reactions (with moving loads omitted between moment origin and the free end) should be as large as possible, and the moving load should cover the whole of the other arm.

Stresses in Members in Tower at Center Pier.

The greatest stresses in the members L_5C_6, C_5U_6, and C_5C_6 in the center panel are next to be found. These members carry the shear past the center for all unbalanced loads on either arm; hence the stresses in them will occur

when there exists the greatest amount of unbalanced load; i.e., when the moving load covers one arm while the other arm is free of it. This also follows clearly from the fact, demonstrated in Art. 102, that all load in the arm, or span l_1 causes R_3 to be negative and R_2 positive; and hence, since the stresses in these members result from the downward pull of R_3, any load in the arm, or span, l_3 would reduce the negative R_3 by balancing some load in l_1, and thus correspondingly reduce the stresses in question. Those considerations are concomitant with complete continuity over the center.

On page 308 are found values of R_1 and R_4 for a unit panel load at each panel-point of either arm, and by using these in eq. (7*a*) of Art. 102, the following values of R_3 result, remembering that P is unity, that $c=6.707$, and that n has the values 0.2, 0.4, 0.6, and 0.8 respectively:

For P at L_1, $R_3=-0.6044$
" " " L_2, $R_3=-1.0579$
" " " L_3, $R_3=-1.2095$
" " " L_4, $R_3=-0.9076$

The unbalanced moving loads will therefore be placed on the left arm with the heaviest concentrations at L_2 and L_4. R_3 then becomes

$-58,500\times0.6044=-\ 35,400$ pounds
$-80,600\times1.0579=-\ 85,100$ "
$-54,600\times1.2095=-\ 66,000$ "
$-84,200\times0.9076=-\ 76,400$ "

Total $=-262,900$ "

The reaction R_4 at L_{11} will be found for the same position of loading to be + 2,400 pounds.

The parallel tension braces C_6U_5, and L_6C_5, sloping upward to the left, will be found to act, and since the

center panel or span is divided into two stories, the upper half of the center post L_5U_5 will sustain a stress different from that in the lower half. Again, the stresses in the two center posts L_5U_5 and L_6U_6 will not only differ in amount, but will also be different in kind.

Since all these stresses are found for one position of the loading, a single stress diagram for that part of the structure shown in Fig. 20 will suffice to determine all

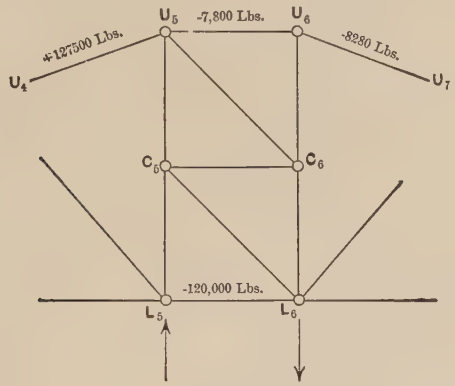


FIG. 20.

the stresses. The stresses given in the figure, U_4U_5, U_5U_6, U_6U_7, and L_5L_6, have been found already. Panel-point U_6 is first treated, since at that point there are but two unknown stresses; then points U_5, C_6, and C_5 are taken in the order indicated. The following values were scaled from the diagram:

$U_5U_6 = -7{,}800$ pounds.
$U_5C_6 = C_5L_6 = +180{,}000$ pounds.
$C_5C_6 = -126{,}000$ pounds.
$L_5C_5 = -303{,}000$ "
$C_5U_5 = -176{,}000$ "
$L_6C_6 = +133{,}000$ "
$C_6U_6 = +\ \ 3{,}000$ "

As a check, it is evident that the vertical components of the members C_5L_6, L_6C_6, and L_6U_7 must equal the reaction $R_3 = -262{,}900$.

The vertical component of C_5L_6 is $+130{,}000$ pounds; L_6C_6 is $+133{,}000$. The stress in L_6U_7 is found by the method of moments to be $\frac{2{,}400 \times 0.8}{3.19} = -600$ pounds, while its vertical component is -450 pounds. Therefore $(130{,}000 + 133{,}000) - 450 = +262{,}550$ pounds, a satisfactory check.

This completes the computations for all the moving-load stresses, and Table I shows them, together with those due to the fixed load, as found in Art. 103.

TABLE I.

Member.	Fixed-load Stress in Pounds.	Moving-load Stress in Pounds.	Member.	Fixed-load Stress in Pounds.	Moving-load Stress in Pounds.
L_0L_1	− 9,800	+107,500	U_1L_1	− 16,300	+ 84,200
L_1L_2	− 40,700	+107,500	L_1U_2	+ 45,100	+ 26,300
L_2L_3	− 89,900	+ 98,800	U_2L_2	− 38,200	− 18,300
L_3L_4	−155,000	+ 26,900 − 20,900	L_2U_3	+ 73,600	+ 83,800
			U_3L_3	− 59,200	− 59,800
L_4L_5	−155,000	+ 26,900 − 20,900	L_3U_4	+100,000	+158,000
			U_4L_4	+ 16,700	+ 84,200
L_5L_6	−186,000	−120,000	U_4L_5	− 47,500	−180,000
L_0U_1	+ 14,000	−154,000	L_5C_5	− 73,600	+133,000 −303,000
U_1U_2	+ 9,800	−142,000			
U_2U_3	+ 40,700	−142,000	C_5U_5	− 73,600	+ 3,000 −176,000
U_3U_4	+ 89,960	− 99,000			
U_4U_5	+197,900	− 8,280 +127,500	C_5C_6	0	−126,000
			$L_5C_5 = C_5U_6$	0	+180,000
U_5U_6	+186,000	− 7,800 +115,000	U_1L_2	0	+ 67,500

By comparison of these results with those of Table II on page 322, it is observed that the assumption of complete continuity produces web stresses either slightly greater in every instance (except those due to the moving load on one arm only) than the hypothesis of partial continuity, or identical with it. The chord stresses are also

usually slightly greater, or equal, excepting, also, those due to moving load on one arm only in the case of partial continuity, which are materially greater. Toward and at the ends of the span, therefore, the assumption of a simple non-continuous span gives much larger stresses, both in chords and web, than are found under the supposition of continuity, as would be anticipated. With these exceptions, however, the differences are unimportant, and either method could be used with indifference, permitting the use of that which results in the least labor of computation.

A great difference is seen, however, in the magnitude of the stresses both for the center posts and central diagonals, and in the labor of their computation. C_5C_6, C_5U_6, and L_5C_6 suffer no stress under partial continuity, but are subjected to heavy stresses if the continuity is complete. The posts also suffer heavy compression in the latter case, and a comparatively light compression in the former, due only to the fixed load. The lower stories of the same members are also subjected to heavy tension when the continuity is complete.

The negative or downward reaction, R_3, at L_6 has been found to be $-262{,}900$ pounds, while the fixed-load upward reaction is 103,000 pounds plus the lower-chord fixed load at $U_6L_6 = \left(\frac{20.5 + 27.5}{2}\right) 1{,}700 = 21{,}000$ pounds, plus the weight of the drum, etc., which may be assumed to be concentrated at the same point, perhaps 20,000 pounds. Hence the amount of anchorage which would have to be provided under the assumption of complete continuity is

$$262{,}900 - (103{,}000 + 21{,}000 + 20{,}000) = 118{,}900 \text{ pounds},$$

and unless this were supplied, each foot of U_5L_5 and U_6L_6 would, in turn, rise and fall with varying conditions of moving load, so that not only destructive hammering would

take place, but also the condition of those spans continuous over two supports would be displaced by that of two unequal spans continuous over one support, thus vitiating the computations underlying the design of the trusses.

The realization of the conditions requisite for the case of complete continuity, therefore, involves considerably increased difficulty and expense in connection with the design of the center portion.

Inasmuch as the main truss members have been found to sustain essentially the same stresses in either case, except where those under partial continuity largely exceed the others, and inasmuch as all drawbridge formulæ drawn from the theory of the continuous beam involve the uniformity of the moment of inertia of all normal sections, thereby incurring a considerable error in at least some of the resulting computations, it is much more rational and in harmony with better judgment to use the methods of partial continuity in all cases. These and similar considerations have led engineers generally to adopt the partial-continuity assumption in the construction of swing-bridges.

It is to be observed that the unbalanced uplift at the foot of U_5L_5 and the stresses in C_5U_6, C_5C_6, and L_5C_6 will increase rapidly as the length of the center panel or span decreases; and, hence, that it is advisable to make that center panel as long as possible. The method of computation will be in no way changed if the center panel is designed with one store only instead of two.

The resultant stresses in this case are not very different from those in Table II of p. 322, except in those members already indicated. In actual practice, the entire lower chord, and the upper-chord panels U_1U_2, U_2U_3, and U_3U_4 would all be designed with compression cross-sections—i.e., they would be counterbraced. The lower story of

L_5C_5 must also be counterbraced, and ordinary construction would make it so even if it were not required.

The existence of the four central diagonals C_5U_6, C_6U_5, L_5C_6, and L_6C_5, designed to transfer shear, results in some ambiguity in the upper-chord stresses at the center. As the moving load passes on the bridge, one pair of those diagonals, C_6U_5 and L_6C_5, receive their greatest stresses, and then they may either be supposed to be relieved with the further progress of the train, or they may be supposed to hold essentially their greatest stresses while the other pair gradually take the same condition, as the moving load covers the entire structure. Either supposition fulfills the requisites for equilibrium, but the former will give the greatest possible stress in U_5U_6, and was used in the preceding computations, because the stress in the same upper-chord member will be decreased under the latter supposition by an amount corresponding to the horizontal components of the stresses in the diagonals C_5U_6 and L_5C_6. This condition of ambiguity cannot be avoided under the assumption of complete continuity, but disappears if the continuity is assumed to be partial only. The proper method, therefore, is to compute the greatest possible stress in each member, and use it in the design, and this has been done in the present case.

The reactions R_1 and R_2 at L_0 and L_5, respectively, remain to be written, but they can be taken directly from results already found in the preceding article.

For this purpose, and for the reasons fully explained in Art. 103, the moving load will be taken as uniform and at 2,880 pounds per linear foot for each truss. The greatest end reaction will exist with the moving load over the entire structure, and it has been shown on p. 325 that its amount will be

$$R_1 = 119{,}500 + \frac{79{,}200}{2} + 20{,}000 = 179{,}100 \text{ pounds.}$$

The half-panel load is that adjacent to the end, which does not affect the trusses, but forms a part of the reaction to be supported by the truss ends, while the 20,000 pounds is the wheel concentration. Since the moving load on each arm produces a negative or downward reaction at the opposite side of the drum, the greatest reaction R_2 will be produced with the moving load on the adjacent arm only. The reaction R_1, due to the four panel loads at the panel-points L_1, L_2, L_3, and L_4, will be

$$R_1' = (.7161 + .4532 + .2322 + .0741) \times 79,200 = 116,500 \text{ lbs.}$$

Hence

$$R_2' = (4 \times 79,200) - 116,500 + \frac{24}{27.5} \times 79,200 + 20,000$$
$$= 289,000 \text{ pounds.}$$

The third term in the second member is for the half-panel adjacent to the center and the central panel, while the 20,000 pounds is for the driving-wheel concentration, as already explained. As shown on page 325, the total fixed load at L_5 is 124,000 pounds; hence the total reaction desired is

$$R_2 = 289,000 + 124,000 = 413,000 \text{ pounds.}$$

The Omission of Counters.

If all counters are omitted, it will usually be necessary to counterbrace some of the main web members. In the present case L_1U_2 will be most in need of such treatment. The position of moving load required to give its maximum compression to L_1U_2 is the same as that used in finding the greatest tension in U_1L_2 on page 341. The moment is 1,063,000 pounds × panel lengths, and the stress therefore

$$L_1U_2 = -\frac{1,063,000}{15.3} = -69,800 \text{ pounds.}$$

Since the fixed-load tension is 45,100 pounds, this member must be counterbraced.

With the same position of the loading, there will be found

$$L_2U_2 = +\frac{1{,}063{,}000}{22} = +48{,}300 \text{ pounds},$$

and this member also requires counterbracing.

For the greatest compression in L_2U_3 and tension in U_3L_3, the advance load of 17,500 pounds must be placed at L_2, with the heavy concentrations at L_3 and L_4 and the right arm also covered.

The moments due to the component parts of the reaction at L_0, taken about a point 20 panels from L_0, are:

$$\begin{aligned}
+17{,}500 \times 0.453178 \times 20 &= +158{,}000 \\
+84{,}200 \times 0.232204 \times 20 &= +392{,}000 \\
+54{,}600 \times 0.074153 \times 20 &= +\ 81{,}000 \\
+\ 1{,}870 \times 20 &= +\ 37{,}400 \\
\text{Total} &= +668{,}400
\end{aligned}$$

From this moment is to be subtracted the negative moment due to the advance load itself, or $17{,}500 \times 22 = 385{,}000$. The final moment is 283,400 pounds $\times$ panel lengths, and therefore

$$U_3L_3 = +\frac{283{,}400}{23} = +12{,}300 \text{ pounds.}$$

$$L_2U_3 = -\frac{283{,}400}{16.4} = -17{,}200 \quad \text{“}$$

Both of these results are so small in comparison with the opposite fixed-load stresses, that no counterbracing is required.

Art. 105.—Tables and Diagrams.

The labor of stress computations by the methods given in the three preceding articles can be much reduced by the aid of the tables and diagrams which follow. They are devised for the purpose of showing the reactions at

TABLE I.

$n=\frac{z}{l}$	R_1			R_4		
	c 8.822	c 6.643	c 4.395	c 8.822	c 6.643	c 4.395
0	1.	1.	1.	0.0	0.0	0.0
.05	0.92754	0.92823	0.92951	.001143	.001424	.001899
.10	.8554	.85679	.85933	.002269	.002826	.00377
.15	.78398	.78601	.78976	.003360	.004186	.005583
.20	.71355	.71621	.72112	.004401	.005482	.007311
.25	.64447	.64771	.65371	.005372	.006692	.008926
.30	.57708	.58085	.58784	.006257	.007794	.010396
.35	.51171	.51596	.52382	.007039	.008768	.011694
.40	.44871	.45336	.46195	.007701	.009593	.012795
.45	.38841	.39338	.40256	.008226	.010247	.013667
.50	.33115	.33634	.34593	.008595	.010706	.01428
.55	.27727	.28257	.29239	.008792	.010952	.014607
.60	.2271	.23241	.24223	.008801	.010963	.014623
.65	.18098	.18617	.19578	.008604	.010718	.014295
.70	.13926	.14419	.15333	.008182	.010192	.013595
.75	.10226	.1068	.11519	.00752	.009367	.012494
.80	.07033	.07431	.08368	.006601	.008222	.010967
.85	.04379	.04706	.05309	.005421	.006752	.009006
.90	.02301	.02537	.02974	.003919	.004882	.006512
.95	.00829	.009576	.011945	.002123	.0026437	.003527
1.00	0.00	0.00	0.00	0.00	0.00	0.00

either extremity of a three-span, rim-bearing drawbridge for a square crossing and for arms of equal length. The ratios, c, of either arm to the center span or panel, as shown immediately preceding eq. (3) of Art. 102, are given in Table I, with the greatest, least, and mean values, taken from fifteen drawbridges as they have actually been designed. They cover, therefore, a range that will include nearly all practical cases of pin-connected structures. The columns of the table show the reactions R_1 and R_4 for a unit panel load placed successively at distances from

the free end of the arm l_1, which vary by $0.05l_1$. These reactions due to unit panel loads are computed from eqs. (5*a*) and (6*a*) of Art. 102, with the values of c given at the heads of the columns of the table. As an example, if the unit load be placed at the distance $0.2l_1$ from the free end of the arm, the reaction R_1 will have the values of 0.71355 and 0.72112 for the values of $l \div l_2 = c = 8.822$

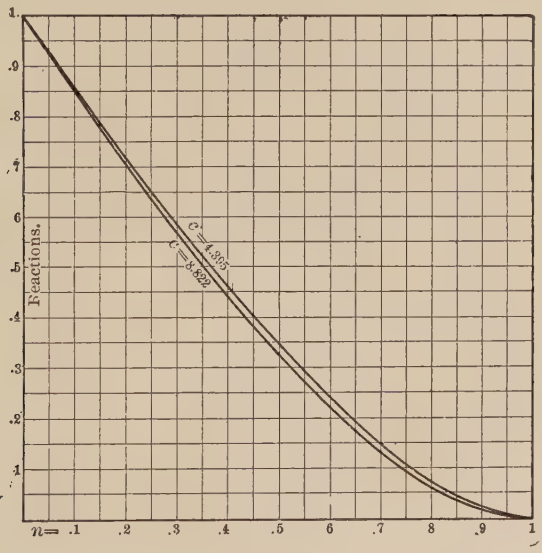


FIG. 21.

and 4.395 respectively. The corresponding reactions, R_4, will be 0.004401 and 0.007311. The reactions for any panel load will be found by simply multiplying those given in the table by the actual panel load in pounds or tons, as the case may be.

It will be observed that the reactions vary little between the limits of range of the values of c. Hence, for values materially outside of those limits, the corresponding reactions may be assigned by the aid of Figs. 21 and 22 with sufficient accuracy. Those figures exhibit the results

given in Table I. Fig. 21 gives the reactions R_1, and Fig. 22 the reactions R_4. The three curves so nearly coincide in Fig. 21 that only those for the two values of $c=8.822$ and $c=4.395$ are shown. For practical working,

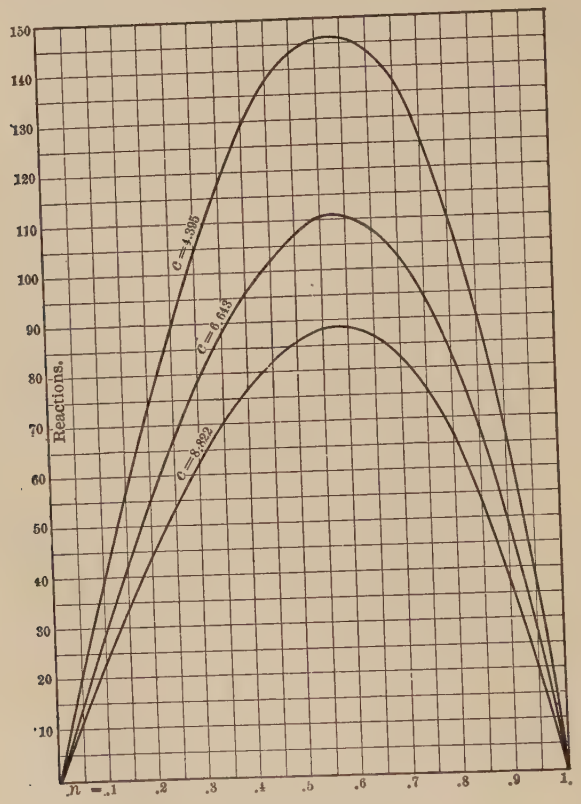


FIG. 22.

that figure should be drawn to at least double the size shown. In Fig. 21 the tabular values of the reactions R_1 have been taken full size, while those for R_4, in Fig. 22, have been multiplied by 10,000. The figures will enable either of the reactions to be read at a glance for any length

of panel in any length of arm. They, as well as Table I, are, therefore, perfectly general for a wide range of the value of c.

After the reactions are read from the table or the diagrams, moments for either the web or chord stresses can readily be written; and from these moments the stresses themselves will at once result in accordance with the methods of the preceding articles. If the chords are parallel, moments will not be required for the web stresses, as the latter can be at once written from the shears.

In many cases, particularly if the chords are parallel, the values given in Tables II to VIII will be found convenient for 7- to 19-panel drawbridges, in which the center panel is equal in length to the others. They have been computed by Mr. Frank C. Osborn, consulting engineer, of Cleveland, Ohio, for his own practice, who has kindly given the privilege of using them in this connection. They express the shears and moments on the basis of each panel load being unity, and of each panel being one unit in length, being precisely the quantities already used in the preceding methods of influence lines. Each actual shear will therefore be found by multiplying each tabular shear by the actual panel load, and each actual moment will be found by multiplying each tabular moment by the product of the actual panel load by the actual panel length. These tables also show the greatest shears and moments at the various panel-points. The reactions due to the various panel loads, by the aid of which the shears and moments are obtained, are computed from eqs. (5a) and (6a) of Art. 102, except those which belong to the simple spans, which, of course, follow the law of the lever. If the chords are not parallel, the web stresses must be determined by the method of moments, as illustrated in the two preceding articles. Even if the length of the center panel should differ to some extent from that of the others, Table I

and Figs. 21 and 22 show that the values in Tables II to VIII will not be sensibly changed.

TABLE II.

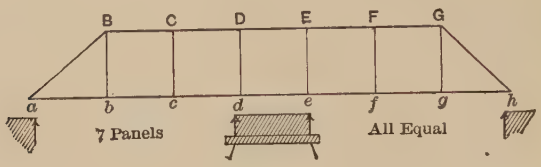


FIG. 23.

NOTE.—Shear in panel *ab*=reaction at *a*, and Shear *cd*=reaction at *d*.

Loads at	Shear in Panel			Moment at		
	ab	*bc*	*cd*	*b*	*c*	*d*
b and *g*	+0.568	−0.432	−0.432	−0.568	+0.136	−0.296
c and *f*	+0.210	+0.210	−0.790	+0.210	+0.420	−0.370
Maximum	+0.778	+0.210	.....	+0.778	+0.556	.....
	.....	−0.432	−1.222	.....	.....	−0.666
As a simple span	+1.000	+0.333	.....	+1.000	+1.000	.....
	.....	−0.333	−1.000	.....	.....	.....

TABLE III.

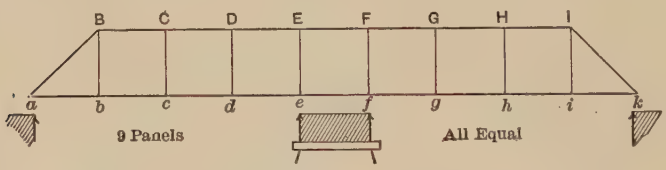


FIG. 24.

NOTE.—Shear in panel *ab*=reaction at *a*, and Shear *de*=reaction at *e*.

Loads at	Shear in Panel				Moment at			
	ab	*bc*	*cd*	*de*	*b*	*c*	*d*	*e*
b and *i*	+0.665	−0.335	−0.335	−0.335	+0.665	+0.330	−0.005	−0.340
c and *h*	+0.364	+0.364	−0.636	−0.636	+0.364	+0.728	+0.092	−0.544
d and *g*	+0.131	+0.131	+0.131	−0.869	+0.131	+0.262	+0.393	−0.476
Maximum	+1.160	+0.495	+0.131	.....	+1.160	+1.320	+0.485	.....
	.....	−0.335	−0.971	−1.840	.....	.....	−0.005	−1.360
As a simple span	+1.500	+0.750	+0.250	.....	+1.500	+2.000	+1.500	.....
	.....	−0.250	−0.750	−1.500	.....	.....	.....	.....

TABLE IV.

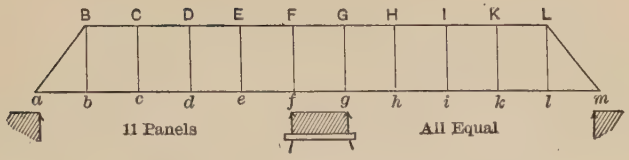


FIG. 25.

NOTE.—Shear in panel ab = reaction at a, and Shear ef = reaction at f.

Loads at	Shear in Panel					Moment at				
	ab	*bc*	*cd*	*de*	*ef*	*b*	*c*	*d*	*e*	*f*
b and *l*	+0.726	−0.274	−0.274	−0.274	−0.274	+0.726	+0.452	+0.178	−0.095	−0.369
c and *k*	+0.471	+0.471	−0.529	−0.529	−0.529	+0.471	+0.942	+0.412	−0.117	−0.646
d and *i*	+0.252	+0.252	+0.252	−0.748	−0.748	+0.252	+0.505	+0.757	+0.009	−0.738
e and *h*	+0.089	+0.089	+0.089	+0.089	−0.911	+0.089	+0.178	+0.268	+0.357	−0.554
Maximum	+1.538	+0.812	+0.341	+0.089	.....	+1.538	+2.077	+1.615	+0.366	.....
	.....	−0.274	−0.803	−1.551	−2.462	.....	.....	.....	−0.212	−2.307
As a simple span	+2.000	+1.200	+0.600	+0.200	.....	+2.000	+3.000	+3.000	+2.000	.....
	.....	−0.200	−0.600	−1.200	−2.000	.....	.....	.....	.....	.....

TABLE V.

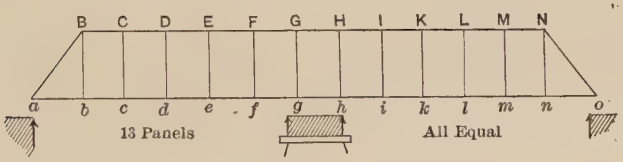


FIG. 26.

NOTE.—Shear in panel ab = reaction at a, and Shear fg = reaction at g.

Loads at	Shear in Panel					
	ab	*bc*	*cd*	*de*	*ef*	*fg*
b and *n*	+0.768	−0.232	−0.232	−0.232	−0.232	−0.232
c and *m*	+0.548	+0.548	−0.452	−0.452	−0.452	−0.452
d and *l*	+0.350	+0.350	+0.350	−0.650	−0.650	−0.650
e and *k*	+0.185	+0.185	+0.185	+0.185	−0.815	−0.815
f and *i*	+0.065	+0.065	+0.065	+0.065	+0.065	−0.935
Maximum	+1.916	+1.148	+0.600	+0.250	+0.065	.....
	.....	−0.232	−0.684	−1.334	−2.149	−3.084
As a simple span	+2.500	+1.667	+1.000	+0.500	+0.167	.....
	.....	−0.167	−0.500	−1.000	−1.667	−2.500

TABLE V—*Continued.*

Loads at	Moment at					
	b	*c*	*d*	*e*	*f*	*g*
b and *n*	+0.768	+0.537	+0.305	+0.074	−0.158	−0.390
c and *m*	+0.548	+1.097	+0.645	+0.193	−0.259	−0.710
d and *l*	+0.350	+0.700	+1.050	+0.400	−0.250	−0.900
e and *k*	+0.185	+0.370	+0.556	+0.741	−0.074	−0.889
f and *i*	+0.065	+0.129	+0.194	+0.259	+0.323	−0.612
Maximum	+1.916	+2.833	+2.750	+1.667	+0.323	.....
	.....	.....	.....	.....	−0.741	−3.501
As a simple span	+2.500	+4.000	+4.500	+4.000	+2.500	.....

TABLE VI.

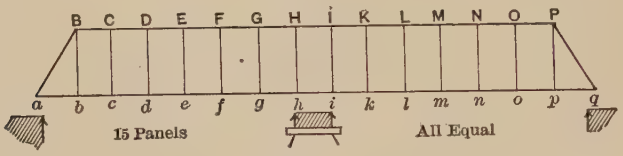


FIG. 27.

NOTE.—Shear in panel *ab*=reaction at *a*, and Shear *gh*=reaction at *h*.

Loads at	Shear in Panel						
	ab	*bc*	*cd*	*de*	*ef*	*fg*	*gh*
b and *p*	+0.799	−0.201	−0.201	−0.201	−0.201	−0.201	−0.201
c and *o*	+0.606	+0.606	−0.394	−0.394	−0.394	−0.394	−0.394
d and *n*	+0.427	+0.427	+0.427	−0.573	−0.573	−0.573	−0.573
e and *m*	+0.270	+0.270	+0.270	+0.270	−0.730	−0.730	−0.730
f and *l*	+0.142	+0.142	+0.142	+0.142	+0.142	−0.858	−0.858
g and *k*	+0.049	+0.049	+0.049	+0.049	+0.049	+0.049	−0.951
Maximum	+2.293	+1.494	+0.888	+0.461	+0.191	+0.049	.....
	.....	−0.201	−0.595	−1.168	−1.898	−2.756	−3.707
As a simple span	+3.000	+2.143	+1.429	+0.857	+0.429	+0.143	.....
	.....	−0.143	−0.429	−0.857	−1.429	−2.143	−3.000

Loads at	Moment at						
	b	*c*	*d*	*e*	*f*	*g*	*h*
b and *p*	+0.799	+0.600	+0.398	+0.198	−0.003	−0.203	−0.404
c and *o*	+0.606	+1.212	+0.819	+0.425	+0.031	−0.363	−0.757
d and *n*	+0.427	+0.855	+1.282	+0.709	+0.137	−0.436	−1.009
e and *m*	+0.270	+0.540	+0.811	+1.081	+0.351	−0.379	−1.109
f and *l*	+0.142	+0.283	+0.425	+0.566	+0.708	−0.150	−1.009
g and *k*	+0.049	+0.098	+0.148	+0.197	+0.246	+0.295	−0.656
Maximum	+2.293	+3.588	+3.883	+3.176	+1.473	+0.295	.....
	.....	.....	.....	.....	−0.003	−1.531	−4.944
As a simple span	+3.000	+5.000	+6.000	+6.000	+5.000	+3.000	.....

TABLE VII.

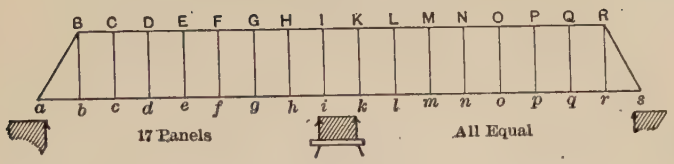


FIG. 28.

NOTE.—Shear in panel ab=reaction at a, and Shear hi=reaction at i.

Loads at	Shear in Panel							
	ab	*bc*	*cd*	*de*	*ef*	*fg*	*gh*	*hi*
b and *r*	+0.823	−0.177	−0.177	−0.177	−0.177	−0.177	−0.177	−0.177
c and *q*	+0.651	+0.651	−0.349	−0.349	−0.349	−0.349	−0.349	−0.349
d and *p*	+0.489	+0.489	+0.489	−0.511	−0.511	−0.511	−0.511	−0.511
e and *o*	+0.342	+0.342	+0.342	+0.342	−0.658	−0.658	−0.658	−0.658
f and *n*	+0.215	+0.215	+0.215	+0.215	+0.215	−0.785	−0.785	−0.785
g and *m*	+0.112	+0.112	+0.112	+0.112	+0.112	+0.112	−0.888	−0.888
h and *l*	+0.039	+0.039	+0.039	+0.039	+0.039	+0.039	+0.039	−0.961
Maximum	+2.671	+1.848	+1.197	+0.708	+0.366	+0.151	+0.039	.....
	.....	−0.177	−0.526	−1.037	−1.695	−2.480	−3.368	−4.329
As a simple span	+3.500	+2.625	+1.875	+1.250	+0.750	+0.375	+0.125	.....
	.....	−0.125	−0.375	−0.750	−1.250	−1.875	−2.625	−3.500

Loads at	Moment at							
	b	*c*	*d*	*e*	*f*	*g*	*h*	*i*
b and *r*	+0.823	+0.646	+0.470	+0.293	+0.116	−0.061	−0.238	−0.415
c and *q*	+0.651	+1.303	+0.954	+0.605	+0.257	−0.092	−0.441	−0.789
d and *p*	+0.489	+0.979	+1.468	+0.957	+0.447	−0.064	−0.575	−1.086
e and *o*	+0.342	+0.684	+1.026	+1.369	+0.711	+0.053	−0.605	−1.263
f and *n*	+0.215	+0.430	+0.644	+0.859	+1.074	+0.289	−0.497	−1.282
g and *m*	+0.112	+0.224	+0.336	+0.448	+0.559	+0.671	−0.217	−1.105
h and *l*	+0.039	+0.077	+0.116	+0.155	+0.193	+0.232	+0.271	−0.691
Maximum	+2.671	+4.343	+5.014	+4.686	+3.357	+1.245	+0.271	.....
	.....	.....	.....	.....	.....	−0.217	−2.573	−6.631
As a simple span	+3.500	+6.000	+7.500	+8.000	+7.500	+6.000	+3.500	.....

TABLE VIII.

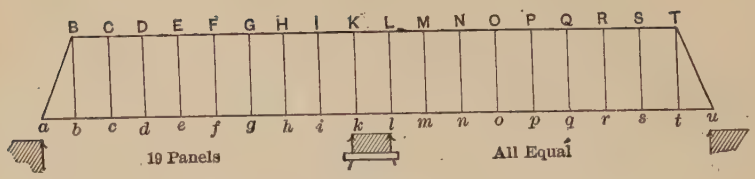


FIG. 29.

NOTE.—Shear in panel ab = reaction at a, and Shear ik = reaction at k.

Loads at	Shear in Panel								
	ab	*bc*	*cd*	*de*	*ef*	*fg*	*gh*	*hi*	*ik*
b and *t*.....	+0.842	−0.158	−0.158	−0.158	−0.158	−0.158	−0.158	−0.158	−0.158
c and *s*.....	+0.687	+0.687	−0.313	−0.313	−0.313	−0.313	−0.313	−0.313	−0.313
d and *r*.....	+0.540	+0.540	+0.540	−0.460	−0.460	−0.460	−0.460	−0.460	−0.460
e and *q*.....	+0.403	+0.403	+0.403	+0.403	−0.597	−0.597	−0.597	−0.597	−0.597
f and *p*.....	+0.280	+0.280	+0.280	+0.280	+0.280	−0.720	−0.720	−0.720	−0.720
g and *o*.....	+0.175	+0.175	+0.175	+0.175	+0.175	+0.175	−0.825	−0.825	−0.825
h and *n*.....	+0.091	+0.091	+0.091	+0.091	+0.091	+0.091	+0.091	−0.909	−0.909
i and *m*.....	+0.031	+0.031	+0.031	+0.031	+0.031	+0.031	+0.031	+0.031	−0.969
Maximum.	+3.049	+2.207	+1.520	+0.980	+0.577	+0.297	+0.122	+0.031	.....
	.....	−0.158	−0.471	−0.931	−1.528	−2.248	−3.073	−3.982	−4.951
As a simple span...	+4.000	+3.111	+2.333	+1.667	+1.111	+0.667	+0.333	+0.111	.....
	.....	−0.111	−0.333	−0.667	−1.111	−1.667	−2.333	−3.111	−4.000

Loads at	Moment at								
	b	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*k*
b and *t*.....	+0.842	+0.684	+0.526	+0.368	+0.210	+0.052	−0.106	−0.264	−0.423
c and *s*.....	+0.687	+1.374	+1.061	+0.748	+0.435	+0.122	−0.191	−0.504	−0.814
d and *r*.....	+0.540	+1.080	+1.620	+1.160	+0.700	+0.240	−0.220	−0.680	−1.142
e and *q*.....	+0.403	+0.806	+1.209	+1.612	+1.015	+0 418	−0.179	−0.776	−1 375
f and *p*.....	+0.280	+0.560	+0.840	+1.120	+1.400	+0.680	−0 040	−0.760	−1 482
g and *o*.....	+0.175	+0.350	+0.525	+0.700	+0.875	+1.050	+0.225	−0.600	−1.428
h and *n*.....	+0.091	+0.182	+0.273	+0.364	+0.455	+0.546	+0.637	−0.272	−1 184
i and *m*.....	+0.031	+0.062	+0.093	+0.124	+0.155	+0.186	+0.217	+0.248	−0.719
Maximum.	+3.049	+5.098	+6.147	+6.196	+5.245	+3.294	+1.079	+0 248	.....
	.....	.....	.....	.....	.....	.....	−0.736	−3.856	−8.567
As a simple span...	+4.000	+7.000	+9.000	+10.000	+10.000	+9.000	+7.000	+4.000	.....

Art. 106.—Example of Swing-bridge with Ends Simply Resting on Supports—One Support at Center.

The general principles fundamentally involved in this case are not different from those already employed. All the fixed load of the bridge is carried to the central point of support, whether the bridge is open or closed, the end supports furnishing reactions for the moving load only.

The truss to be taken as an example is the one shown in Fig. 30, in which the arms are of equal length.

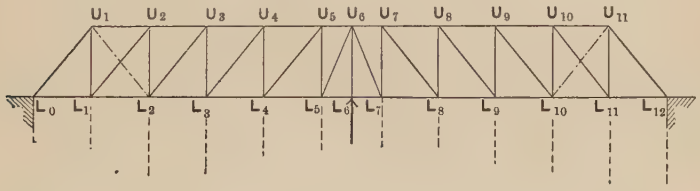


FIG 30.

The general formulæ to be used for the reactions at L_0, L_6, and L_{12}, and for the bending moment at the center, are eqs. (11), (12), (13), and (10) respectively of Art. 102. These equations may be written as follows, remembering that $l_1 = l_2 = l$, and $M_2 = M$:

$$M = -\frac{1}{4l^2}\left\{ \overset{1}{\Sigma} P(l^2 - z^2)z + \overset{2}{\Sigma} P(l^2 - z^2)z \right\}. \quad . \quad (1)$$

$$R_1 = \frac{1}{l}\left\{ \overset{1}{\Sigma} P(l - z) + M \right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$R_2 = \frac{1}{l}\left\{ \overset{1}{\Sigma} Pz + \overset{2}{\Sigma} Pz - 2M \right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$R_3 = \frac{1}{l}\left\{ \overset{2}{\Sigma} P(l - z) + M \right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It is to be remembered that z is measured from L_0 or L_{12} according as the left or right arm is considered.

The following are the data to be used:

Total length $= L_0L_{12} = 2l = 2L_0L_6 = 2L_6L_{12} = 144$ feet.
Uniform depth $= L_1U_1 = L_6U_6 = 16$ feet.
Panel length $= L_0L_1 = L_1L_2 = \text{etc.} = 13$ feet.
$L_5L_6 = L_6L_7 = 7$ feet.
Angle $L_0U_1L_1 = \alpha$.
" $L_5U_6L_6 = \beta$.

$\tan \alpha = 0.8125$; $\sec \alpha = 1.29$.
$\tan \beta = 0.4375$; $\sec \beta = 1.09$.

$$\frac{1}{4l^2} = \frac{1}{20{,}736} = 0.00004823$$

Total fixed weight per foot of bridge = 1,200 lbs.
Upper-chord panel fixed weight $= W =$ 5,500 lbs. per truss
Lower- " " " " $= W' =$ 10,000 " " "
Uniform panel moving load $= w =$ 19,500 " " "

The moving load traverses the lower chord, and the weight of the floor system is taken at nearly 350 pounds per foot.

On account of the extra weight of the locking apparatus, the fixed weight at L_0, or L_{12}, will be taken at 3,000 pounds per truss, and will be denoted by w_1.

As is clear from the figure, all inclined web members, except the end posts, act in tension only, while the verticals are compression members.

FIXED-LOAD STRESSES.

These stresses are found most expeditiously by graphic methods, and their determination requires no detailed explanation.

LIVE-LOAD STRESSES.

General Considerations.

As the ends L_0 and L_{12} are neither latched down nor lifted up, either arm is a single truss simply supported at

each end, for all moving loads which rest upon it, *so long as there are no moving loads on the other arm.* For exactly the same reasons, therefore, as those given in Art. 103, *any counter, as U_1L_2, will sustain its greatest stress when the moving load extends from its foot to the center, if on other moving load rests on the bridge.*

It must still be borne in mind that in connection with any counter-stress, the stress in the vertical which cuts its upper extremity is to be found, for such a one may be the greatest stress in the vertical.

The general expression for the shear in any web member may be written

$$s = S - n(W + W') - n'w,$$

in which S is the shear at one extremity of the arm and n and n' the numbers of fixed and moving panel weights respectively between the same end of the arm and the web member in question. In considering the main web members, S will be taken adjacent to the center, and, in the present example, at an indefinitely short distance from L_6 in the arm L_0L_6.

For a given condition of loading in L_0L_6, it is evident that the smaller is R_1 the greater will be S; but eq. (1) shows that M is always negative. Hence so long as $\overset{1}{\Sigma}P(l-z)$ remains the same, eq. (2) shows that R_1 decreases as M increases (numerically).

Again, eq. (1) shows that M will have its greatest numerical value, other things remaining the same, when $\overset{2}{\Sigma}P(l^2-z^2)z$ has the greatest value possible; i.e., when the moving load covers the whole of the arm L_6L_{12}. With a given value, therefore, for $\overset{1}{\Sigma}P(l-z)$, R_1 will be the least possible when the moving load covers the whole of the other arm, or the whole of L_6L_{12}; consequently S will be the greatest under the same conditions. Having found under

what circumstances S is the greatest, precisely the same reasoning used in the preceding acticles shows that s will be the greatest, under the same circumstances, when n' is zero.

Any inclined main web member, then, will sustain its greatest tensile stress when the moving load extends from its foot to the free extremity of the arm in which it is found, and covers at the same time the whole of the other arm.

Any vertical web member, unless acting as a counter, will sustain its greatest compression in connection with the greatest tension in the inclined main web member which cuts its upper extremity.

In seeking the greatest main web stresses, it may happen that the reaction R_1 becomes zero; this, however, changes nothing in the method.

Since either arm is a simple truss for all moving loads resting on it (supposing none on the other), every such load tends to cause the same kind of stress throughout the same chord. Consequently, as in the previous article, *the greatest tension in the lower chord, and compression in the upper, will exist when the moving load covers one arm only.* These stresses will only be found in that portion of the arm adjacent to its free extremity.

The greatest chord stresses of the same kind as those caused by the fixed load with the bridge open, can be found with the least labor by the aid of influence lines.

The maximum live-load stresses caused by the moving load on one arm only are to be found by the methods of Arts. 70 and 72 of Chap. IV, since each arm of the bridge is a simply supported truss of 72 feet span. The maximum stresses caused by moving loads on both arms, the bridge being then a continuous structure of two spans, will be found by the aid of influence lines. In using those lines it will be necessary to determine the reactions R_1 at L_0 caused by unit loads of one pound at the various panel-

points. By the aid of the equations and data already given, the following values at once result:

R_1 for Unit Panel Loads on L_1L_{12}.

Load at	z	$l-z$	$(l^2-z^2)z$	M in Foot-Pounds.	R_1 in Pounds.
L_{11}	13	59	65,195	−3.144	−0.0437
L_{10}	26	46	117,208	−5.653	−0.0785
L_9	39	33	142,857	−6.890	−0.0957
L_8	52	20	128,960	−6.219	−0.0864
L_7	65	7	62,335	−3.006	−0.0418
				Sum	=−0.3461

R_1 for Unit Panel Loads on L_0L_5.

Load at	z	$\frac{l-z}{l}$	$\frac{M}{l}$	R_1 (from eq. (2)) in Pounds.
L_2	13	0.8194	−0.0437	+0.7757
L_2	26	0.6389	−0.0785	+0.5604
L_3	39	0.4594	−0.0957	+0.3637
L_4	52	0.2778	−0.0864	+0.1914
L_5	65	0.0972	−0.0418	+0.0554
			Sum	=+1.9466

It should be noted that z is measured from L_0 and from L_{12} for the left and right arms respectively.

Live-load Stresses in the Web Members.

Since the chords of this truss are parallel and horizontal, the maximum stresses in the web members may be obtained by finding the greatest shears in the proper panels and multiplying them by the secants of their inclinations to a vertical line. The influence lines for these shears are shown in Fig. 32, and may be drawn by the aid of the reactions just found.

Members L_2U_3 and U_3L_3.

The greatest shear in the panel L_2L_3 will furnish the maximum stresses in L_2U_3 and U_3L_3. The influence line for shear is $AB''C''D'E'F'GH'J'M$ and it is obtained as follows:

A unit load of 1 pound at L_1 causes a reaction at L_0 of 0.7757 pound, but the shear caused by this load in

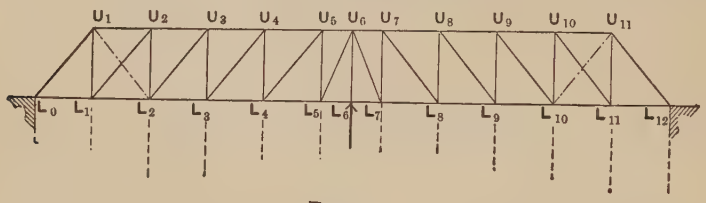


FIG. 31.

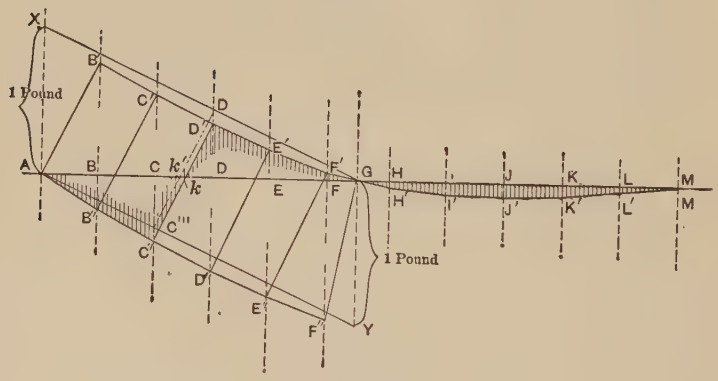


FIG. 32.

the panel L_2L_3 is this reaction minus the load itself. Therefore the influence ordinate BB'' has a value of $0.7757 - 1.00 = -0.2243$ pound. Since this value is negative it is laid off downwards from the base line.

Similarly, then, $C''C = 0.5604 - 1.0 = -0.4396$ pound. $D'D$, however, is a positive quantity and equal to the full

reaction caused at L_0 by a load at L_3, since the load at L_3 is situated on the right of the section taken through the panel.

The influence ordinates on the span L_6L_{12} are all negative and are therefore laid off downwards from the base line. Inspection of the diagram shows at once that for maximum compression in L_2U_3, the span L_6L_{12} should be covered by moving load extending from L_6 to the point of intersection of $C''D'$ with the base line at k. The area is shown shaded as $kD'E'F'G$ and when multiplied by the intensity of the moving load (pounds per linear foot) it will give the maximum shear. Since full panel loads are treated in this problem, the values of the ordinates below L_3, L_4, and L_5 must each be multiplied by a full panel load, viz., 19,500 pounds. This product will exceed slightly that found for the more exact position of the loading extending to k.

The influence line for shear, in the same panel L_2L_3 when the arm L_0L_6 is a simple span, is drawn by erecting an ordinate at A equal to 1 pound and then joining X and G by a straight line. Similarly, the ordinate GY below L_6 is equal to 1 pound, and Y is connected with A by the straight line YA. The points of intersection D''' and C''' of these lines, which are reaction influence lines, with the ordinates below L_2 and L_3 are then connected by a straight line. The influence line for the shear causing compression in L_2U_3 is then $GD'''k'$, and it is at once evident that the area inclosed by this line is greater than that found in the treatment of the bridge as continuous. The greatest counter or compressive stress in L_2U_3 will be obtained, therefore, for the span L_0L_6 treated as a simple, non-continuous truss, by extending the load from G to k'.

The treatment is so obviously applicable to the other web members that they need not be further considered.

In all cases the greatest counter-stresses in the web members occur with a similar position of loading.

By precisely the same reasoning the greatest main web stresses are all found by covering the right arm completely, and the left arm from L_6 to the panel in question. In the case of L_2U_3, for instance, the area $AB''C''k$ plus the area $GI'K'M$ is greater than the area $AC'''k'$; hence the latter loading need not be considered.

The value of the shear is always obtained by multiplying the ordinates at the panel-points by the loads at those points, or by multiplying the area included within the influence line by the intensity of the loading.

Live-load Stresses in the Chord Members.

The method of sections is employed in determining these stresses by dividing the maximum moments at the proper panel-points by the proper lever-arms. Instead of drawing stress influence lines, therefore, moment influence lines showing the variation of moments at the centers of moments will be employed. It is to be noted that if the chords of the truss are not parallel, the shear influence lines fail to furnish the stresses in the web members. In such cases, moment influence lines similar to those to be presented here must be drawn.

Moment Influence Line, Center of Moments L_2.

This influence line for the truss considered continuous over two spans is shown in Fig. 34 as $AB''C''D''E''G \ldots I''K'' \ldots M$. The ordinates at the panel-points are obtained by means of the reactions already found on p. 365.

For a load of 1 pound placed at L_1, the reaction at L_0 is 0.7757 pound. The lever-arm of the reaction is 2 panel lengths, but from this moment there must be subtracted the negative moment of the weight itself, acting on the left of the section, multiplied by its lever-arm of one panel.

That is, the moment is $0.7757 \times 2 - 1 \times 1 = +0.4514$ expressed in pounds × panels, and the ordinate BB'' is drawn with that value.

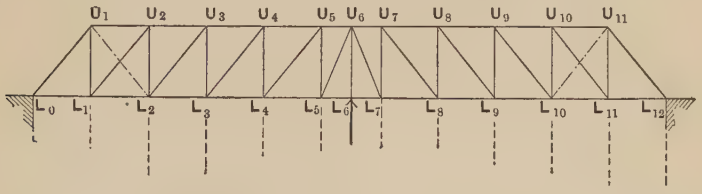


FIG. 33.

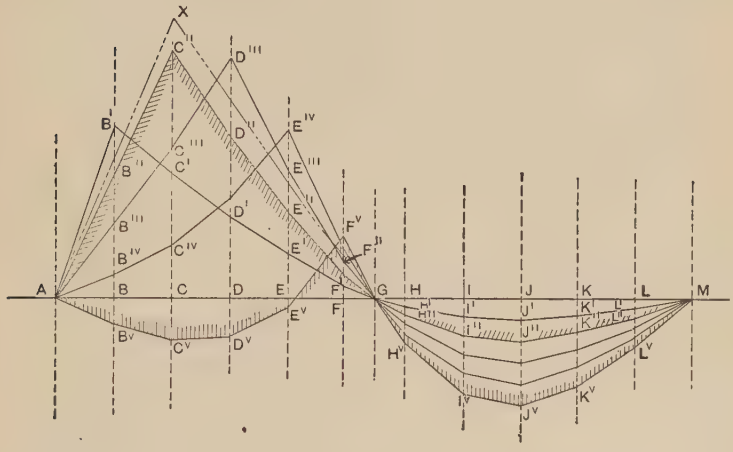


FIG. 34.

The values of the other ordinates are found as follows:

$$CC'' = +0.5604 \times 2 = +1.1208$$
$$DD'' = +0.3637 \times 2 = +0.7224$$
$$EE'' = +0.1914 \times 2 = +0.3828$$
$$FF'' = +0.0554 \times 2 = +0.1108$$
$$G = 0$$
$$HH'' = -0.0418 \times 2 = -0.0836$$
$$II'' = -0.0864 \times 2 = -0.1728$$
$$JJ'' = -0.0957 \times 2 = -0.1914$$
$$KK'' = -0.0785 \times 2 = -0.1570$$
$$LL'' = -0.0437 \times 2 = -0.0874$$

The influence line for the moment at the same point, if the span L_0L_6 be considered simply supported, is shown as AXG. The ordinate CX is found by placing a load of 1 pound at L_2 and multiplying the reaction at L_0, viz., $\frac{4}{12}\frac{6}{}$ pound, by 2 panel lengths; therefore $CX = 1.28$. It is to be remembered that the moment influence lines for simple spans consist of two straight lines. Since the area AXG is greater than the area found for the structure considered continuous, it is evident that the maximum compression in the upper chord U_2U_3 and maximum tension in the lower chord L_1L_2 will occur with each arm of the truss considered as simply supported. This statement applies to every chord member. These stresses are easily found and need no further explanation.

In order to find the greatest tension in the upper chord and compression in the lower, the right arm must be covered by the live load and sufficient load placed on the left arm to prevent the panel-point L_0 rising from its support. The maximum upward reaction at L_0 (p. 365) has been found to be -0.3461 pound for panel weights of 1 pound. If it be possible to assume that a unit panel weight is just entering at L_0 when this uplift occurs, this negative reaction, which is smaller than one panel load, may be considered to cause stress in all the chord members of the truss. The greatest tension in U_2U_3 therefore occurs with the right arm loaded, and it is obtained by dividing the sum of the products of the ordinates HH'', II'', etc., by the true panel weights, by the proper lever-arm, in this case the depth of truss.

Since the negative moment areas shown in Fig. 34 are seen to increase as the centers of moments approach L_6, it is evident that the greatest tension stresses in the upper chord are near L_6.

It may, however, happen that loads on the left span

also add to the tension in the upper chord, and as an illustration, the member U_5U_6 will be treated.

Influence Line for Center of Moments at L_5.

The center of moments for L_4L_5 and U_5U_6 falls at L_5, and the moment influence line for the bridge considered as a two-span continuous truss is $AB^VC^VD^VE^VF^VG \ldots J^V \ldots M$.

The values of the ordinates at the panel-points are found as follows:

$$\begin{aligned}
B^VB &= +(0.7757\times 5) - (1\times 4) = -0.1215\\
C^VC &= +(0.5604\times 5) - (1\times 3) = -0.1980\\
D^VD &= +(0.3637\times 5) - (1\times 2) = -0.1815\\
E^VE &= +(0.1914\times 5) - (1\times 1) = -0.0430\\
F^VF &= +(0.0554\times 5) = +0.2770\\
H^VH &= -(0.0418\times 5) = -0.2090\\
I^VI &= -(0.0864\times 5) = -0.4320\\
J^VJ &= -(0.0957\times 5) = -0.4785\\
K^VK &= -(0.0785\times 5) = -0.3925\\
L^VL &= -(0.0437\times 5) = -0.2185
\end{aligned}$$

These moments are expressed in terms of pounds × panel lengths for loads of 1 pound at the panel-points.

As is clear from this influence line, the greatest tension in U_5U_6 occurs with the right arm completely covered, and with that portion of the left arm covered extending from L_0 to L_4.

In Fig. 34 are shown the moment influence lines for moments at every panel-point. The actual work of constructing these lines is not as tedious as it may appear, for the ordinates of the right arm are all direct multiples of one another, and the simplest multiplication only is necessary for the ordinates of the left arm.

The determination of the actual stresses is left as a

useful exercise in acquiring facility with these methods. It will be found that the portions $U_1U_6U_{11}$, L_0L_4, and L_8L_{12} of the chords must be counterbraced. L_0U_1, $L_{12}U_{11}$, U_1L_1, and $U_{11}L_{11}$ of the web members need the same treatment.

It may happen that with a moving panel load at L_1 the reaction at L_0 will be negative in the search for main web stresses. In such a case the method of operation is simply an extension of that used in the example. If the numerical value of this negative reaction is equal to or less than a moving panel load (which may rest at L_0), a weight equal to this reaction is to be taken as hung from L_0, and the panel load (moving) at L_1 is to be taken as hung from that point, while the arm L_0L_6 is to be considered as an overhanging one. If the reaction, however, is greater than a moving panel load, then a load is to be taken as hanging both from L_0 and L_1 with the overhanging condition of the arm.

A whole panel moving load is taken at L_0 for prudential reasons. If the load were of uniform density, then a half panel moving load would be taken at L_0.

Negative reactions by the formula for any number of moving panel loads near the end are to be treated in exactly the same way; for it is to be remembered that negative reactions in an actual truss, in this case, cannot exist.

It may be urged that the case of partial continuity, taken in a preceding article, should be treated according to the principles developed in this by taking the middle span equal to zero instead of using balanced loads.

Making such an assumption, however, would be a departure from the real state of the truss. The *safest* way would be to determine the greatest stresses by both methods and select the greatest of the two sets of results.

Differences would be found only in the upper-chord tension, lower-chord compression, and main web stresses.

The case just treated really includes that of a center-bearing turntable with two points of support at the center, as shown in Fig. 35. L_5L_6 is free to "rock" on the central point L_c, and as the motion is always small, L_cL_5 (horizontal distance) is essentially equal to L_cL_6 (also horizontal) at all times. From this it results that the reaction at L_5 will always be equal to that at L_6; consequently the diagonals L_5L_6 and L_6U_5 must be introduced.

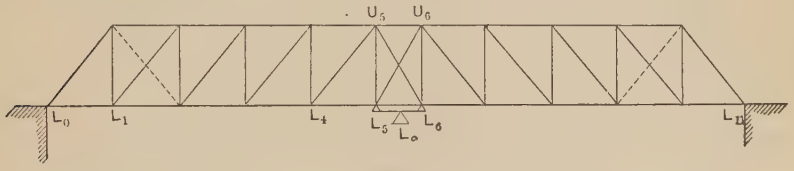


FIG. 35.

As L_5L_6 is really a part of the truss, attached to and moving with it, the whole bridge, L_0L_{11}, is simply a continuous truss of two spans supported on the fixed point L_c. All the conclusions and formulæ, therefore, of this article apply to it directly. R_2 will be the reaction at L_c and M will be the moment over the same point.

According to the principles established, L_5U_6 will receive its greatest stress when L_0L_c only carries moving load; for loads placed on both spans partially or wholly balance each other over the center. Since the load on L_5 is always equal to that on L_6, and also to a half of the reaction at L_c, there will result:

$$\text{Stress in } L_5U_6 = \frac{R_2' \times \sec U_6L_5U_5}{2},$$

in which R_2' is the reaction at L_c due to the moving load on L_0L_c only, considered as a simple truss. The greatest stress in L_0U_5 is obviously equal to that in L_5U_6.

The greatest stress in L_5U_5 (equal to L_6U_6) is found, as before, by putting the moving load on L_cL_{11} and L_0L_4.

Art. 107.—Swing-bridge with Ends Latched Down.

General Considerations.

It has already been stated that the object of fitting the ends of a swing-bridge with a latching apparatus is to enable those ends to resist a negative reaction, or, in other words, to prevent their rising from the points of support. All "hammering" of the ends will thus be prevented.

It has further been shown in Art. 102 that if there are *always* two points of support at the center for each system of triangulation the ends will never tend to rise. It was also observed in the preceding article that with a pivot or center-bearing turntable, the bridge always presents the case of continuity with two spans only, whatever may be the number of *apparent* points of support at the center.

It will then only be necessary to consider the one case of continuity with a single point of support between the extremities of the bridge.

In such a structure with loads on one arm only, that arm is not treated as a simply supported, non-continuous span, but the use of the formulæ, eqs. (1), (2), (3), and (4) of Art. 102, is required for the determination of the reactions at the three points of support.

In this particular only then does the treatment of this form of swing-bridge differ from that with ends simply supported. Since the greatest moving-load stresses found for one arm only in that case were the upper-chord compression, lower-chord tension, and counter-stresses in the web members, only those quantities will be affected by latching down the end points of support.

Let Fig. 36, which is the same truss and shows the same data as Fig. 30, be considered. By inspection of

the influence lines for shear shown in Fig. 32 it will be evident that for the greatest stress in any inclined counter, the moving load must extend from the center point of support to the foot of the counter in question. This is precisely the condition used before, but the reactions in this case are to be found by the theorem of three moments,

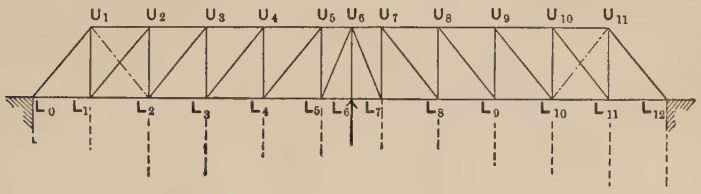


FIG. 36.

since the span L_0L_6 is not to be considered a simple span. The values of these counter-stresses will evidently be less than in the case of ends simply supported.

As usual, the stress in the vertical which cuts the upper extremity of the counter must be found, for it may be the greatest in that member.

Precisely the same reasoning used previously shows that *the greatest stress in any main web member (inclined) exists when the moving load covers the whole of one span, and that portion of the other included between the free end and the foot of the member considered.*

The stress in the vertical which cuts the upper extremity of the inclined web member is to be found with the same condition of loading; it will usually be the greatest possible.

The maximum compression in the upper chord and tension in the lower chord for the left arm can be most easily found by the use of the influence lines for moments shown in Fig. 34. Except for the chord members adjacent to the center point of support, the left arm must be entirely covered, but in the case of the member L_4L_5, for instance, for which the moment influence line is $AB^VC^V \ldots$, the panel-points L_1 to L_4 only should be covered.

For the greatest counter-stresses in these same chord members, those panel-points of the span must be covered which cause the greatest stresses in any given panel opposite in kind to the main stresses determined as above. The moment influence lines of Fig. 25 illustrate clearly the panel-points to be covered.

The greatest negative reaction at the extremity of one arm will exist when the whole of the other is covered by the moving load. The tabulation of p. 365 shows its value to be $0.3461 \times 19{,}500 = 6{,}750$ pounds. The quantity 19,500 pounds represents one panel moving load.

The resistance of the latching apparatus must be sufficient to oppose this with a proper safety factor.

It has been found by experience to be difficult to construct a satisfactory latching device, although it may be done by a sufficiently deep anchorage in the pier masonry at the ends of a draw-span. Unless extraordinary care is taken the exertion of the up-pull in blows tends to shake the masonry and weaken or destroy the anchorage, resulting in costly construction and maintenance. For these reasons, latching down the ends of a drawbridge is seldom done, lifting the ends being the usual procedure to prevent hammering.

The stresses due to the fixed load differ in no way from those of any of the previous cases. They are found most expeditiously by graphical methods, the bridge being considered open and the arms being treated as simple cantilevers.

The observations which were made at the end of Art. 106 relating to a pivot or center-bearing turntable over which there are two points of support for the truss apply exactly as they stand to this case. The value of R_2' must, however, be found by eq. (3) of Art. 102.

If the two arms of this type of bridge are unequal in length, so that the condition $l_1 = l_2 = l$ involved in eqs. (1),

(2), (3), and (4) does not exist, then the formulæ to be used are eqs. (10), (11), (12), and (13) of Art. 102. They are to be used, however, in precisely the same manner.

Art. 108.—Swing-bridge with Ends Lifted.

General Considerations.

In the preceding article there was noticed the method of prevention of "hammering" by latching down the ends of a swing-bridge of two spans. It was also observed there that the necessity of such an arrangement could only exist in the case of continuity with two spans. For precisely the same reasons the necessity of lifted ends can exist, in the event of continuity, with two spans only. The ends of partially continuous swing-bridges with two points of support at the center are also usually lifted so that the deformation of the structure produced by moving load on one arm will not cause hammering at the end of the other arm.

It is plain that if the ends of a swing-bridge are pressed upward by forces exceeding the greatest negative reactions determined for latched ends by the formulæ of the preceding articles, there can be no hammering, for the ends can never leave their seats or supports. The lifting of the ends therefore should be done by upward forces at least equal to the negative reactions determined for latched ends. Those lifting forces should be somewhat greater in order to afford a proper margin of safety for live-load shocks or other contingencies.

The greatest stresses in a swing-bridge with ends lifted may be considered as composed of stresses caused concurrently by the following conditions of loading:

I. The dead-load stresses found with the bridge open and swinging.

II. The stresses caused by the required uplift at each end.

III. The live-load stresses caused by the moving load on one arm, the swing-bridge being considered continuous over the two spans.

IV. The moving-load stresses caused by load on both arms, the structure being considered continuous; and,

V. The moving-load stresses caused by the load on one arm only, that arm being considered a simply supported span.

It is evident that conditions III and V cannot concur. It is more usual, as it is more reasonable, to omit condition III entirely. It is then only necessary to determine separately the stresses for conditions I, II, IV, and V, and to combine those which, acting concurrently, result in the greatest values of the two kinds of stress.

In general, the combination of cases I and IV cause the maximum stress of one kind and the combination of stresses caused by loadings II and V those of the opposite kind.

From this it at once follows that the conditions determining the positions of the moving load for the greatest stresses when the ends are lifted are exactly the same as those given in the preceding articles. Considering the through-bridge already shown in Fig. 36, the greatest stresses in the members which slope downward from the upper chord and toward the ends are found when the moving load extends from the center to the lower extremities of such members, under the conditions of loading of either case III or V. These stresses will be compressive, and in order to find the greatest tensile stress in any web member sloping downward from the upper chord and toward the center, the moving load must extend from the end of that arm to its lower extremity, and at the same time cover the whole of the other arm. This implies the condition of loading of case IV.

As already stated, any two web members which inter-

sect in that chord which does not carry the moving load, take their greatest stresses together.

Example.

As an illustration, the center-bearing swing-bridge shown in Fig. 37 will be treated. There is given in Plate IV in detail the stress sheet of this bridge as actually designed by the American Bridge Company. Each arm is divided into 4 panels of 24 feet $8\frac{1}{2}$ inches each, the distance between the centers of the bearings at the extremities of the arms being 197 feet 8 inches. The uniform height of trusses is 27 feet.

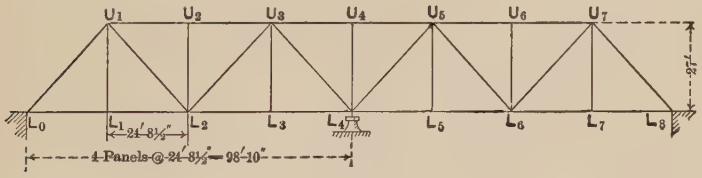


FIG. 37.

The moving load is Cooper's standard E 50 and the fixed load has been taken to weigh 400 pounds per foot of span for the track, 480 pounds for the floor, and 1,020 pounds for the trusses.

The secant of $U_1L_2U_2 = \sec \alpha = 1.35$ and $\tan \alpha = 0.915$.

Each lower-chord fixed panel load per truss will weigh 17,300 pounds, and each corresponding upper-chord load 6,300 pounds. The end-panel fixed load will be taken to weigh 12,000 pounds on account of the extra weight of the locking apparatus.

The uplift to be exerted has been taken at 30,000 pounds per truss. The greatest negative reaction may be found as follows by the aid of eq. (11) of Art. 102.

$$R = \frac{M_2}{l_1} = -\frac{1}{4{l_1}^3}\{\overset{2}{\Sigma} P(l^2 - z^2)z\}.$$

The following table shows the uplift at L_0 for unit loads of 1 pound placed at the panel-points shown.

Load Placed at	z in Feet.	$(l-z)$ in Feet.	$(l^2-z^2)z$.	M.	R.
L_7	24.7	74.1	226,000	5.78	$-.0586$
L_6	49.4	49.4	362,000	9.27	$-.0939$
L_5	74.1	24.7	316,000	8.08	$-.0818$
				Sum	$=-.2343$

If a panel moving load be taken at $24.7 \times 2{,}500 = 61{,}700$ pounds, the greatest negative reaction becomes $61{,}700 \times 0.2343 = 14{,}500$ pounds. As already noted, the uplift has been taken at 30,000 pounds, which affords ample margin for the exigencies of moving trains.

Some specifications require the uplift to be a prescribed percentage of the fixed load of the bridge. The method illustrated in the example appears, however, to furnish more rational results. The distance which the end must be lifted in order to give the desired uplift can be found by the methods of Chapter VIII. In that chapter there will be found treated in detail the deflection Δ of one end of a swing-bridge subjected to a downward load, P, at the end. If an uplift, U, were desired, the upward deflection $\Delta' = \frac{U}{P}\Delta$ must be reached.

This upward deflection may be produced by wedges operated by toggles or screws, by direct screw devices, or by hydraulic machinery. The first method is the one generally employed.

The conditions of loading taken for the bridge under consideration are those already indicated as cases I, II, IV, and V, and the resulting stresses are all shown on the stress sheet, Plate IV. The stresses of cases I and II are most easily found by graphical methods, as case I is the

simple problem of the cantilever with fixed loads, and case II the same problem with the exception that the end panel load of 12,000 pounds has been changed to an upward load of 30,000 pounds.

The stresses of case V are those found as if each arm were a simple non-continuous span, while those of case IV result from using the values of the reactions shown in the preceding table. They require no further explanation.

The specifications for which the bridge was designed required the addition of an impact stress, which is also shown on the stress sheet.

The areas of cross-section and the composition of the members are fully set forth, as are also the design of the stringers and floor-beams, and that of the center cross-girder, with draw both open and closed. The center cross-beam, as will be seen, is a box girder.

When the bridge is closed wedges are driven under the center cross-girder directly below the posts L_4U_4, so that none of the live load of the bridge will rest upon the center pin.

Art. 109.—Final Observations on the Preceding Methods.

Although particular forms of triangulation have been chosen for the various examples in the different cases of swing-bridges, yet the conclusions reached and the principles established are perfectly general. They are applicable to any form of triangulation, and to either the deck or through form of bridge; they also apply whether the two arms are of the same length or of unequal length, the panels being either uniform or irregular. It is only necessary to bear in mind what may be called the "local" circumstances of any given case; these do not, however, affect the general principles. As a single illustration, if the bridge is of the "deck" form, those web members

which intersect in the lower chord take their greatest stresses together; if of the "through" form, those which intersect in the upper chord take their greatest stresses together.

Art. 110.—Partially Continuous Swing-bridges Treated by the Method of Deflections.*

The method of treatment of partially continuous swing-bridges illustrated in the preceding articles has the merit of simplicity. The results obtained by it are closely approximate and sufficiently accurate for all ordinary purposes. The assumption that moving load on one arm only may be treated as load on a non-continuous span is not quite true when the ends of the trusses are latched down instead of being lifted up, although the error involved is not important. Nor are the values of the reactions rigorously exact, since a constant moment of inertia of cross-section of the truss is assumed in the special form of the theorem of three moments employed in establishing those values for the condition of continuity. In such a case the methods of deflections may be employed for original computations or for checking results found by the other method.

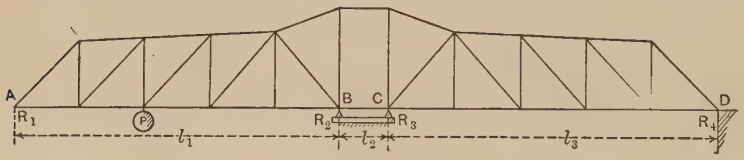


FIG. 38.

It will be shown in Art. 120 that the deflection of any point, such as A in the partially continuous swing-bridge shown in Fig. 38, may be written by using the following notation.

* Before reading this and the succeeding articles it is necessary to be familiar with the theory of the deflections of trusses as set forth in Chapter VIII.

w = deflection (usually in inches) produced at the given point by the loading;

s = intensity of stress in pounds per square inch in any member due to the loading, such as the panel load P;

z = total stress produced in any member by a unit load (usually 1 pound) applied at the point where the deflection w is desired;

l = length (usually in inches) of any piece or member;

E = coefficient of elasticity in pounds per square inch;

a = the area of cross-section of any member in square inches.

The general expression for the deflection is then

$$w = \sum \frac{zsl}{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this case it will first be supposed that the truss is supported at B, C, and D only, the arm AB overhanging. It is also supposed that the end D is latched down. The deflection of the free end A produced by a load P at any point is

$$w' = \sum \frac{zsl}{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This summation must extend from A to D, A being the point of application of the unit load producing the stresses z. The upward reaction R_1 is that force applied at A which will raise the point A to its original elevation, usually but not necessarily identical with that of D, while the load P is acting. It should be observed that the upward reaction R_1 requires, in analyzing the deflection due to it, the action of an equal force upward at D, and hence, that in determining the upward deflection w'' of A produced by R_1 there will be two equal summations, supposing the

arms to be equal in length, similar to eq. (1), i.e., for each arm. Inasmuch as the reaction R_1 is applied at the end of the arm, the intensity of stress s in eq. (1) will be $s=\frac{R_1 z}{a}$. The upward deflection of A produced by R_1 will then be

$$w''=-w'=-2\sum^{I}\frac{R_1 z^2 l}{Ea}. \quad . \quad . \quad . \quad . \quad (3)$$

The sign $\sum^{I}$ indicates that the summation is to extend from A to a vertical line midway between B and C. By substituting the value of w' from eq. (2) in eq. (3),

$$R_1=\frac{\Sigma zsl}{2\sum^{I}\frac{z^2 l}{a}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If the load P is at the distance kl from the left end of the left arm, the downward reaction R at D due to that load, since no shear can be transferred through the center panel BC, will be

$$Rl=P(1-k)l. \quad \therefore \; R=P(1-k). \quad . \quad . \quad . \quad (5)$$

That portion of Σzsl in eq. (4) belonging to the right arm is, therefore,

$$\sum^{3}P(1-k)\frac{z^2 l}{a}=P(1-k)\sum^{3}\frac{z^2 l}{a}. \quad . \quad . \quad . \quad (6)$$

Eq. (4) will then take the form

$$R_1=\frac{\sum^{I}zsl+P(1-k)\sum^{3}\frac{z^2 l}{a}}{2\sum^{I}\frac{z^2 l}{a}}. \quad . \quad . \quad . \quad . \quad (7)$$

$\overset{3}{\Sigma}$ indicates the summation extending from D to midway between B and C, just as $\overset{1}{\Sigma}$ indicates that the summation must extend from A to the same midway vertical line.

When R_1 becomes known by eq. (7), then

$$R_2 = P - R_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The bending moment in the center panel BC then takes the value

$$M = R_1 l - P(1-k)l = -R_4 l. \quad . \quad . \quad . \quad . \quad (9)$$

$$\therefore R_4 = -R_3 = P(1-k) - R_1. \quad . \quad . \quad . \quad . \quad (10)$$

$$= R_2 - Pk. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The application of these equations obviously requires some approximate method at least to determine the stress s, as the latter depends on the areas of cross-section of the members or on the reactions sought. The method set forth in the preceding articles may best be used for this purpose, as the following example will illustrate:

Example.

Let it be required to determine the reaction R_1 at L_0 in the partially continuous truss already treated in Art. 12 for a load of 1 pound placed at L_2. The cross-sections of the members will be assumed found from the stresses shown in Table II, Art. 103, by employing the intensities of stress for live and fixed load specified by Cooper. The lengths of the members as well as the areas of cross-section are fully shown in the second and third columns of the table on page 387, but it is to be noted that since eq. (7), to be used in determining R_1, requires a summation extending half-way between L_5 and L_6, only one half the lengths of L_5L_6 and U_5U_6 are to be taken.

In the fourth column of Table I there is shown the stress z in each member for a load of 1 pound hanging at L_0, the arm L_0L_5 being supposed overhanging, and the point L_{11} latched down.

In the next column is shown the stress S in each member of the left arm for the same conditions of support just given, but with the load of 1 pound hanging at L_2. The next column contains the intensities s of stress found by dividing S by the areas a given in the second column.

In the next column are given the products zsl, all being positive except U_4L_5; and in the last column are found the products $\frac{z^2l}{a}$, all being positive. Summing the last two columns, and inserting these values in eq. (7), there is found, since $P(1-k)=0.4$,

$$R_1=\frac{563.8+0.4(1,310.5)}{2(1,310.5)}=0.453.$$

The method previously used (theorem of three moments) gave to R_1 for the same position of loading the value 0.453178. The agreement is remarkably close. In any case the theorem of three moments may be relied on to furnish values sufficiently accurate for all purposes of design.

If, as is sometimes done, those portions of the deflections caused by the web members be neglected, the value of R_1 becomes

$$R_1=\frac{403+0.4(957.6)}{2(957.6)}=0.412.$$

This differs from the more accurate value by about 10 per cent.

After the values of the reactions have been found, the partially continuous structure is to be treated precisely as in Art. 103, since the end of the right arm is not

latched down. Loads on one arm are to be treated as unbalanced, or as if on a simply supported span. Loads simultaneously on both arms are to be divided into balanced and unbalanced portions, as before. If the structure is completely continuous, and it is desired to employ the method of deflections, the equations of the succeeding article are to be used.

TABLE I.

Member.	a Area of Cross-section in Square Inches.	l Length of Member in Inches.	z Stress in Pounds Caused by Load of 1 Pound at L_0.	S Stress in Pounds Caused by Load of 1 Pound at L_2.	$s=\frac{S}{a}$ Pounds per Square Inch.	zsl	$\frac{z^2l}{a}$
L_0L_1	14	330	−0.96	.....	.....	.....	+ 21.7
L_1L_2	14	330	−1.82	.....	.....	.....	+ 78.0
L_2L_3	20	330	−2.62	−0.91	−0.045	+ 38.9	+113.0
L_3L_4	23	330	−3.36	−1.73	−0.075	+ 83.0	+162.0
L_4L_5	23	330	−3.36	−1.73	−0.075	+ 83.0	+162.0
L_5L_6	29	123*	−3.18	−1.97	−0.068	+ 26.6	+ 42.8
L_0U_1	38	471	+1.38	.....	.....	.....	+ 23.6
U_1U_2	30	331	+0.95	.....	.....	.....	+ 9.9
U_2U_3	30	331	+1.82	.....	.....	.....	+ 36.5
U_3U_4	35	331	+2.62	+0.91	+0.026	+ 22.5	+ 64.9
U_4U_5	22	351	+3.40	+2.09	+0.095	+113.0	+184.0
U_5U_6	21	123*	+3.18	+1.97	+0.094	+ 36.8	+ 59.2
U_1L_1	14	336	−0.95	.....	.....	.....	+ 21.6
L_1U_2	6	482	+1.28	.....	.....	.....	+132.0
U_2L_2	10	352	−0.91	.....	.....	.....	+ 29.1
L_2U_3	12	494	+1.22	+1.35	+0.112	+ 74.8	+ 61.2
U_3L_3	12	368	−0.86	−0.95	−0.079	+ 25.0	+ 22.7
L_3U_4	21	506	+1.14	+1.25	+0.059	+ 34.0	+ 31.4
U_4L_4	12	384	0	0	0	0	0
U_4L_5	38	506	+0.25	−0.38	−0.010	− 1.3	+ 8.3
L_5U_5	15	504	−1.19	−0.70	−0.046	+ 27.5	+ 47.6
						$\Sigma zsl=563.8$	$\Sigma\frac{z^2l}{a}=1310.5$

* These lengths are one half the total lengths of the members.

Art. 111.—Completely Continuous Swing-bridges Treated by the Method of Deflections.

The method of deflections may readily be applied to completely continuous swing-bridge trusses in the same general manner followed for those partially continuous, but the practical results of its actual application to a given case are much less satisfactory. Reference will be

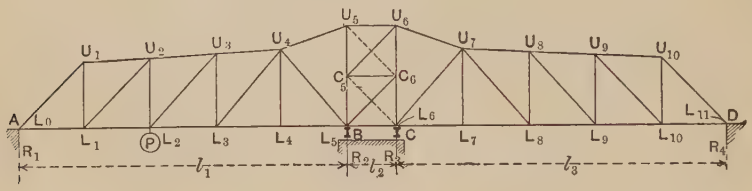


FIG. 39.

made to Fig. 39, a skeleton diagram of a swing-bridge truss continuous over three spans.

Let S be the actual stress (total) in any member of the truss conditioned as in use;

E " the coefficient of elasticity;

a " the area (usually in square inches) of cross-section of any member;

l " the length (usually in inches) of any member;

z_B " the stress (total) in any member due to load unity acting downward at B, supposing the truss to be simply supported at A and D only;

z_C " the stress (total) in any member due to load unity acting downward at B, supposing the truss to be simply supported at A and D only. In finding the stresses z_B and z_C, therefore, the truss is assumed to be non-continuous with the span AD;

s be the stress (total) in any member produced by a load P acting at any point as if the truss were non-continuous with the span AD;

R_2 " the reaction at B;

R_3 " the reaction at C.

The stress S may then be given the value

$$S = s - z_B R_2 + z_C R_3. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The deflection of B due to the stress S will be

$$\Delta_B = \sum_A^D z_B \frac{sl}{Ea}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Similarly, the expression for the deflection of C will be

$$\Delta_C = \sum_A^D z_C \frac{Sl}{Ea}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

As indicated, the summations must extend throughout the entire length of the trusses, i.e., from A to D.

In the case of a swing-bridge the deflections at the points of support B and C, Fig. 39, are each equal to zero.

By substituting the value of S from eq. (1) in eqs. (2) and (3), therefore,

$$\sum_A^D \frac{z_B sl}{Ea} - R_2 \sum_A^D \frac{z_B^2 l}{Ea} + R_3 \sum_A^D \frac{z_B z_C l}{Ea} = 0; \quad . \quad (4)$$

$$\sum_A^D \frac{z_C sl}{Ea} - R_2 \sum_A^D \frac{z_B z_C l}{Ea} + R_3 \sum_A^D \frac{z_C^2 l}{Ea} = 0. \quad . \quad (5)$$

The stresses s, z_B, and z_C appearing in these equations due to the loads P and unit loads, respectively, are easily found by treating the trusses as non-continuous and having the span AD. Trial values of the areas of cross-section, a, of the members must, however, be assumed before an application of the formulæ can be made, and that constitutes one of the unsatisfactory features of the method. After the computations are complete and the areas are

determined from them, if the difference between the latter and those assumed are too great the computations must be revised with new values of a.

Equations (4) and (5) constitute two of the four equations of condition required to find the reactions R_1, R_2, R_3, and R_4. The other two are

$$R_1 + R_2 + R_3 + R_4 = P, \quad . \quad . \quad . \quad . \quad (6)$$

$$-P \cdot x + R_1 l_1 = R_4(l_2 + l_3) - R_3 l_2; \quad . \quad . \quad . \quad (7)$$

x indicates the distance between the load and the center of moments B.

When eqs. (4) and (5) are applied to an actual case all the summations Σ will take numerical values and two equations each expressing R_2 in terms of R_3, or R_3 in terms of R_2, will at once result.

The preceding equations may also be applied in the case of a swing-bridge continuous over one central point of support. In that case R_3 disappears from eq. (4) and that equation becomes

$$\Delta_B = \sum_A^C \frac{z_B \cdot s \cdot l}{E \cdot a} - R_2 \sum_A^C \frac{z_B^2 \cdot l}{E \cdot a} = 0. \quad . \quad . \quad . \quad (8)$$

The ends of the span are indicated by A and C. Therefore

$$R_2 = \frac{\sum_A^C \frac{z_B \cdot s \cdot l}{E \cdot a}}{\sum_A^C \frac{z_B^2 \cdot l}{Ea}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If these formulæ be applied to an actual swing-bridge design, it will almost certainly be found that erratic values for the reactions result. This will necessitate a revision of the assumed cross-sections a. As the results of the applications of the formulæ are highly sensitive to variations in their assumed sectional areas, this process of

successive revisions and applications of formulæ will be found laborious and, on the whole, unsatisfactory.

As a working method in the design of swing-bridges the use of the theorem of three moments possesses many substantial advantages and is as nearly accurate as any yet developed.

Art. 112.—Determining the Uplift Required at the End of a Swing-bridge to Prevent Hammering.

In Fig. 40 there is shown in outline one arm of the draw-span of the 145th Street bridge over the Harlem River, New York City,* for which it is desired to determine the amount of deflection of the free end L_0 caused by a load of 100,000 pounds hanging at that point. In Fig. 40 there is shown in detail on each member the length of that member in inches, the area of its cross-section in square inches, and the stress z caused by a load of one pound hanging at the point L_0. There are also shown in the figure, although those quantities are not required in the following demonstration, the fixed-panel loads found from the following estimated weights. The concrete and asphalt was estimated to weigh 50 pounds per square foot of roadway and $33\frac{1}{3}$ pounds per square foot of sidewalk. The machinery at each end was assumed to weigh 16,000 pounds for the three trusses, the bridge having three parallel trusses. The hand railing was assumed to weigh 40 pounds per linear foot and the grating was assumed at 60 pounds per linear foot of each truss. Finally, the weight of the steel work included in the fixed load was taken from the estimates of the design. The weights indicated in Fig. 40 are for one truss only.

* The authors acknowledge their indebtedness for the detailed stress and deformation sheets of this bridge to Mr. O. E. Hovey, Engineer in Charge of Design, American Bridge Co., and Mr. H. S. Prichard.

The coefficient of elasticity E has been taken at 30,000,000 pounds per square inch, which is a little higher than ordinarily assumed, in order to allow for the influence of lattice bars whose areas are not included in the cross-sections of the members. The formula which will deter-

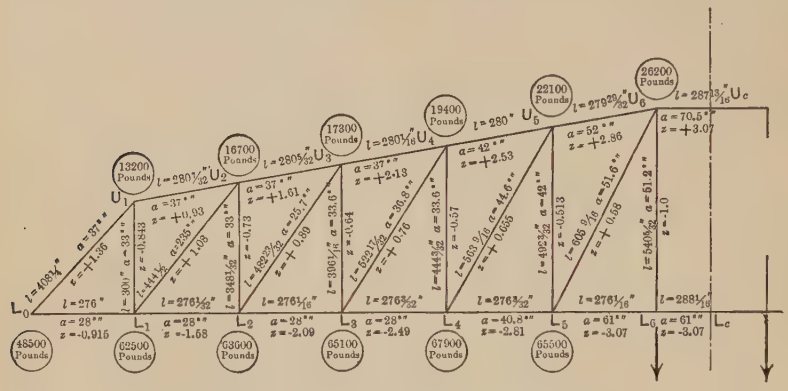


FIG. 40.

mine the vertical deflection of the point L_0 when any load P is hanging from that point is, as before,

$$\Delta = \sum_{L_0}^{L_c} \frac{szl}{E},$$

where z is the stress in any member due to the unit load at L_0, s the intensity of stress in any member due to the loading of 100,000 pounds hanging at L_0 and l the length of any member. As is evident in this case $s = \frac{100{,}000z}{a}$, a being the area of cross-section. Consequently the formula for the deflection takes the following form:

$$\Delta = \sum_{L_0}^{L_c} \frac{100{,}000 \cdot z^2 \cdot l}{E \cdot a} = \frac{1}{300} \sum_{L_0}^{L_c} \frac{z^2 l}{a}. \quad . \quad . \quad (1)$$

It is assumed that the truss is symmetrically loaded about the center line L_c, there being also a load of 100,000 pounds at the right end L_0.

By substituting the quantities shown in Fig. 40 in Eq. (1) the following tabulation and summation may be found.

TABLE I.

Member.	$\frac{z^2 l}{300.a}$	Member.	$\frac{z^2 l}{300.a}$
L_0U_1	.068	L_0L_1	.027
U_1U_2	.022	L_1L_2	.082
U_2U_3	.065	L_2L_3	.143
U_3U_4	.114	L_3L_4	.204
U_4U_5	.142	L_4L_5	.178
U_5U_6	.147	L_5L_6	.142
U_6U_C	.064	L_6L_C	.074
L_1U_2	.073		
L_2U_3	.049	Sum =	0.850
L_3U_4	.027		
L_4U_5	.018	½ Sum =	0.425
L_5U_6	.013		
U_1L_1	.021		
U_2L_2	.019		
U_3L_3	.016		
U_4L_4	.014		
U_5L_5	.010		
U_6L_6	.035		
Sum =	0.917		

Resultant = 0.917 + 0.425 = 1.342 inches.

It is to be noted that in this bridge the lower chord is practically continuous with a solid buckle-plate floor; it was therefore assumed that half of the stress s in the bottom chord was taken by the solid floor and half by the bottom chord proper. For this reason only one half of the deflections due to the lower chord have been included in the summation of the quantities in Table I.

The deflection is found to be 1.342 inches. If it is desired that the end of the bridge be subjected to an uplift of 44,300 pounds, in order to provide against hammering at the ends, the distance through which the lifting arrangements must raise the end of the truss is then

$$\frac{44{,}300}{100{,}000} \times 1.342 = 0.59 \text{ inch.}$$

Art. 113. Deflection of the End of an Open Swing-bridge.

By the Analytical Method.

The swing-bridge of the previous article will be considered. The only stresses needed are those due to the fixed load. Table I furnishes the stresses z caused by a

TABLE I.

Member.	z In Pounds.	S In Pounds.	a In Square Inches.	l In Inches.	$\frac{Sl}{Ea}$ In Inches.	$\frac{z.S.l}{E.a}$ +	$\frac{z.S.l}{E.a}$ −
L_0U_1	+1.36	+ 63,600	37	408	+0.023	+.0313	
U_1U_2	+0.93	+ 44,300	37	280	+0.011	.0102	
U_2U_3	+1.61	+137,300	37	280	+0.035	.0564	
U_3U_4	+2.13	+263,300	37	280	+0.067	.1427	
U_4U_5	+2.53	+418,000	42	280	+0.093	.2353	
U_5U_6	+2.86	+593,000	52	280	+0.106	.3032	
U_6U_c	+3.07	+768,000	70.5	144	+0.051	.1566	
L_0L_1	−0.915	− 43,300	28	276	*−0.009	.0082	
L_1L_2	−1.58	−135,000	28	276	*−0.030	.0474	
L_2L_3	−2.09	−259,000	28	276	*−0.057	.1191	
L_3L_4	−2.49	−411,000	28	276	*−0.090	.2241	
L_4L_5	−2.81	−586,000	40.8	276	*−0.089	.2501	
L_5L_6	−3.07	−768,000	61	276	*−0.177	.2364	
L_6L_c	−3.07	−768,000	61	144	*−0.041	.1259	
					Sum=	1.9369	
L_1U_2	+1.08	+147,000	23.5	444.5	+0.093	0.1004	
L_2U_3	+0.89	+218,700	25.7	483	+0.137	.1219	
L_3U_4	+0.76	+286,700	36.8	522.5	+0.137	.1041	
L_4U_5	+0.655	+350,000	44.6	563.5	+0.148	.0969	
L_5U_6	+0.58	+403,300	51.6	605.5	+0.158	.0916	
U_1L_1	−0.843	− 55,000	33	300	−0.016	.0135	
U_2L_2	−0.73	−117,300	33	348	−0.041	.0299	
U_3L_3	−0.64	−176,700	33.6	396	−0.069	.0442	
U_4L_4	−0.57	−237,000	33.6	444	−0.105	.0599	
U_5L_5	−0.513	−295,300	42	492	+0.115	.....	−.0590
U_6L_6	−1.0	−488,300	51.2	540	−0.172	.1720	
					Sum=	+.8342	
						−.0590	
						+0.7754	inch

* In determining these deformations of the lower chord, two-thirds of the stress was assumed as taken by the chord members proper and one third by the solid floor, the bridge having a buckle-plate floor covered with asphalt.

load of 1 pound hanging from the open end L_0; the fixed-load stresses S, the areas of cross-section a, and the lengths l of all the members and the necessary products required by the following equation:

$$\Delta = \sum_{L_c}^{L_0} \frac{z \cdot S \cdot l}{E \cdot a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The total deflection at the end will therefore be $1.9369 + 0.7754 = 2.7123$ inches, the first part indicating the deflection due to the chords, and the second that due to the web members.

Art. 114. Deflection of the End of an Open Swing-bridge.

By the Graphical Method.

The Williot diagram * is peculiarly adopted to finding the deflections of the open end of a swing-bridge and the diagram for the truss of Art. 113 is shown in Figs. 41 and 42.

In the upper figure, there is shown along each member the deformation λ, whether tension or compression, to which that member is subjected by the fixed load. This quantity is obtained from the usual stress-strain relation $E = \frac{S \cdot l}{a \cdot \lambda}$, E being the coefficient of elasticity and S the fixed-load stress in the member whose length is l and whose area is a. The lower figure is begun by assuming the point L_c as fixed, and determining the displacements of L_6 and U_6. L_6 is moved a distance 0.081 inch to the right, and U_6 the distance 0.102 to the left, and 0.172 inch downward. Having determined these displacements, L_5 may be located by drawing lines through the new positions of U_6 and L_6

* See the authors' "Influence Lines," pages 160 to 175, for the detailed explanation.

parallel to U_6L_5 and L_6L_5 with lengths equal to the distortions of those members and erecting perpendiculars

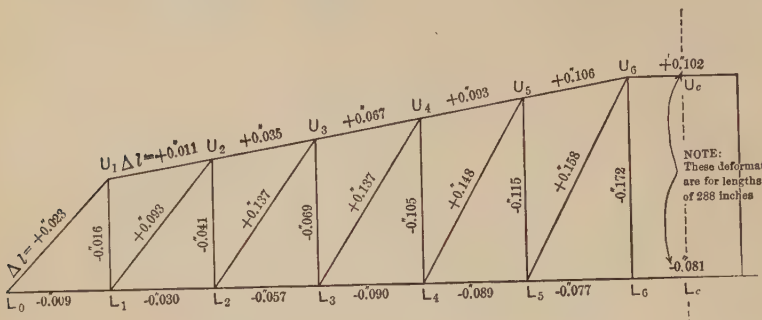


FIG. 41.

at their ends. The intersection of these perpendiculars will locate L_5. In the same manner, U_5, L_4, U_4, etc., are located. The final vertical deflection of L_0 is then found to be 2.81 inches, which furnishes a satisfactory check on the value found analytically. The graphical method also shows a horizontal displacement of 0.4 inch.

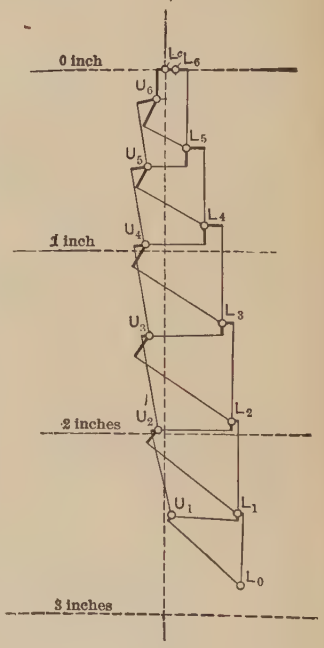


FIG. 42.

Art. 115. Turntables.

The drums of rim-bearing turntables should be of sufficient depth to prevent upward deflection from materially disturbing a uniform distribution of load over the rollers. The upward pressure of the latter constitutes an upward loading on the lower flange of the drum, and the points on the upper flange of the latter, at which the truss load is

applied, form the supporting points of the continuous-drum girder. The loading on the lower drum flange should be as nearly uniform as possible, and in order to secure that result the drum depth should be as great as practicable. It is also necessary that the truss load should be carried to the upper drum flange at as many equidistant points as may be found practicable. In long and heavy draw-spans it will be necessary to carry the truss loads to the drum through a combination of transverse and longitudinal girders, in order to secure the requisite number of points of application on the upper flange. A calculation of the girder strength of the drum can be made by assuming that its segments are beams with a span length equal to the distance between points of application of the truss loads on the upper flange, and that they are loaded with the uniform roller pressure. Although the drum is continuous, these beams should be considered non-continuous, for they are not straight, and a failure to secure the assumed uniformity of loading may essentially destroy the advantages of continuity. The results of all such computations must, however, be strongly tempered by the experience and judgment of the engineer. The drum section must be such as to avoid any appreciable deflection; its depth should never be less than one third the distance between adjacent points of support on the upper flange, and one half is better practice. If the total truss load does not require nearly all the rollers which the circumference of the drum affords room for the rule may be proportionately modified, but not otherwise.

The Thames River bridge, carrying about 2,400,000 pounds dead load on the rollers, has eight equidistant points of support on its drum, with 32 feet diameter and 5 feet depth.

The 500 feet single-track Arthur Kill draw-span has about 1,450,000 pounds of dead load resting at eight

equidistant points on a drum 27½ feet in diameter and 3½ feet deep.

The Central bridge across the Harlem River at New York has two concentric drums 44 feet and 36 feet in diameter with sixteen points of support on each. The total truss weight is about 3,800,000 pounds, and the depth of the drum is 5 feet. The total truss weight of the Kingsbridge Road bridge across the Harlem Ship Canal at New York is about 1,500,000 pounds, and it is carried at twelve equidistant points of support on a drum 39 feet in diameter and 5 feet deep.

It is usually easy to secure eight equidistant points of support on the drum of an ordinary single-track draw-span, with a depth of drum of at least 30 to 36 inches, and such an arrangement should be required. The resulting distribution of load on the rollers will be found satisfactory.

Art. 116. Power Required to Turn a Drawbridge.

The power required to be exerted by an engine to turn a drawbridge is expended in three parts. One portion is used, at the beginning of the operation of opening or closing, in developing the maximum velocity possible, and is stored for a short time as the actual energy of the structure in motion; it subsequently performs work against some brake arrangement by which the bridge is brought to rest. A second portion is used in performing work against the entire frictional resistance of the moving parts of the structure and machinery; while the third and last portion is required to overcome the wind resistance when the wind blows against one arm with a total pressure which is not balanced by that against the other.

The rolling friction, or, rather, the entire friction, of a drawbridge in motion, has been determined by Mr. Theo-

dore Cooper for the Second Avenue double-track railroad bridge over the Harlem River at New York; and for the Thames River double-track railroad bridge at New London, Conn., by Messrs. Boller and Schumacher; and the results of these investigations can be found in the Transactions of the American Society of Civil Engineers for December, 1891. The total moving weight of the Second Avenue bridge was 880,000 pounds, and that of the Thames River bridge 2,500,000 pounds. The length of the latter is 500 feet, and the diameter of the drum 32 feet. Mr. Cooper found the coefficient of frictional resistance for the Second Avenue bridge to be 0.0038—i.e., a force of 3.8 pounds per 1,000 pounds of total weight moved would have to be applied tangentially at the center line of the track (or to the drum) in order to overcome the total friction.

Messrs. Boller and Schumacher found a coefficient of about 0.004 for the Thames River bridge, or 4 pounds per 1000 pounds moved. The greater part of the "total friction" is the rolling friction at the drum. These two instances are the most valuable rim-bearing drawbridge investigations of the kind ever made in this country, and as the workmanship, fitting of track, etc., were of an unusually excellent character, it is probably advisable to take the coefficient of friction for ordinary draw-spans at 0.01, or 10 pounds per 1,000 pounds of weight moved.

In the Transactions of the American Society of Civil Engineers for 1874, Mr. C. Shaler Smith gave the results of a number of less complete but very interesting tests of rim-bearing draw-spans in about the ordinary conditions of workmanship and running order. He found the total friction to vary from 4 to 8 pounds per 1,000 pounds of weight moved.

The power required to give the desired velocity of rotation to a drawbridge will depend upon the time allowed for opening or closing. Draws operated by power are usually

opened or closed in one to three minutes. Small draws operated by hand will consume three to eight minutes.

If drawbridges are operated against an unbalanced wind pressure, the necessary power increases very rapidly. Seven eighths, or even nine tenths, of the total capacity of a well-proportioned drawbridge engine may be exerted against a wind pressure when the structure is moved in a moderately high wind. A comparatively small amount of power is required to overcome the friction.

In the 500 feet double-track Thames River bridge, with moving parts weighing about 2,500,000 pounds, not more than 5 or 6 horse-power at most was expended in developing the acceleration and overcoming the total friction. An unbalanced wind pressure of 5 pounds per square foot on one arm would have required only a little less than 30 horse-power to turn the draw against it, or double that amount for 10-pound wind.

The computation of the work required to turn a drawbridge will require its moment of inertia to be taken about a vertical axis through the center of the drum. It will be sufficient for this purpose to consider the trusses, lateral bracing, floor system, and track as a homogeneous prism with length l and width w. This portion of the weight is, for all practical purposes, five sixths the total moving weight W for single-track railroad spans, and seven eighths the same total weight for double-track spans. Hence the moment of inertia I' of this portion of the weight will be—if g is the approximate constant, 32.2, for gravity—for single-track spans:

$$I' = \frac{\frac{5}{6}W(w^2+l^2)}{12g} = \frac{5W(w^2+l^2)}{72g}. \quad . \quad . \quad . \quad (1)$$

Or for double-track spans,

$$I' = \frac{7W(w^2+l^2)}{96g}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The moment of inertia of the drum, rollers, etc., can be considered concentrated at the distance R = radius of the drum from the axis. Hence the moment of inertia, I'', of this portion of the weight will be for single-track spans:

$$I'' = \frac{WR^2}{6g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Or for double-track spans:

$$I'' = \frac{WR^2}{8g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence the total moment of inertia will be

$$I = I' + I''. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

GENERAL.

The circumstances under which a swing or draw bridge is turned involve the variable motion of the moving mass, the friction of the machinery, rolling friction in the case of a rim-bearing turntable, and the action of the wind as well as other resisting agencies. It is therefore impossible to write an exact expression for the entire work expended in turning the bridge, but the following analysis gives a correct general idea of the problem and it is probably as accurate as anything that can be written.

It will be assumed that the total period of the turning motion is divided into three parts, one of acceleration, one of uniform motion, and one (the last) of retardation. This involves no essential error.

The following notation will be employed:

NOTATION.

T = time required to turn the bridge from beginning to end of motion;
cT = period of acceleration;
dT = " " constant velocity;
eT = " " retardation;
W = total weight of the moving structure;
β = total angle through which the bridge is turned;
ρ = radius of gyration of the total moving mass;
$I = \frac{W\rho^2}{g}$ = moment of inertia of the total moving mass;
r = radius of the pitch circle of the circular rack;
r_0 = " " " center of wind pressure;
α = greatest angular velocity;
v = " velocity at distance ρ from center;
a = uniform acceleration during the period cT.

$$\therefore\ a = \frac{v}{cT} = \frac{\alpha\rho}{cT}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

R = radius of the drum;
R' = " " " pin of a center-bearing swing-span.

Work of Acceleration.

It is known from an elementary demonstration in mechanics that the actual or kinetic energy, or simply work stored in the rotating bridge moving with the velocity $v = \rho\alpha$, is

$$E = \frac{Wv^2}{2g} = \frac{I\alpha^2}{2} = F_0 \cdot cT \cdot \frac{\rho\alpha}{2}, \quad . \quad . \quad . \quad . \quad (7)$$

where F_0 is the force which, if applied at the center of inertia located by ρ, will produce the acceleration a. Hence

$$F_0 = \frac{I\alpha}{\rho cT} = \frac{W\rho\alpha}{cTg} = \frac{Wa}{g}. \quad . \quad . \quad . \quad . \quad (8)$$

The force F assumed to be acting at the pitch circle is

$$F = \frac{\rho}{r} F_0 = \frac{I\alpha}{rcT} = \frac{\rho}{r} \frac{Wa}{g}. \quad . \quad . \quad . \quad . \quad (9)$$

The work performed by either the force F or F_0 in accelerating the motion of the bridge and giving to the latter its greatest velocity is shown by eq. (7). The distance turned through on the pitch circle of the circular rack is

$$\frac{r}{\rho} \cdot cT \cdot \frac{\rho\alpha}{2} = \frac{rcT\alpha}{2}. \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Work Performed Against Friction.

If f is the amount of frictional resistance in pounds per 1,000 pounds of total moving weight of the structure, supposed applied at the drum, and if P is the number of thousand pounds in that total moving weight, then the work performed against friction in the whole operation of opening or closing the draw-span will be

$$E' = fP\frac{R}{r} \cdot r\beta = fPR\beta. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The frictional resistance may without sensible error be assumed to be constant throughout the motion of either opening or closing the bridge. As has already been stated, f may be taken at 10 pounds per 1,000 pounds of total weight of structure where the latter is properly cared for, although it may sometimes fall as low as 5 pounds.

If a swing-span is center-bearing, the total load being carried on a center pin whose radius is R', the work performed against the friction of the pivot in turning the bridge is the same as if the entire friction were concentrated at $\frac{2}{3}R'$ from the center of the pivot or pin. If f'

be the coefficient of friction for the pivot, W being the total supported load, the frictional work performed at the pivot in opening or closing the bridge will be

$$f'W \cdot \tfrac{2}{3}R'\beta = \tfrac{2}{3}f'WR'\beta. \quad . \quad . \quad . \quad . \quad . \quad (12)$$

To this must be added, however, the friction of the machinery. If f_1 represent the total frictional resistance as a single force in pounds per 1,000 pounds of total turning weight applied tangentially to the circumference of the pin, the expression for the work performed will be found by simply writing f_1 for $\frac{2}{3}f'$ and P for W in eq. (12). That total frictional work will be

$$E'' = f_1 PR'\beta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In eq. (13), as in eq. (11), P is the number of thousand pounds in the total weight W. According to the investigations of C. Shaler Smith, f_1 may be taken at 90 pounds.

Work Performed Against Wind Pressure.

If p is the intensity of unbalanced wind pressure against the total exposed surface, A, of one arm of the structure, the total unbalanced wind pressure will be pA, and as its center is r_0 from the center of rotation, the work performed will be

$$E''' = pA \cdot r_0\beta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

It is implicitly assumed in this equation that the wind pressure acts normally to the exposed surface throughout the entire motion of opening or closing. While this is probably never exactly true, the assumption is as good as any that can be made.

Total Work Performed for Rim-bearing Bridge.

The total work performed in opening or closing a swing-bridge supported on a rim-bearing turntable is given by eqs. (7), (11), and (14).

$$Work = \frac{I\alpha^2}{2} + fPR\beta + pA \cdot r_0\beta. \quad . \quad . \quad . \quad (15)$$

With the units ordinarily employed this expression for the work will be found in foot-pounds. If the time T is given in seconds, remembering that a horse-power is 550 foot-pounds of work performed per second, the greatest horse-power required will be

$$\text{H.P.} = \left[\frac{I\alpha^2}{2c} + \beta(fPR + pAr_0)\right]\frac{1}{550T}. \quad . \quad . \quad (16)$$

This horse-power is required only during the period of acceleration. During the period of retardation all power is shut off and the brakes are applied. While the motion is uniform the H.P. is that required to overcome friction and the wind, and it is expressed by the last two terms of the second member of eq. (16)

Total Work Performed for a Center-bearing Bridge.

Both the total work performed and the greatest horse-power required are the same in this case as in the preceding except that the second term of the second member is taken from eq. (13) instead of eq. (11). Hence

$$Work = \frac{I\alpha^2}{2} + f_1PR'\beta + pAr_0\beta \quad . \quad . \quad . \quad (17)$$

and

$$\text{H.P.} = \left[\frac{I\alpha^2}{2c} + \beta(f_1PR' + pAr_0)\right]\frac{1}{550T}. \quad . \quad . \quad (18)$$

Greatest Force at Pitch Line of Circular Rack.

This force is easily found by dividing each part of the expressions for work in eqs. (15) and (17) by its proper path. That path for $\frac{I\alpha^2}{2}$ is $\frac{r\alpha}{2}cT$ and $r\beta$ for the remaining two terms of the second member of each equation. Hence the greatest force desired for *rim-bearing turntable is*

$$\text{Greatest force} = \frac{I\alpha}{rcT} + fP\frac{R}{r} + pA\frac{r_0}{r}. \quad . \quad . \quad . \quad (19)$$

For center-bearing turntable:

$$\text{Greatest force} = \frac{I\alpha}{rcT} + f_1P\frac{R'}{r} + pA\frac{r_0}{r}. \quad . \quad . \quad (20)$$

It is possible that the brakes may be applied so strongly as to produce a force at the pitch circle greater than that given by eq. (19) or (20), although that is not probable. The path passed over on the pitch circle during the period of retardation will be $\frac{r\alpha}{2}eT$. Again, the force of friction shown in the second members of eqs. (19) and (20) will aid retardation. Hence the greatest brake force at the pitch circle will be for a rim-bearing turntable:

$$\text{Greatest brake force} = \frac{I\alpha}{reT} + pA\frac{r_0}{r} - fP\frac{R}{r}. \quad . \quad (21)$$

And for a center-bearing turntable:

$$\text{Greatest brake force} = \frac{I\alpha}{reT} + pA\frac{r_0}{r} - f_1P\frac{R'}{r}. \quad . \quad (22)$$

The gear-teeth of the circular rack and driving pinion must be designed to resist the greatest force given by

eqs. (19), (20), (21), and (22). The shafting, gear, and other parts of the turning machinery must be so arranged and designed also as to transmit stresses corresponding to the greatest forces and horse-power expressed by the preceding equations.

Periods of Acceleration and Retardation.

The periods into which the whole period of turning may be divided may obviously vary. There is no necessary relation among them. The period of acceleration may continue far enough to be followed immediately by the period of retardation or, as is frequently the case, a period of uniform motion may intervene.

Ordinarily the total angle of turning of a swing-span is 90°, but if it is a skew-bridge that total angle will depend on the angle of skew.

Art. 117. Motors for Moving Swing-bridges.

The motive power for turning all the early swing-bridges not turned by hand was derived from small steam-engines housed either in the drum below the bridge floor or preferably in a small building or room supported between the truss, over the center, at a sufficient elevation to afford the requisite head room for traffic. This power, although satisfactory for the purpose, had the disadvantages of occupying considerable space and requiring care and disposition of fuel and ashes. Gas- and oil-engines were next used and in many locations they were material improvements over the steam-engine. Where electric power is available at reasonable cost it is now the best type of power and is much used. It requires the least space and is the most convenient to manipulate. Nor is any care for fuel or ashes needed. If electric power is not available gas- or oil-engines may advantageously be employed.

Art. 118. Application of Formulæ to Thames River Bridge.

The total weight moved in this double-track railway bridge (at New London, Connecticut) has already been stated to be 2,500,000 pounds. Of this total weight 308,000 pounds may be considered to rotate with or as a part of the turntable, leaving 2,192,000 pounds as the weight of the trusses, lateral bracing, track, and similar parts. The data of this bridge to be used in the preceding equations are:

Extreme length $= l = 500$ feet;
" width $= w = 30$ "
Diameter of drum $= 2R = 32$ feet;
" " pitch circle of rack $= 34$ feet 4 inches;
$\beta = 90° = 1.5709$.

Assume $T = 2$ minutes $= 120$ seconds. There will then be assumed

$$c = 0.25, \quad d = 0.45, \quad \text{and} \quad e = 0.3.$$

The following equation can then be written:

$$\left(0.25 \times 120 \times \frac{\alpha}{2}\right) + \left(0.45 \times 120 \times \alpha\right) + \left(0.3 \times 120 \times \frac{\alpha}{2}\right) = 1.5709.$$

$$\therefore \ \alpha = 0.01806.$$

This value of α is the greatest angular velocity reached in turning.

Eqs. (2) and (4) adapted to this case then give:

$$I' = \frac{2{,}192{,}000(900 + 250{,}000)}{12 \times 32.2} = 1{,}423{,}390{,}000;$$

$$I'' = \frac{308{,}000 \times 256}{32.2} = 2{,}464{,}000;$$

$$\therefore \ I = I' + I'' = 1{,}425{,}854{,}000;$$

$$\therefore \ \rho = \sqrt{\frac{1{,}425{,}854{,}000 \times 32.2}{2{,}500{,}000}} = 135.5 \text{ feet.}$$

An unbalanced wind pressure of 4 pounds per square foot over an exposed surface of 8 square feet per linear foot will be assumed. Hence

$$A = 8 \times 250 = 2{,}000 \text{ square feet} \quad \text{and} \quad r_0 = 125 \text{ feet}.$$

The value of f will be taken at 10 pounds and P will be 2,500. Substituting the preceding quantities in eq. (15),

$$\begin{aligned} \textit{Work} &= 232{,}557 + 628{,}360 + 1{,}570{,}900 \\ &= 2{,}431{,}817 \text{ ft.-lbs.} \end{aligned}$$

Eq. (16) then gives the greatest horse-power:

$$\text{H.P.} = \frac{3{,}129{,}488}{66{,}000} = 47.4.$$

During the period of uniform motion the horse-power becomes

$$\frac{2{,}199{,}260}{66{,}000} = 33.3.$$

The greatest force at the pitch circle is at once found by substituting the preceding quantities in eq. (19):

$$\textit{Greatest force} = 50{,}000 + 23{,}255 + 58{,}160 = 131{,}415 \text{ pounds}.$$

If for any sufficient reason the period of retardation should be as small as $0.125T$, eq. (21) shows that the greatest brake force at the pitch circle of the rack would be

$$\begin{aligned} \textit{Greatest brake force} &= 100{,}000 + 58{,}160 - 23{,}255 \\ &= 134{,}905 \text{ pounds}. \end{aligned}$$

This is a little larger than the above "*Greatest Force.*" The brakes would be applied so strongly only to meet some emergency.

Art. 119. Examples of Power.

The nominal horse-powers of steam-engines fitted to a number of railroad and heavy city draw-spans which have proved to be very satisfactorily operated are given in the tabulation below:

Bridge.	Weight of Moving Parts.	Engines.
	Pounds.	H.P.
500 feet double-track railway	2,500,000	40
400 feet city bridge	4,200,000	50
270 feet city bridge	1,800,000	40
400 feet double-track railway	2,000,000	35
300 feet double-track railway	1,250,000	20
362 feet single-track railway	684,000	20
217 feet double-track railway	600,000	20

The tendency has been toward an increase of engine power, in consequence of some of the earlier and smaller engines having shown insufficient capacity in winds and other contingencies of drawbridge operation.

The preceding considerations apply to rim-bearing turntables only, which were at one time exclusively used for all draw-spans over a length sufficiently great to require the structure to be of the through type.

The center- or pin-bearing type, in which the entire moving weight of the structure is carried on a center pin or pivot, was formerly used for short spans only, but it is now employed for heavy swing-bridges. This center pin or pivot may vary from eight to twenty-four or more inches in diameter, so that the pressure per square inch of bearing surface will not take greater values than about two thousand pounds to twenty-five hundred pounds. The center pin, usually steel, generally rests upon flat disks of phosphor-bronze in order to reduce friction and wear. The bearing faces of the pin and disks should

be channeled or grooved to allow entrance for a heavy lubricant.

In the Transactions of the American Society of Civil Engineers for 1874, Mr. C. Shaler Smith gave, as the result of a number of tests of center-bearing draw-spans, the frictional resistance, *if exerted at the circumference of the center pin*, at $\frac{9}{100}$ of the total weight turned. If W is that weight in pounds, and F the force of friction supposed exerted in the circumference of the pin, then

$$F = 0.09W. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Man-power for Small Spans.

If, again, t is the time in minutes required by one man to open the draw or close it, and if d is the diameter of the center pin in feet, Mr. Smith gave

$$t = 0.09W\frac{\pi d}{4 \times 10{,}000}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The time given by eq. (7) will usually be insufficient for the requirements of one man, even if no other consideration than that of friction be involved.

In the contingency of an unbalanced wind pressure on one arm of the draw, one man may not be able to put the bridge in motion, nor to hold it against the wind. Hence none but the smallest draws in unimportant situations are made dependent on one man-power.

CHAPTER VIII.

DEFLECTION OF FRAMED STRUCTURES AND OUTLINE OF THE THEOREM OF LEAST WORK.

Art. 120. General Method.

The elastic deflection of a bridge or roof-truss depends upon the stretching or compression of its various members in consequence of the tensile or compressive forces to which they are subjected. Any method by which the deflection is found, therefore, must involve these elastic changes of length. There are a number of methods which give the desired expressions, but probably the simplest as well as the most elegant procedure is that which reaches the results sought through the consideration of the work performed in the truss members in producing their elastic lengthenings and shortenings.

The general features of this method can readily be shown by reference to Fig. 1. It may be desired to find the deflection of any point, as J, of the lower chord, produced both by the dead and live load which the truss carries. It is known from what has preceded that every member of the upper chord will be shortened and that every member of the lower chord will be lengthened; and also that generally the vertical web members will be shortened and the inclined web members lengthened. If there can be obtained an expression giving that part of the deflection of J which is due to the change of length of any one member of the truss independently of the others,

then that expression may be applied to every other member in the entire truss, and by taking the sum of all these effects, the desired deflection will at once result. While this expression will be found for some one particular member, it will be of such a general form that it may be used for any truss whatever; it will be written for the upper-chord member BC in Fig. 1.

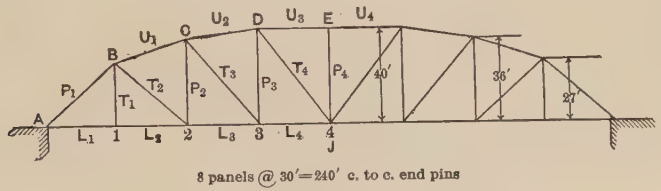


FIG. 1.

The general problem is to determine the deflection of the point J when the bridge carries both dead and moving load over the entire span. The general plan of procedure is first to find the stresses due to this combined load in every member of the truss, so that the corresponding lengthening or shortening is at once shown. The effect of this lengthening and shortening for any single member BC in producing deflection at J is then determined; the sum of all such effects for every member of the truss is next taken, and that sum is the deflection sought. In this case the vertical deflection will be found, because that is generally desired in connection with bridge structures, but precisely the same method and essentially the same formulæ are used to find the deflection in any direction whatever. The following notation will be employed:

Let w = deflection in inches at any panel-point or joint of the truss;

P = any arbitrary load or weight in pounds supposed to be hung at the point where the deflection is desired and acting as if gradually applied. This load may be taken as unity;

Z = stress in pounds produced in any member of the truss by P;

S = stress in pounds produced in any member of the truss by the combined dead and moving loads;

l = length in inches of any member of the truss in which Z or S is found;

A = area of cross-section of the same member in square inches;

E = coefficient of elasticity in pounds per square inch.

S or Z may be either tension or compression, and the formulæ will be so expressed that tension will be made positive and compression negative.

The change of length of the chord member BC produced by a stress gradually increasing from zero to S is $\frac{S}{AE}l$. If it be supposed that BC is a spring of such stiffness that it will be compressed by the gradual application of Z exactly as much as the shortening of the actual member by the stress S, the deflection of the point 4 with the weight P hung from it, and due to that compression alone, will be precisely the same as that due to the actual shortening of BC by the combined dead and moving loads.

It is known by one of the elementary principles of mechanics that, since P acts along the direction of the vertical deflection w, the work performed by the weight P over that deflection is equal to the work performed by Z over the change of length l. Hence

$$\frac{l}{2}P \cdot w = \frac{l}{2} \cdot Z \cdot \frac{Sl}{AE},$$

or

$$w = \frac{Z}{P} \cdot \frac{Sl}{AE}. \quad \ldots\ldots \quad (1)$$

The quantity $Z \div P$ is the stress produced in the member by a unit load applied at a joint or point where the deflection is desired. Again, $S \div A$ is the stress per unit of area, i.e., intensity of stress, in the member considered due to the actual dead and moving loads. For brevity let these be written

$$\frac{Z}{P}=z \quad \text{and} \quad \frac{S}{A}=s; \quad \text{then} \quad w=z\frac{sl}{E}. \quad . \quad . \quad (2)$$

If the influence of every member of the truss is similarly expressed, the value of the total deflection produced by the dead and moving loads will be

$$w=\sum\frac{zsl}{E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The sign of summation Σ indicates that the summation is to extend over all the web and chord members of the truss.

Art. 121. Application of Method for Deflection to Triangular Frame.

Before applying those equations to the case of Fig. 1, it is best to consider a simple case, i.e., that of the tri-

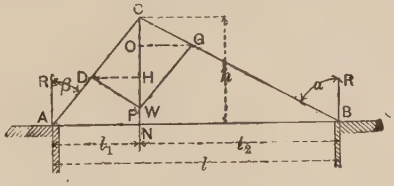


FIG. 2.

angluar frame shown in Fig. 2. The reactions are

$$R=\frac{l_2}{l}W \quad \text{and} \quad R'=\frac{l_1}{l}W. \quad . \quad . \quad . \quad . \quad (4)$$

The stresses in the various members are:

In CB, $S = \frac{l_1}{l} W \cdot \sec \alpha$;

" CA, $S = \frac{l_2}{l} W \cdot \sec \beta$;

" AB, $S = \frac{l_2}{l} W \tan \beta = \frac{l_1}{l} W \cdot \tan \alpha$.

Also, $CB = h \sec \alpha$; area of section $= A_1$.
$CA = h \cdot \sec \beta$; " " " $= A_2$.
$AB = l$; " " " $= A_3$.

In this instance it is simplest to take $P = W$. Eq. (3) then gives

$$w = \left(\frac{l_1^2}{l^2} \frac{h \sec^3 \alpha}{A_1} + \frac{l_2^2}{l^2} \cdot \frac{h \sec^3 \beta}{A_2} + \frac{l_2^2}{l^2} \cdot \frac{l \tan^2 \beta}{A_3} \right) \frac{W}{E}. \quad . \quad (5)$$

Let it be supposed that

$l = 25$ feet $= 300$ inches;
$h = 8$ feet 4 inches $= 100$ inches;
$l_2 = 16$ feet 8 inches $= 200$ inches and $l_1 = 100$ inches;
$\tan \beta = 1$; $\sec \beta = 1.414$;
$\sec \alpha = 2.24$;
$W = 10{,}000$ pounds.

If the bars are all supposed to be of yellow-pine timber, there may be taken

$E = 1{,}000{,}000$ pounds;
$A_1 = 10'' \times 12'' = 120$ square inches;
$A_2 = 10'' \times 10'' = 100$ square inches;
$A_3 = 10'' \times 12'' = 120$ square inches.

The insertion of these quantities in eq. (5) gives the deflection

$$w = 0.01042 + 0.01253 + 0.01111 = 0.034''. \quad . \quad (6)$$

Eq. (6) is so written as to show the portion of the deflection due to each member of the frame.

In applying either eq. (2) or (3) care must be taken to give each stress and its corresponding strain (lengthening or shortening) the proper sign. As the formulæ have been written and used, a tensile stress and its resulting stretch must each be written positive, while a compressive stress must be written negative. This holds true for both the stresses Z and S (or z and s). The magnitude of the assumed load P is a matter of indifference, since the stress Z will always be proportionate to it and the ratio $P \div Z$ will therefore be constant. P is frequently taken as unity; or, as in the case just given, it may have any value that the conditions of the problem make most convenient.

Art. 122. Application of Method for Deflection to Truss.

In making application of the deflection formulæ to any steel railroad truss similar to that shown in Fig. 1, it will be first necessary to determine the stresses in all its members due to the dead and moving loads, since the deflection under the moving load is sought. These loads will be considered uniform, and that is sufficiently accurate for any railroad bridge. The moving train load will be taken as covering the entire span, assumed, for a single-track railroad, 240 feet in length between centers of end pins. There are eight panels of 30 feet each, and the depth of truss at center is 40 feet. Other truss dimensions are as shown in Fig. 1. The dead loads, or own weight, are taken at 400 pounds per linear foot of span for the rails and other pieces that constitute the track; at 400 pounds per linear foot for the steel floor-beams and stringers, and 1,600 pounds per linear foot for the weight of trusses and bracing. The moving train load will be

taken at 4,000 pounds per linear foot. This will make the panel loads for each truss as follows:

Lower-chord dead load. . .	$30 \times 800 = 24{,}000$	lbs. per panel
Lower-chord moving load.	$30 \times 2000 = 60{,}000$	" " "
Total load on lower chord.....	$84{,}000$	" " "
Upper-chord dead load. .	$30 \times 400 = 12{,}000$	" " "

The structure is a through-bridge, hence all moving loads rest on the lower chord.

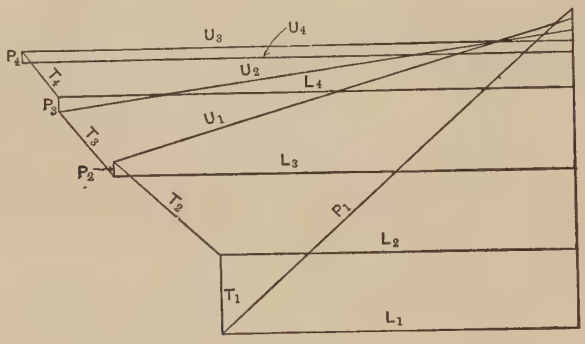


FIG. 3.

The stresses in the truss members due to the combined uniform dead and moving loads are best found by the graphical method. One diagram only is needed to determine all the stresses, and it is shown in Fig. 3. This diagram is drawn accurately to scale, and the stresses measured from it are shown in Table I.

The stresses in all the truss members due to the unit load hung at J are readily found by the single diagram shown in Fig. 4, also carefully drawn to scale. These stresses measured from the diagram are given in Table I as indicated by the column z; they are also represented in eq. (3) by the letter z. The quantity s in eq. (3) is the

intensity of the stress (pounds per square inch of cross-section of member) produced by the combined dead and moving loads in each member. As shown, these stresses

TABLE I.

	S	s	z	l	w
L_1	+373,300	+12,000	+0.555	360	+0.08563
L_2	+373,300	+12,000	+0.555	360	+0.08563
L_3	+480,000	+12,000	+0.833	360	+0.1284
L_4	+540,000	+12,000	+1.125	360	+0.1736
P_1	−502,300	− 9,000	−0.748	472	+0.1132
U_1	−501,000	− 9,500	−0.870	376	+0.1108
U_2	−544,800	−10,000	−1.135	363	+0.1472
U_3	−576,000	−10,000	−1.50	360	+0.1928
T_1	+ 84,000	+ 9,000	0	324	.....
T_2	+143,500	+10,000	+0.3738	472	+0.0629
P_2	− 12,000	− 1,000	−0.250	432	+0.00386
T_3	+ 93,720	+ 7,400	+0.456	562	+0.0677
P_3	+ 12,000	+ 1,000	−0.35	480	−0.0060
T_4	+ 60,000	+ 4,800	+0.625	600	+0.0643
P_4	− 12,000	− 1,000	0	480	.....

Deflection for $\frac{1}{2}$ truss members = 1.2300 inches.
Deflection at $J = 2 \times 1.2300 = 2.4600$ inches.

are least in the web members near the center of the span, and greatest in chord members. The lengths in inches of the truss members are shown in the proper column of the table. It will be observed that all counter web mem-

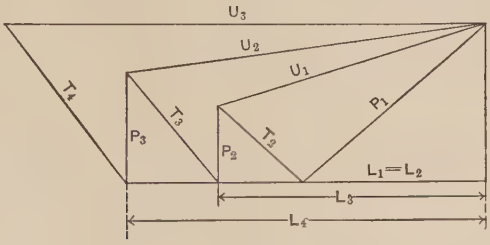


FIG. 4.

bers are omitted, as they are not needed for the uniform load. The coefficient of elasticity (E) is taken at 28,000,000 pounds. The quantities represented by the second mem-

ber of eq. (3) are computed from these data, and they appear in the last column of the table, the sum of which gives the desired deflection in inches. The elements of the table indicate how much of the deflection is due to the chords and to the web members, and they show that disregarding the latter would lead to a considerable error.

As the deflection is usually desired in inches, the lengths of members must be taken in the same unit.

Art. 123. Method of Least Work.

The so-called theorem or principle of "Least Work" is closely related to the subject of elastic deflection just considered in its availabilty for furnishing equations of condition, in addition to those of a purely statical character, in cases where indetermination would result without them. This principle of least work is expressed in the simple statement that when any structure supports external loading, the work performed in producing elastic deformation of all the members will be the least possible. Although this principle may not be susceptible of a complete and general demonstration, it may be shown to hold true in many cases if not all. The hypothesis is entirely reasonable and furnishes elegant solutions in many useful problems.

The application of this principle requires the determination of expressions for the work performed in the elastic lengthening and shortening of pieces subjected either to tension or compression, and for the work performed in the elastic bending of beams carrying loads at right angles to their axes. Both of these expressions can be simply found.

Let it be supposed that a piece of material whose

length is L and the area of whose cross-section is A is either stretched or compressed by the weight or load S applied so as to increase gradually from zero to its full value. The elastic change of length will be $\frac{SL}{AE}$, E being the coefficient of elasticity. The average force acting will be $\frac{1}{2}S$, hence the work performed in producing the strain will be

$$\frac{1}{2}\frac{S^2L}{AE}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

It will generally be best, although not necessary, to take L in inches. The expression (7) applies either to tension or compression precisely as it stands.

To obtain the expression for the work performed by the stresses in a beam bent by loads acting at right angles to its axis, a differential length (dL) of the beam is considered at any normal section in which the bending moment is M, the total length being L. Let I be the moment of inertia of the normal section, A, about an axis passing through the center of gravity of the latter, and let k be the intensity of stress (usually the stress per square inch) at any point distant d from the axis about which I is taken. The elastic change produced in the indefinitely short length dL when the intensity k exists is $\frac{k}{E}dL$. If dA is an indefinitely small portion of the normal section, the average force or stress, either of tension or compression, acting through the small elastic change of length just given, can be written, by the aid of the equation $M = k\frac{I}{d}$, as

$$\frac{1}{2}k \cdot dA = \frac{Md}{2I} \cdot dA. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Hence the work performed in any normal section of the member, for which M remains unchanged, will be, since $\int k \cdot dA \cdot d = M$,

$$\int \frac{M}{2IE} kd \cdot dA \cdot dL = \frac{M^2}{2EI} dL. \quad . \quad . \quad . \quad . \quad (9)$$

The work performed throughout the entire piece will then be

$$\int \frac{M^2}{2EI} dL. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Each of the expressions (7) and (10) belongs to a single piece or member of the structure. The total work performed in all the pieces subjected either to direct stress or to bending, and which, according to the principle of least work, must be a minimum, is found by taking the summation of the two preceding expressions,

$$e = \frac{1}{2E} \sum \frac{S^2 L}{A} + \frac{1}{2E} \sum \int \frac{M^2}{I} dL = \text{minimum}. \quad . \quad (11)$$

In making an application of eq. (11) it is to be remembered that S is the direct stress of tension or compression in any member, and that M is the general value of the bending moment in any bent member expressed in terms of the length L.

Art. 124. Application of Method of Least Work to a General Problem.

The problem which generally presents itself in the use of eq. (11) is the finding of an equation which expresses the condition that the work expended in producing elastic deformation shall be a minimum, some particular stress in the structure or some external load or force being the

variable. If t represent that variable, then the desired equation of condition will be found by simply placing the first differential coefficient of e in eq. (11) equal to zero:

$$\frac{de}{dt} = \frac{1}{E}\left(\sum \frac{S}{A} \cdot \frac{dS}{dt} \cdot dL + \sum \int \frac{M}{I} \frac{dM}{dt} \cdot dL\right) = 0. \quad (12)$$

The solution of eq. (12) will give a value of t which will make the work performed as expressed in eq. (11) a minimum. This method is not a difficult one to employ in such cases as those of drawbridges and stiffened suspension bridges. In the latter case particularly it is of great practical value.

Art. 125. Application of Method of Least Work to Trussed Beam.

The method of least work may be illustrated by the application of the preceding equations to the simple truss shown in Fig. 5. The pieces BC and GD are supposed to

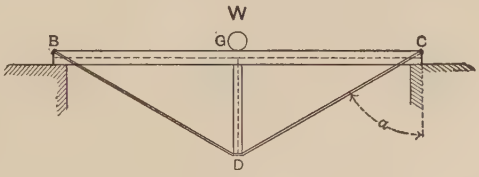


FIG. 5.

be of yellow-pine timber, the former 10 inches by 14 inches (vertical) in section and the latter 8 inches by 10 inches, while each of the pieces BD and DC are two $1\frac{5}{8}$-inch round steel bars. The coefficient of elasticity E will be taken at 1,000,000 pounds for the timber and 28,000,000 for the steel. The length of BC is 360 inches; GD, 96 inches; $BD = 96 \times 2.13 = 204.5$ inches.

$$\tan \alpha = 1.875 \quad \text{and} \quad \sec \alpha = 2.13.$$

The weight W resting at G is 20,000 pounds. A part of this weight is carried by BC as a simple timber beam, while the remainder of the load will be carried on the triangular frame BCD acting as a truss, the elastic deflection of the latter throwing a part of the load on BC acting as a beam. According to the principle of least work the division of the load will be such as to make the work performed in straining the different members of the system a minimum.

That part of W which rests on BC as a simple beam may be represented by W_1, while W_2 represents the remaining portion carried by the triangular frame. As G is the center of the span, the beam reaction at either B or C is $\frac{1}{2}W_1$. Hence the general value of the bending moment in either half of the beam at any distance x from either B or C is

$$M = \tfrac{1}{2}W_1 x.$$

Hence

$$M^2 \cdot dL = \tfrac{1}{4}W_1^2 \cdot x^2 \cdot dx.$$

As there is but one member acting as a beam, whose moment of inertia I is constant, the second term of the second member of eq. (11) becomes, by the aid of the preceding equation,

$$\frac{1}{2EI}\int M^2 \cdot dL = \frac{1}{EI}\int_0^{\frac{l}{2}} \frac{1}{4}W_1^2 \cdot x^2 \cdot dx = \frac{1}{EI}\,\frac{W_1^2 \cdot l^3}{96}. \quad (13)$$

The numerical elements of the expression for the work done in the members of the triangular frame are:

Member.	Stress.	Length.	Area of Section.
BC.	$\frac{1}{2}W_2 \cdot \tan \alpha$	360 inches $= l$	140 square inches
DC.	$\frac{1}{2}W_2 \cdot \sec \alpha$	204.5 "	4.14 " "
DG.	W_2	96 "	80 " "

$$I = \frac{10 \times 14^3}{12} = \frac{27{,}440}{12} = 2{,}286.7.$$

The substitution of those quantities in the first term of the second member of eq. (11) will give

$$\frac{1}{2E}\sum\frac{S^2L}{A}=\frac{1}{2{,}000{,}000}\left(\frac{W_2^2\cdot\tan^2\alpha\cdot 360}{4\times 140}+\frac{W_2^2\cdot 96}{80}\right)$$
$$+\frac{2}{56{,}000{,}000}\,\frac{W_2^2\cdot\sec^2\alpha\cdot 204.5}{4\times 4.14}=0.00000373W_2^2.$$

The substitution of numerical quantities in eq. (13) gives

$$\frac{1}{EI}\,\frac{W_1^2l^3}{96}=0.000213W_1^2.$$

Or, since $W-W_2=W_1$,

$$e=0.00000373W_2^2+0.000213(W-W_2)^2. \quad (14)$$

Hence

$$\frac{de}{dW_2}=0.00000746W_2-0.000426(W-W_2)=0. \quad (15)$$

The solution of this equation gives

$$W_2=0.893W=19{,}660 \text{ pounds.}$$
$$W_1=340 \text{ pounds.}$$

It is interesting to observe that the first term of the second member of eq. (15) is the deflection of the point of application of W_2 as a point in the frame, while the second term is the deflection of the point of application of W_1 considered as a point of the beam. In other words, the condition resulting from the application of the prin-

ciple of least work is equivalent to making the elastic deflections of W_1 and W_2 equal. Indeed, eq. (12) expresses the equivalence of deflections whenever the features of the problem are such as to involve concurrent deflections of two different parts of the structure.

Art. 126. Removal of Indetermination by Methods of Least Work and Deflection.

The indetermination existing in connection with the computations for such trusses as those shown in Figs. 6 and 7 can be removed by finding equations of condi-

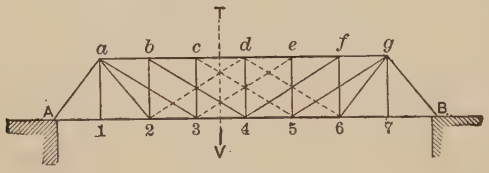


FIG. 6.

tion by the aid of the method of deflections. It is evident that the component systems of bracing of which such trusses are composed must all deflect equally. Hence expressions may be found for the deflection of those component trusses, each under its own load. Since these deflections must be equal, equations of condition at once result. A sufficient number of such equations, taken with those required by statical equilibrium, can be found to solve completely

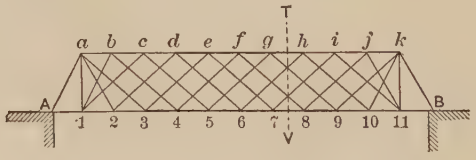


FIG. 7.

the problem. Such methods, however, are laborious, and the ordinary assumption of each system carrying wholly

the loads resting at its panel-points is sufficiently near for all ordinary purposes.

The method of least work can be conveniently used for the solution of a great number of simple problems, like that which requires the determination of the four reactions under the four legs of a table carrying a single weight or a number of weights, and many others of the same character.

CHAPTER IX.

STRESSES CAUSED BY WIND.

Art. 127. Stresses in the Sway-bracing of a Through-bridge.

The construction of the upper and lower sway-bracing of a bridge must, as far as the jar and oscillation of a moving load are concerned, be a matter of judgment; but the stresses due to the action of the wind may be determined with sufficient accuracy.

Although a moving train will partially shelter one truss, it seems no more than prudent, with the ordinary open style of American bridge, to consider the action of the wind as existing constantly, during the passing of a train, over the whole of the projection of each truss in the bridge on a plane normal to the direction of the wind. If this is considered excessive, however, for low trusses, that portion of the windward truss sheltered by the train may be omitted. In any case, the intensity of the wind, and the surface against which it acts, are determined by the specifications under which the bridge will be designed.

Example.

Let Fig. 1 and Fig. 2 represent a single cancellation railway truss-bridge with vertical and diagonal bracing, and let the wind be supposed to blow in the direction shown by the arrow, which is normal to the planes of the trusses. Primed letters belong to the truss $DC'N'O'$, but all are not shown.

As the truss is a "through" one, all wind pressure against the floor system will act in the lower chord.

With the wind pressure between thirty and forty

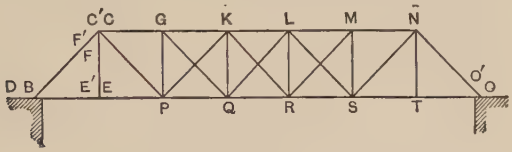


FIG. 1.—Elevation of Trusses.

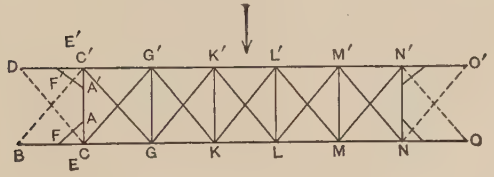


FIG. 2.—Plan of Trusses.

pounds per square foot, the following loads may be taken at the various panel-points:

At C', G', K', L', M', N', C, G, K, L, M, N	0.53	ton
" D and O'	0.27	"
" intermediate points	0.53	"
" B and O	0.27	"
" intermediate points	4.52	tons

The amount 4.52 tons * includes the pressure against the train, which is taken at 300 pounds per foot of track. The panel length is 21 feet, hence the panel train load is $21 \times 300 = 6{,}300$ pounds $= 3.15$ tons. The wind pressure against the floor system is assumed to be 0.84 ton per panel, while the panel pressure against each truss is 0.53 ton. The sum of the three quantities is 4.52 tons.

The panel train loads (3.15 tons) constitute a continuous, moving load; the wind pressure against the trusses and floor system, however, forms a *fixed load.*

* The loads and stresses in this example are expressed in tons to illustrate a variation in units.

The following are the truss dimensions, including the lengths of the braces AF and $A'F'$:

Panel length = 21 feet. Height of truss = 24.00 feet.
Width, BD = 14 " BC = 31.89 "
CF = 7 " AC = 4.04 "
FA = 8.08 feet.
Normal from C on $FA = CF \times \sin 30° = 3.5$ feet.

Let H represent half the total wind pressure concentrated in the two upper chords; this will be resisted (if the bridge is not blown bodily off the abutments or piers) by an equal force of friction developed at the feet B and D, or O and O' of the end posts. Let H' and H'' be the forces developed at D and B, respectively. These horizontal forces will tend to overturn the trusses in a vertical plane normal to the axis of the bridge. A vertically upward reaction, V, will be developed at B, and an equal downward one (a portion of the weight of the truss $DC'N'O'$) at C'. Considering the left end of the truss, the following three equations of equilibrium must be fulfilled:

$$H' + H'' + H = 0. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$V + V' = 0. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$(H' + H'') \times 24 + V \times 21 = 0. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The vertical force acting at C' is represented by V'.

These three equations are not sufficient for the determination of the four quantities H', H'', V, and V'. The forces H' and H'' are therefore indeterminate in magnitude, except in this respect, their sum must be equal and opposite to H.

With the form of portal bracing shown in Fig. 1, it will be assumed in this article that $H'' = 0$ and $H' = -H$. Other and better forms of portal, together with other

assumptions in regard to the horizontal reactions H' and H'', will be given in succeeding articles.

From the data already given,

$$H = 6 \times 0.525 = 3.15 \text{ tons.}$$

Hence by eqs. (3) and (2)

$$V = 3.6 \text{ tons} = -V'.$$

The vertical force V is an increase of the upward reaction at B, but it is a downward reaction at D, i.e., a decrease of the upward reaction at the latter point. The wind therefore may be taken as holding the trusses in equilibrium against the upward vertical force, V, acting at B and an equal downward force, V, at D. The force V, therefore, produces tension throughout BO, and compression throughout CN, while V' produces equal compression in DO' and tension in $C'N'$. Hence

$$BO = C'N' = -DO' = -CN = 3.6 \times \tfrac{21}{24} = 3.15 \text{ tons.}$$

This stress neutralizes an equal amount of each chord stress due to the vertical loading in the leeward truss, but it adds to those stresses in the windward truss.

The following stresses in the members of the portal of the bridge may now be written:

$$\begin{aligned} (A'F') &= H \times 31.89 \div 3.5 = +19.2 \quad \text{tons} \\ (A'C') &= -19.2 \times \sin 30^\circ = -\ 9.6 \quad \text{“} \\ (F'C') &= -19.2 \times \cos 30^\circ = -16.65 \quad \text{“} \end{aligned}$$

The greatest bending moment in DC' exists at F', and is

$$M_2 = 3.15 \times (31.89 - 7.00) = 78.41 \text{ ft.-tons.}$$

The stress in BC is

$$(BC) = -\frac{31.89}{21} \times H = -\frac{31.89}{24} \times V = -4.79 \text{ tons.}$$

The compressive stress in DC', due to the vertical loading, is relieved by the same amount.

The greatest bending moment in CC' exists at A', and has for its value, as there is a point of contraflexure* halfway between A and A',

$$M_3 = 3.15\frac{31.89}{7}(7 - 4.04) = 42.43 \text{ foot-tons.}$$

Both moments M_2 and M_3 produce bending in the plane of the portal.

The end post DC' must be able to resist with a proper safety factor, at F', the bending moment M_2. The sway-brace CC' must be able to resist the moment M_3 at *both* the points A and A'.

The ordinary truss stresses in the sway-bracing remain to be found.

In both upper and lower sway-bracing the inclined members are in tension only, while those normal to the planes of the trusses (in the direction of the wind) sustain compression only.

Upper Sway-bracing.

In the upper chord the truss $C'GMN'$ has the two points of support C' and N'. The following trigonometric quantities will be required:

$$\text{Angle } G'KK' = 56° 20' = \alpha; \quad \tan \alpha = 1.5; \quad \sec \alpha = 1.8.$$

* See the authors' "Influence Lines," page 54, for the determination of the location of the point of contraflexure.

The upper web stresses are the following:

$$
\begin{aligned}
(KK') &= \qquad\qquad\qquad\qquad = -0.53 \text{ ton} \\
(G'K) &= +2 \times 0.53 \times \sec \alpha = +1.91 \text{ tons} \\
(G'G) &= -3 \times 0.53 \qquad\quad = -1.59 \text{ ``} \\
(GC') &= +4 \times 0.53 \times \sec \alpha = +3.82 \text{ ``} \\
(C'C) &= \qquad\qquad\qquad\qquad = +0.53 \text{ ton}
\end{aligned}
$$

The resultant upper-chord stresses are the following:

$$
\begin{aligned}
(C'G') &= -4 \times 0.53 \times \tan \alpha \qquad\quad = -3.18 \text{ tons} \\
(G'M') &= -2 \times 0.53 \times \tan \alpha - 3.18 = -4.77 \text{ ``} \\
(GK) &= +4 \times 0.53 \times \tan \alpha \qquad\quad = +3.18 \text{ ``} \\
(KL) &= +2 \times 0.53 \times \tan \alpha + 3.18 = +4.77 \text{ ``}
\end{aligned}
$$

Lower Sway-bracing.

The resultant lower web stresses are the following, remembering that the train * pressure is a moving load, and that D and O' are the supporting points for the lower sway-truss:

$$
\begin{aligned}
(Q'R) &= +\ 6 \times 0.45 \times \sec \alpha \qquad\qquad\quad = +\ 4.86 \text{ tons} \\
(QQ') &= -\ 6 \times 0.45 \qquad\qquad -0.53 = -\ 3.23 \text{ ``} \\
(P'Q) &= +10 \times 0.45 \times \sec \alpha + 1.90 \times \sec \alpha = +11.52 \text{ ``} \\
(PP') &= -10 \times 0.45 \qquad -2.43 \qquad\quad = -\ 6.93 \text{ ``} \\
(E'P) &= +15 \times 0.45 \times \sec \alpha + 3.80 \times \sec \alpha = +18.99 \text{ ``} \\
(EE') &= -15 \times 0.45 \qquad -4.33 \qquad\quad = -11.08 \text{ ``} \\
(DE) &= +21 \times 0.45 \times \sec \alpha + 5.70 \times \sec \alpha = +27.27 \text{ ``}
\end{aligned}
$$

The quantity 0.45 will be at once recognized as $3.15 \div 7$. The counters $S'T$ and $R'S$ are not required to resist wind stresses, but should never be omitted, in order that the general stiffness of the bridge may be increased; their cross-sections may be the same as that of $Q'R$.

* The train is taken as passing from right to left.

The lower resultant chord stresses are the following:

$$(DE') = -(3\times 1.90 + 3\times 3.15)\tan\alpha = -22.73 \text{ tons}$$
$$(E'P') = -(2\times 1.90 + 2\times 3.15)\tan\alpha - 22.73 = -37.89 \text{ ``}$$
$$(P'S') = -(1.90 + 3.15)\tan\alpha - 37.89 = -45.47 \text{ ``}$$
$$(EP) = -(DE') + 3.15 = +25.88 \text{ tons}$$
$$(PQ) = -(E'P') + 3.15 = +41.04 \text{ ``}$$
$$(QR) = -(P'S') + 3.15 = +48.62 \text{ ``}$$

If the wind blows in the opposite direction to that assumed, the chord stresses which have been determined for $C'N'$ will be found in CN, and vice versa. Precisely corresponding changes are to be made in the lower chords. The stresses in the sway-struts would not be changed. That diagonal in each panel which is not stressed in the preceding instance would sustain a tensile stress exactly equal to that already found in the other diagonal.

It is therefore necessary to make calculations for but one direction of the wind.

So far as equilibrium is concerned, in the preceding investigation, there might be taken $H'' = -H$ and $H' = 0$. In such a case BC would be subjected to a bending moment at F equal to $-M_2$, and the bending moment in CC', at A, would be $-M_3$, while the stresses in FA, AC, and CF would be respectively $-(F'A')$, $-(A'C')$, and $-(C'F')$. For these reasons all parts of the portal should be built to sustain the stresses and moments which have been found when affected by opposite signs.

It should be remembered that the strut CC' is subjected to combined direct stress and bending to the respective amounts that have been found.

Those portions of the lower sway-struts EE', PP', etc., extending from the windward rail to the lower chord BO (with the direction of the wind first assumed) are each subjected to a compressive stress, in addition to those

already found, nearly equal to an amount to be determined in the following manner: Let N represent the number of panels in the sway-truss and n the number of any strut, from the farther end of the truss, counting the end itself zero; i.e., for PP', n will be 5. In the case taken $N=7$. The amount desired will then be *the panel train wind load multiplied by* $\frac{n}{N}$, *added to the panel wind pressure against the floor system;* or, in the example,

$$3.15\times\frac{n}{7}+0.84=0.45\times n+0.84.$$

This compression in the struts arises from the fact that the wind pressure against train and floor system is not applied at panel-points, but *on the struts* (floor-beams in through-spans) *between their ends*, and that a panel load of the former must be added to the ordinary strut stress which exists with the head of the train at the strut considered.

This involves, however, a very small error on the side of safety, since the pressure is divided between the two rails. Considering both directions of the wind, it will be seen that these struts are subjected to this amount of compression from end to end in addition to the regular truss stresses.

All the preceding wind stresses are to be combined with those due to the vertical loading, wherever they act in the same piece, in the manner prescribed in the specifications for any particular structure.

If the portals are vertical, the stresses (BO) and $(C'N')$, due to the moment M_1, will be zero; also the span of the upper sway-truss will be equal to that of the lower. No other changes will occur.

If the bridge is a deck structure, when practicable the ends of the chords should be secured directly to the piers

or abutments, as no bending will then take place in the end posts. If this is not feasible, the calculations will be precisely the same as those already indicated, with possible changes of signs in some of the stresses in the end posts or braces. In deck-bridges, however, the wind pressure against floor system and train will be found in the upper chord.

The method of treatment which has been exemplified is, therefore, perfectly general and sufficient for all cases.

It is now frequently specified that all wind pressure against a bridge shall be treated as moving load. This treatment is necessary with the train wind load, and as all wind pressure over extended areas is exceedingly variable, it is not unreasonable to treat the entire wind load as moving. The resulting error is small and on the side of safety. The practice is to be commended.

Art. 128. Transverse Sway-braces in Deck-bridges.

In deck-bridges, tension sway-braces (contained in planes normal to the trusses) are introduced, extending from either chord of one truss to the diagonally opposite one in the adjacent truss. So far as pure equilibrium is concerned, when horizontal sway-trusses are present, these are superfluous; but they are highly efficient in their influence on lateral stability. The actual stresses in these members in any given case are indeterminate, but their greatest possible values are easily fixed. Let the total wind pressure exerted at a pair of opposite panel-points in the two upper chords be represented by P', and let α represent the angle between a horizontal line and the tension brace in question. Then the greatest possible stress which is required will be

$$T' = P' \sec \alpha.$$

This assumes that *all* the wind pressure is carried to the lower chords and resisted by the lower sway-truss.

In the case of vertical end posts, where the upper-chord ends are not secured directly to piers or abutments the stresses in the end lateral diagonals become perfectly determinate. In fact, in such a case, the braces AF, $A'F'$, Figs. 1 and 2, become the diagonals in question. Let P represent half the *total* wind pressure in the two upper chords. The tensile stress in either one of these diagonals (if α retains its preceding signification) will then be

$$T = P_1 \sec \alpha.$$

Let P represent the total wind pressure against the bridge, and P'' the total wind pressure against the train when it covers the whole span. Then let W and W' represent the total weight of bridge and train respectively; also let f be the coefficient of friction between the foot of end post and the supporting surface underneath. In order that neither truss shall possibly be moved bodily by the wind, with the bridge empty or covered by a train, there must exist the following relations:

$$P < f\left(\frac{W}{2} - 2V\right), \quad \text{or} \quad P + P'' < f\left(\frac{W + W'}{2} - 2V_1\right).$$

In the cases of through-bridges, or of deck-bridges with upper chords *not* secured to abutments, V_1 is to be found by applying the general form of eq. (3) to *both bridges and train*.

If the ends of the chords are secured to the piers or abutments, the resistances of these fastenings will take the place of the frictional resistances.

Art. 129. Transference of Train Load by Wind Pressure.

One other effect of the wind pressure against the train remains to be noticed. The normal action of this pressure will not permit the train's weight to be distributed between the two chords which carry it, according to the law of the lever. Let p'' (Fig. 3) represent a panel wind pres-

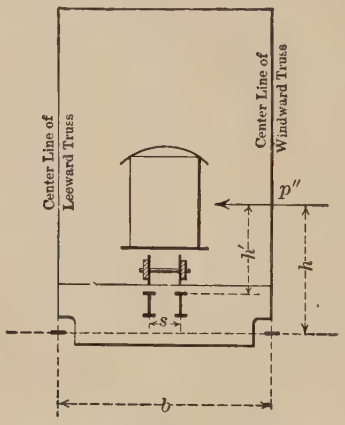


FIG. 3.

sure against the train, and let h represent the height of its center of action above the points of support. Also let b represent the horizontal distance between centers of trusses; then

$$t = \frac{p''h}{b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

will be the amount of load which is transferred from the windward to the leeward truss. In other words, the panel leeward load will exceed the panel windward one by $2t$. If, therefore, w' is a panel moving load, the action of the wind will cause all moving-load truss stresses to be increased by an amount found by multiplying the stresses,

determined without regard to the wind, by $\frac{t}{w'}$. Also, if s is the distance (normal) between two adjacent parallel stringers, the increase of load on one, and decrease of that on the other, will be

$$t_1 = \frac{p''h'}{s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Eq. (5) gives the variation of load on the floor-beam also. Without essential error, h may be measured downward from the center of the body of the car.

Specifications sometimes require calculations to be made with an unloaded bridge. In such a case the methods are precisely the same as the preceding, with the train wind pressure omitted.

Art. 130. Transverse Bracing for Transferring Wind Stresses from One Chord to Another—Concentrated Reaction.

In Art. 127 it has been supposed that the wind pressure is resisted by sway-trusses in the horizontal planes of both upper and lower chord. It may sometimes be desirable to transfer wind pressure to the lower chord, or to the upper.

Through-trusses.

The section of a through-truss bridge in which it is desired to carry all the wind pressure to the lower chord is represented in Fig. 4. AC and ND are posts directly opposite to each other in the two trusses. AN and OB are lateral struts, while AO and BN are lateral ties. Let the wind be supposed to blow from right to left, as shown by the arrow. According to the principles of the preceding article, in consequence of its direction, the wind

will relieve the truss AC of a part of the weight which it carries and add the same amount to that carried by the truss DN.

If the direction of the wind were reversed, the truss DN would be relieved, and AC would receive the increase of loading.

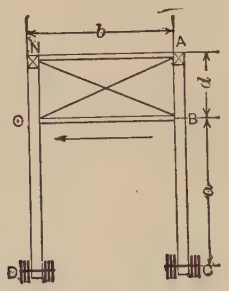


FIG. 4.

Let this relief (or increase) of truss load, per panel, be denoted by w; it will act as though hung from B.

The following notation, also, will be used:

$$AB = d = ON. \qquad BC = a = OD.$$
$$DC = AN = b.$$

F = total wind pressure, per panel (for one truss), on $\frac{1}{2}(AB + BC)$.

F' = total wind pressure, per panel (for one truss), on $\frac{1}{2}AB$.

With the assumed direction of the wind, the tie AO will not be stressed.

In this article, the total horizontal reaction equal to $2(F + F')$ will be taken as concentrated at D.

There will then result:

$$\text{Relief in truss } AC = w = 2\left(\frac{F'(a+d) + Fa}{b}\right). \quad . \quad (1)$$

$$\text{Compression in } AN = -F'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{Tension in } BN \quad = +w \sec ABN. \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{Compression in } BO = -(F + w \tan ABN)$$

$$= -\left(F + 2F' + [F + F']\frac{2a}{d}\right)$$

$$= F - 2(F + F')\left(\frac{a+d}{d}\right). \quad . \quad (4)$$

$$\text{Compression in } ND = -w. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The horizontal force $2(F + F')$ acts at D toward C, producing a bending in DN which has its greatest moment M at O. Hence

$$M = 2(F + F')a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If K is the greatest intensity of compressive stress (due to flexure) in the cross-section of the post DN, at O, d_1 the greatest distance of any compressed fiber from the neutral axis of the cross-section, and I the moment of inertia of the section about an axis passing through its center of gravity and lying in the plane of the truss; then, by the well-known formula,

$$K = \frac{d_1 M}{I}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

At O there will then exist the intensity of compression: $-\left(\frac{w}{q} + K\right)$, in which q is the area of cross-section of the column. The intensity of compression $-\left(\frac{w}{q} + K\right)$ is in addition to the regular truss stresses arising from vertical and wind loads.

If, for lack of head room, a flanged beam only is used, as shown in Fig. 5, instead of the lateral bracing of Fig. 4,

then that beam will be subjected to combined compression and bending. Let $+F$ represent the total wind pressure, per panel, *for both trusses*, on $\frac{1}{2}AB$, and let w represent the release of weight in AB and increase in CD.

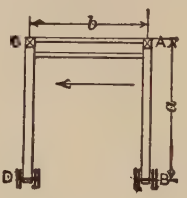


FIG. 5.

Also, let $AB=a$ and $BD=b$. $\frac{1}{2}F$ is to be taken as applied at A and w at the same point. Equal and opposite forces are also to be supposed to act at D.

The moment $M=Fa=wb$ exists at C and gives

$$w=\frac{Fa}{b}.$$

With the direction of wind shown by the arrow, the bending caused by w will increase uniformly from nothing at A to $M=wb$ at C. The bending moment, therefore, to be resisted by this beam AC, *and by the joints between it and the chords A and C*, is

$$M=Fa=wb. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The direct compression in AC is

$$AC=-\tfrac{1}{2}F. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Hence, if q is the area of cross-section of the beam, and if K, I, and d_1 retain the same general signification as in eq. (7), the greatest intensity of compression in the beam (at its ends) will be

$$-\left(\frac{\frac{1}{2}F}{q}+\frac{d_1M}{I}\right). \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The direct compression in CD is

$$CD = -w. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The bending moment in CD at C is

$$M = Fa. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The greatest compressive intensity is found at once by eq. (10), after writing w for $\frac{1}{2}F$, and giving to the remaining notation its general signification.

Deck-trusses.

The two preceding cases are those of through-trusses. In the case of a deck-truss the lateral bracing is of much more simple character; it is shown in Fig. 6. At C and A

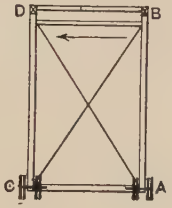


FIG. 6.

are the two lower chords. CA is a lateral strut, while BC and DA are lateral ties. No parts are subjected to bending.

If F is the panel wind pressure (for both trusses) acting along AC, there will result, if all wind pressure in the panel is carried to the upper chord:

$$CA = -\tfrac{1}{2}F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$w = F\frac{DC}{AC}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13a)$$

$$BA = -w. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$BC = +\sqrt{F^2 + w^2}. \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The horizontal component of BC is equal to F, and acts at B. Thus all wind pressure is carried to the upper chord.

The compression BA is in addition to the regular truss stresses induced by the vertical and wind loads. Similarly the stress in BD is increased by F.

By these methods all the wind pressure may be carried to either chord. The truss stresses of the sway-truss in the horizontal plane of that chord have already been found in the preceding article, or rather the methods for finding them and the effect of w on the stresses in the vertical trusses have there been completely given.

The wind has been taken in one direction only; with the other direction, opposite but symmetrically located parts would be stressed by the amounts found.

Art. 131. Transverse Bracing with Distributed Reactions.

In the preceding articles it has been assumed that the horizontal reactions of the wind pressure were concentrated at the extremity (top or bottom, as the case may be) of

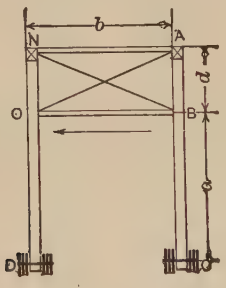


Fig. 7.

one post in the transverse panel considered. This assumption, however, may not be admitted, or some other may be substituted in its place.

Let Fig. 7 represent a transverse panel, with the wind blowing in the direction shown by the arrow.

As before, the following notation will be used:

$$AB = ON = d. \qquad BC = OD = a.$$
$$DC = AN = b.$$

F = total wind pressure, per panel, for one truss on $\frac{1}{2}(AB + BC)$.

F' = total wind pressure, per panel, for one truss on $\frac{1}{2}AB$.

w = relief of load in truss AC.

Instead of concentrating the entire horizontal reaction at D, if n is a quantity less than unity, there will be assumed:

Horizontal reaction at $D = 2n(F + F')$.
" " " $C = 2(1 - n)(F + F')$.

The wind pressures on $\frac{1}{2}BC = \frac{1}{2}OD$ act directly at C and D in the horizontal sway-truss, and consequently will be omitted from consideration.

As in the preceding article,

$$w = \frac{2(F'(a + d) + Fa)}{b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Taking moments about B,

$$AN = -\left[F' + 2(1 - n)(F + F')\frac{a}{d}\right]. \quad . \quad . \quad (2)$$

Taking moments about N,

$$OB = -\left[\frac{2n(F + F')(a + d)}{d} - F\right]. \quad . \quad . \quad . \quad (3)$$

Taking moments about the intersection of AN and OB at the distance infinity (∞) from the figure,

$$BN \infty \cos ABN = +[w \infty + 2n(F + F')a];$$

$$\therefore\ BN = +w \sec ABN = \frac{2(F')a + d(+Fa)}{b} \sec ABN. \quad (4)$$

The stress in BN, therefore, remains the same whatever may be the assumptions in regard to the horizontal reactions.

The bending moment at O, about an axis lying in the plane of the vertical truss, will be

$$M = 2n(F + F')a. \quad (5)$$

Since the windward truss is always relieved of a part of its weight the bending moment $2(1-n)(F+F')a$, at B, will seldom or never be needed.

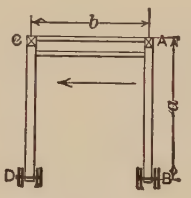


FIG. 8.

The value of M, from eq. (5), put in eq. (7) of the preceding article, and in the expression following that equation, will enable the greatest compressive intensity in the post to be found.

If the transverse panel, Fig. 7, represents the portal of a bridge, the distances AB and BC, or d and a, represent inclined distances in the plane of the portal. F' will (or may) then include, also, the reaction of the horizontal sway-truss whose plane contains AN, while F will include the reaction of the horizontal sway-truss whose plane contains OB, if there is such a sway-truss.

If $n=\frac{1}{2}$, as is sometimes assumed,

$$AN=-\left[F'+(F+F')\frac{a}{d}\right]. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$OB=-\left[F'+(F+F')\frac{a}{d}\right]=AN. \quad . \quad . \quad . \quad (7)$$

If $n=1$ in the formulæ of this article, those of the corresponding cases in the preceding article at once follow.

In Fig. 8 let the notation be as follows:

$$AB=CD=a. \qquad BD=CA=b.$$

Total wind pressure for *both trusses*, per panel, along

$$AC=F.$$

Horizontal reaction at $D=nF$; . . . . (8)

" " " $B=(1-n)F$. . . . (9)

If Fig. 8 represents a portal, F will or may include the reaction of a horizontal sway-truss.

The bending moment on both DC and CA, at C, also on the joint at the same point, is

$$M_1=nFa. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This is the greatest bending in DC and CA of that kind which produces *compression* in the lower flange of the beam CA.

The relief of panel load in the truss AB and increase of that in CD is

$$w=\frac{Fa}{b}.$$

Let x, measured from A, represent any variable portion of CA; then the bending moment at any point of CA is

$$M = (1-n)F \cdot a - wx = (1-n)F \cdot a - \frac{Fa}{b}x. \quad . \quad (11)$$

If $n = \frac{1}{2}$, and if the mid-point of AC be considered, $x = \frac{1}{2}b$. Hence

$$M = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11a)$$

This indicates a point of contraflexure.

For the point or joint A, x becomes equal to 0, while the expression for the moment is

$$M_1' = -(1-n)Fa. \quad . \quad . \quad . \quad . \quad . \quad (12)$$

This is the greatest bending of the kind opposite to M_1, in CA. It is also the greatest bending in AB. The connections at C must resist the moment M_1, while those at A must resist M_1'.

The direct compression in CA is $\frac{1}{2}F$;
" " " " CD " w.

If $n = \frac{1}{2}$, $M_1 = M_1' = \frac{1}{2}Fa$. . . . . (13)

These various bending moments, substituted in eq. (10) of the preceding article for M, will enable the greatest intensities of stress in the members CA, CD, and AB to be at once found.

Art. 132. Stresses in Braced Piers.

The general treatment of stresses in braced piers may be exemplified by that of a single "bent" represented by a skeleton diagram in Fig. 9, in which the horizontal web members are in compression. The plane of the "bent" is vertical and normal to the center line of the truss whose

end rests upon it; or if the track is curved, this plane is normal to it. The bent shown in Fig. 9 may be considered one of a pair, in parallel planes, which, being braced together, compose the complete braced pier. The rect-

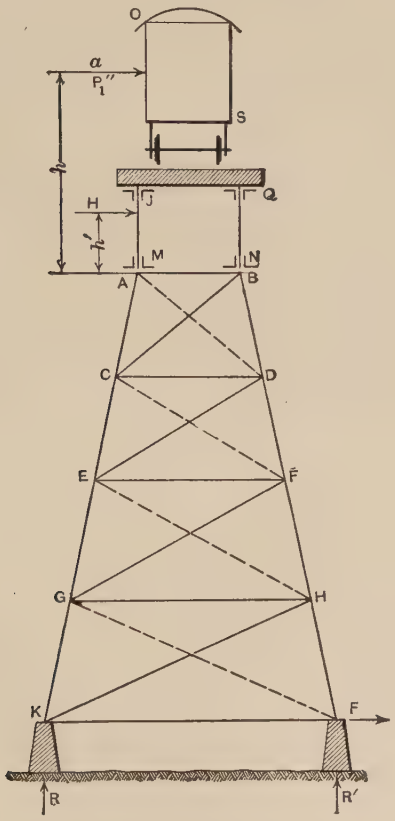


FIG. 9.

angle $JMNQ$ represents a skeleton section of the truss supported by the piers, the lower chords of which rest upon the top of the pier at A and B. A skeleton section of the train is also shown.

The direction of the wind is supposed to be shown by

the arrow a, normal to the track at the top of the pier. If the trusses are loaded with a train, the wind pressure against them and the train will be carried to the top of the piers in the manner shown in Art. 129. The wind will also act against the pier itself.

Wind Stresses in Loaded Structure.

Let the train be supposed to cover the whole of the two spans adjacent to the top of the bent (in all ordinary cases one of these spans will be the distance between two adjacent bents); then let H represent half the total wind pressure against trusses, and P_1'' half that on the train covering the two spans.

The height of the center of action of P_1'' above AB, Fig. 9, is h. Also let $b = AB$. The pressure P_1'' will decrease the train reaction at A and increase that at B by the amount

$$V_1 = \frac{P_1'' h}{b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let h' represent the vertical distance of the center of action of H from the horizontal line AB.

The wind pressure on the trusses $JMNQ$ will also cause a decrease of truss reaction at A, and an equal increase of that at B, which will be denoted by V, and its value will be

$$V = \frac{Hh'}{b}.$$

Consequently if

$$t' = V_1 + V = \frac{P_1'' h}{b} + \frac{Hh'}{b}, \quad . \quad . \quad . \quad . \quad (2)$$

the total horizontal force to be taken as acting at A, and with the wind, will be $(H + P_1'' - 2t' \tan \alpha)$ added to the wind pressure acting directly at A.

It is implicitly supposed that two equal and opposite forces, equal in magnitude and parallel to P_1'', act along AB. One of these forms, with P_1'' itself, the couple $P_1''h$; the other is the wind pressure which, combined with the half-panel pressure at A, and $(H - 2t' \tan \alpha)$, is represented by cd in Fig. 12.

The quantity t' (Fig. 10) is the force of a couple whose lever-arm is b. One force t' is therefore supposed to act

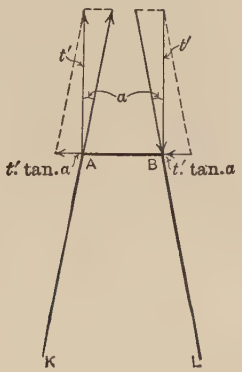


FIG. 10.

at A, and the other at B. V_1 will be considered larger than V; t' will act upward at A and downward at B. If α is the angle between AK or BL and a vertical line, the t' at B will cause a compression in AB equal to $t' \tan \alpha$, while the t' at A will pull to the left by the same amount. Consequently the force $2t' \tan \alpha$ will act on the point A and toward the left.

In Fig. 11, cd represents the force $(H + P_1'' - 2t' \tan \alpha)$ laid down to any desired scale. The small segments measured to the right of d represent the panel wind pressures against the pier at the points C, E, G, and K, while those shown on the left of c represent the panel pressures at D, B, F, H, and L. The panel pressures at A, B, K, and L are half those at the other points.

Stresses Due to Vertical Loading.

These stresses, due to the horizontal wind pressures against structure and train, may be at once scaled from the diagram of Fig. 11 and tabulated. The mode of

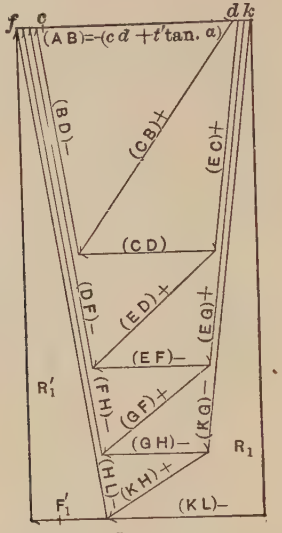


FIG. 11.

combination with the stresses due to the vertical loading will be prescribed in the specifications governing the design of the structure.

If the girders on trusses rest at the tops of the viaduct towers in the manner shown in Fig. 12, there will be no change whatever in the preceding methods or diagram. In this case, however, the wind pressure against the girders or trusses $AMNB$, Fig. 12, will increase the reaction at A and equally decrease that at B. The only result different from the preceding will be a change in the sign of V from plus to minus, so that

$$t' = V_1 - V = \frac{P_1'' h}{b} - \frac{Hh'}{b}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This value of t' is to be used in the expression for the total horizontal force acting at A, i.e. $(H + P_1'' - 2t' \tan \alpha)$.

Wind Stresses on Unloaded Structure.

If the structure carries no train load, the train pressure $P_1'' = 0$, and

$$t' = \pm \frac{Hh'}{b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

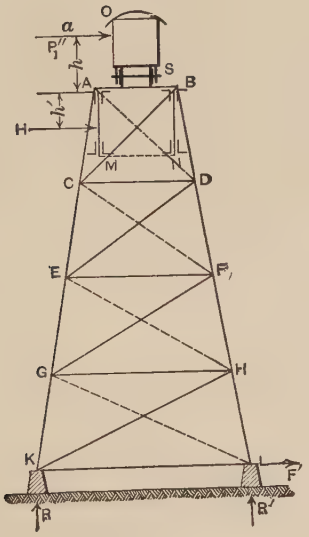


FIG. 12.

The plus sign is to be used in the case shown by Fig. 9, and the minus sign for Fig. 12.

The total horizontal force acting at A then becomes

$$H - 2t' \tan \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

These various values for the total horizontal force at A for the different cases are to be represented by cd in the diagram of Fig. 11.

Let W represent the total dead weight of adjacent girders or trusses and tracks, and W' the total moving load resting at the top of the bent.

Let W_1 represent the panel weight of the pier itself resting at the points C, E, G, D, F, H, $\frac{1}{2}W_1$ will be taken as applied at the points A, B, K, and L; then the resultant reactions at A and B, with the wind blowing, will be, respectively,

$$\frac{W_1+W}{2}-t' \quad \text{and} \quad \frac{W_1+W}{2}+t' \quad . \quad . \quad . \quad (6)$$

when the structure carries no moving load, or

$$\frac{W'+W_1+W}{2}-t' \quad \text{and} \quad \frac{W'+W_1+W}{2}+t'$$

when the moving load is carried by the structure.

In the diagrams and in the equations which follow, positive and negative signs indicate tensile and compressive stresses respectively.

Dead-load Stresses.

The stresses due to the dead loads at A and B and the other panel-points and to the vertical effects at A and B of the wind pressure will be the following:

$$AC'' = -\left(\frac{W}{2}-t'+\frac{W_1}{2}\right)\sec\alpha;$$

$$CE'' = -\left(\text{“} \quad \text{“} +\frac{3W_1}{2}\right) \quad \text{“}$$

$$EG'' = -\left(\text{“} \quad \text{“} +\frac{5W_1}{2}\right) \quad \text{“}$$

$$GK'' = -\left(\text{“} \quad \text{“} +\frac{7W_1}{2}\right) \quad \text{“}$$

$$BD'' = -\left(\frac{W}{2} + t' + \frac{W_1}{2}\right) \sec \alpha;$$

$$DF'' = -\left(\text{“} \quad \text{“} + \frac{3W_1}{2}\right) \text{“}$$

$$FH'' = -\left(\text{“} \quad \text{“} + \frac{5W_1}{2}\right) \text{“}$$

$$HL'' = -\left(\text{“} \quad \text{“} + \frac{7W_1}{2}\right) \text{“}$$

$$AB'' = -\left(\frac{W}{2} + \frac{W_1}{2}\right) \tan \alpha.$$

$$CD'' = -W_1 \tan \alpha;$$
$$EF'' = -\text{“} \quad \text{“}$$
$$GH'' = -\text{“} \quad \text{“}$$

$$KL'' = +\left(\frac{W}{2} - t' + \frac{7W_1}{2}\right) \tan \alpha.$$

The difference between the horizontal component in HL and KL'' is $2t'$ tan α, and it acts toward the right.

Moving-load Stresses.

The stresses due to the moving loads at A and B are uniform throughout AK and BL; they are

$$AK' = BL' = \frac{W'}{2} \sec \alpha.$$

Also the moving-load stresses in AB and KL are

$$AB' = -\frac{W'}{2} \tan \alpha.$$

$$KL = +\frac{W'}{2} \tan \alpha.$$

Resultant Stresses.

In all cases t' is to include wind effects on the train only if the moving load is considered.

The stresses caused by the horizontal wind pressure acting through A, B, C, D, etc., are shown in Fig. 11, as has already been noticed. The diagonals sloping similarly to AD are assumed not to be stressed. The diagram explains itself.

The resultant stresses, finally, are to be found by combining the results of the diagram in Fig. 11 with those expressed by the equations already written. They are the following:

$$\overline{AC} = -\left(\frac{W+W'}{2} - t' + \frac{W_1}{2}\right) \sec \alpha;$$

$$\overline{CE} = -\left(\text{“} \quad \text{“} + \frac{3W_1}{2}\right) \text{“} + CE;$$

$$\overline{EG} = -\left(\text{“} \quad \text{“} + \frac{5W_1}{2}\right) \text{“} + EG;$$

$$\overline{GK} = -\left(\text{“} \quad \text{“} + \frac{7W_1}{2}\right) \text{“} + GK;$$

$$\overline{BD} = -\left(\frac{W+W'}{2} + t' + \frac{W_1}{2}\right) \sec \alpha - BD;$$

$$\overline{DF} = -\left(\text{“} \quad \text{“} + \frac{3W_1}{2}\right) \text{“} - DF;$$

$$\overline{FH} = -\left(\text{“} \quad \text{“} + \frac{5W_1}{2}\right) \text{“} - FH;$$

$$\overline{HL} = -\left(\text{“} \quad \text{“} + \frac{7W_1}{2}\right) \text{“} - HL;$$

$$\overline{AB} = -\tfrac{1}{2}(W+W'+W_1) \tan \alpha - AB;$$

$$\overline{CD} = -W_1 \tan \alpha - CD;$$
$$\overline{EF} = - \text{ “ } \text{ “ } - EF;$$
$$\overline{GH} = - \text{ “ } \text{ “ } - GH;$$

$$\overline{KL} = + \left(\frac{W + W'}{2} - t' + \frac{7W_1}{2} \right) \tan \alpha - KL.$$

The resultant stresses, however, will not ordinarily be combined by simply adding as in the above series of equations, but the specifications for each case will indicate the mode of combination.*

It is not necessary to reproduce the stresses in the oblique web members, since they can be scaled directly from Fig. 11.

All the stresses may be checked by the method of moments in the usual manner, and such checks should always be applied.

The two reactions R and R' are the following:

$$R = \tfrac{1}{2}(W + W' - 2t' + 8W_1) + R_1;$$
$$R' = \tfrac{1}{2}(W + W' + 2t' + 8W_1) + R_1'.$$

It is to be remembered that R_1' is to be taken as *positive* in these expressions; also that $R_1' = -R_1$, as shown in Fig. 11.

Friction at Foot of Bent and Other Special Matters.

The lateral force F_1 to be resisted at the foot of the bent by friction or some special device is the total wind pressure against the train, truss, and bent.

If f' is the coefficient of friction at K and L, Figs. 9 and 12, the lateral resistance of friction offered at K is $f'R$,

* See the specifications of Arts. 49 and 51.

and that at L, $f'R'$. It is supposed that both the reactions R and R' are *upward*, also that both coefficients of friction are the same.

The expression for stress KL has been written on the assumption that all frictional resistance is exerted at L. Strictly, however, the stress in KL may be taken as

$$\overline{KL}_1 = \overline{KL} - f'R,$$

always supposing, numerically, $\overline{KL} > f'R$.

The circumstances of particular cases frequently require calculations to be made with the structure free of moving load, as well as covered with it.

The wind has been taken in but one direction only, but the pier is to be designed for both directions.

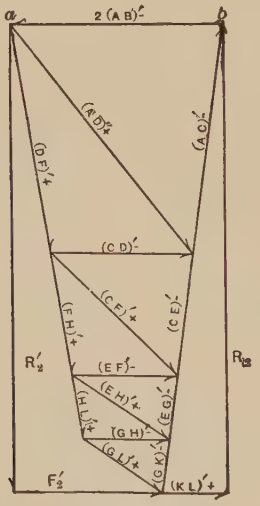


FIG. 13.

It should be stated that $2t'$ tan α may be treated as a single force acting toward the left and along AB, Figs. 9 and 12. It will then give rise to the diagram in Fig. 13,

which shows all the stresses produced by its action. In that figure ad represents $2t' \tan \alpha$. In such a treatment of the question, cd, Fig. 11, would represent $H + P_1''$ added to the half-panel pressure at A. The resultant stresses would then be found by combining the results of the two diagrams with those of the equations. All the results of these two methods will not agree; the latter will give the greatest. This ambiguity cannot be avoided, for it results from the fact that a section through the bent cannot be taken so as to sever three members only.

Art. 133. Double-track Viaducts.

Mr. J. A. Powers, C.E., has called attention to the fact that the web members of a braced tower carrying a double-track railway similar to that shown in Fig. 14

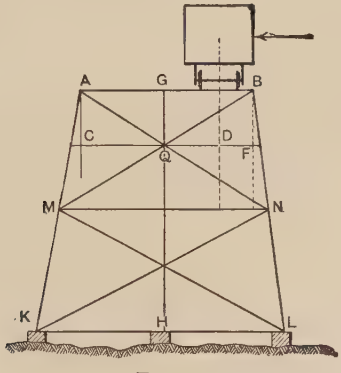


FIG. 14.

will receive their greatest stresses with the windward track only loaded.

The vertical member GH may be supposed to carry its proper proportion of the load which rests on each track. This supposition, however, does not affect the statement made above.

Let the wind have the direction shown by the arrow, and let W', as before, represent the moving weight resting on GB, while w' is that part of W' which is carried to B. If GH acts,

$$w' = \frac{QD}{QF} W'.$$

If GH does not act,

$$w' = \frac{CD}{CF} W'.$$

In the latter case the beam AB will carry $w_1' = \frac{DF}{CF} W'$ to A.

If the angle $FBN = CAM = \alpha$, the force

$$H' = w' \tan \alpha - w_1' \tan \alpha$$

will act along AB as an unbalanced horizontal one. If GH acts, $w_1' \tan \alpha$ becomes equal to zero.

Then in the preceding investigation, there is to be put $(H + H')$ for H, while w' is to be taken as acting vertically down at B, and w_1' or o (as the case may be) at A. The preceding methods and diagrams remain exactly the same as before.

In the formulæ, however, w_1' or o is to be put for the $\frac{W'}{2}$ at A, Fig. 12, and w' for that at B in the same figure. Nothing else is changed.

If W rests on AG and GB at the same time, a horizontal force equal and opposite to H' is developed at A, Fig. 14. Hence H' will be balanced and disappear.

If W' rests on AG alone, with the direction of the wind remaining the same, H' will change its direction, thus giving much smaller web stresses than those existing with W' on GB alone.

If, for any reason, the load on a single-track bent does not rest over its center, H' will have a definite value, and the above considerations must govern the determination of the web stresses. This condition may exist if it becomes necessary to place braced piers under a single-track railway curve.

Art. 134. Stresses Caused by Tractive Force.

The stresses caused by the traction, or pull, of the locomotive, in the members of a braced pier, are simple in character and easily determined.

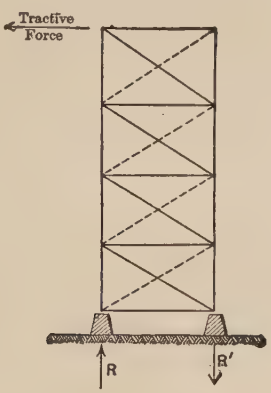


FIG. 15.

In such a case, the tower, as shown in Fig. 15, is simply a cantilever with the traction, or pull, as a single force acting at its extremity. The traction acts along the line of the rails, and the length of the cantilever is the height of the pier. The stresses thus determined are to be combined with those already found.

CHAPTER X.

DETAILS OF CONSTRUCTION.

Art. 135. Classes of Bridges—Forms of Compression Members—Chords Continuous or Non-continuous.

TRUSS-BRIDGE structures are divided into two classes, those with "pin-connections" and those with "riveted connections." In the former class the connection of web members with the chords or with each other is made by a single pin only, as in Fig. 1 (the figure shows simply

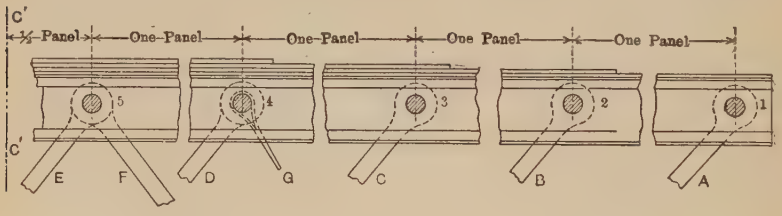


FIG. 1.

the upper-chord and tension web members together with one end post). The pins are shown at 1, 2, 3, 4, and 5 where the tension members join the chord. In the latter class the connections mentioned are made by means of rivets, as shown in Fig. 2. In that figure *A* is a tension, and *B* a compression web member, while *CDE* is a portion of the lower chord.

Screw connections for tension web members and simple abutting connections for compression ends have been used, but are not usually employed at present. A screw con-

nection is formed by passing a tension member through the chord at one of the joints and placing a nut upon

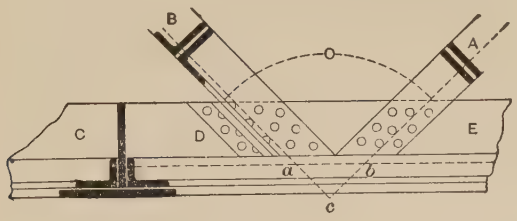


FIG. 2.

the end of it; or again, as in Fig. 3. The method can only be conveniently used when the tension members are of circular cross-section.

An abutting connection for a compression web member is formed by simply abutting either end against the chord,

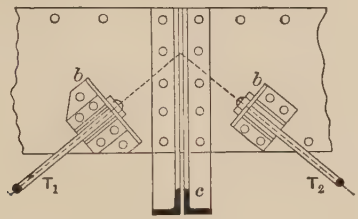


FIG. 3.

which is properly formed for the purpose at the joint. The end of the post or strut may be inserted in the chord, or else a projection of the chord passes into the end of the post; or, again, some simple device is employed for the purpose of keeping the ends of the post in position, and for nothing else, as the entire compressive stress in the post or strut is transmitted through the abutting surfaces.

Occasionally screw, abutting, and pin connections have been combined in a single bridge.

The various forms of cross-sections which have been

used for upper chords and posts, and compression members generally, are almost innumerable. The result always to be kept in view is to place the material as far as practicable from the neutral axis. Figs. 4, 5, 6, and 7 show methods of building up the upper chord, consisting of riveting an upper cover-plate to a pair of channel bars, or to other plates and angles.



FIG. 4.

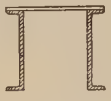


FIG. 5.

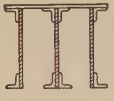


FIG. 6.



FIG. 7.

Chords are continuous or non-continuous according as they are built up in a continuous manner from end to end, or built up of panels abutting against each other at the panel-points. The former are principally used.

Tension members are always of rectangular or circular section. Fig. 16 is a lower-chord "eye-bar." In writing of channel bars, I beams, Z bars, angles, square or round bars and rods, they are indicated as follows: ⊏, I, Z, L, □, O; that is, by skeletons of their sections.

Art. 136. Cumulative Stresses.

Stresses are said to be cumulative in any part of a structure when they are transmitted through that part to other parts, whose whole duty is to sustain them, the part in question being subject at the same time to its own stress. The member in which the stresses are cumulative is, therefore, overstrained to some extent in some one or more portions of it. The two channel bars in Fig. 1 are the portions of that upper chord which are subjected to cumulative stresses. If $C'C'$ is supposed to be the center line of the span, the compressive stress in the chord in-

creases as $C'C'$ is approached from either end in consequence of the components in the direction of the center line of the chord of the stresses in the inclined ties A, B, C, etc. If this increase of stress were provided for by riveting plates to the upper flanges of the two channels, it is evident that those plates would receive their stresses indirectly through the channels. Since the latter are supposed to sustain their own share of direct compressive stress, it is plain that the material in the vicinity of the front of the pin (looking from the center of the pin toward the center of the bridge) would be subjected to a much greater intensity of compressive stress than should exist in the structure. This relates only to the material in front of the pin, and that is the only vicinity in which cumulative stresses would exist if the plates could be so securely riveted to the ⊏s that the whole chord could be depended upon to act as one piece. In practice, however, no such riveted work exists. The ⊏s must inevitably yield to some extent before they would bear sufficiently on the rivets to give to the plates their proper share of the stress. The result would be that not only the material in front of the pin but the whole of the ⊏s, would be overstrained by these cumulative stresses. The only remedy is so to proportion the chord that those parts which are designed to sustain stress shall receive it immediately, and not indirectly through some other part.

Fig. 12, page 472, shows a method of accomplishing this object. The plate *ab* is a light one riveted to the top flanges of the ⊏s, and extends throughout the whole length of the chord. The increase of the areas of cross-sections are obtained by riveting plates to the flat sides of the ⊏s, and by adding an ⌶, if necessary, as shown. The parts of the chord thus receive stresses directly from the pins, and cumulative stresses are obviated.

It is also evident that if the stresses are applied to the

centers of gravity of the cross-sections, or parts of the cross-section, no cumulative stresses will exist.

Cumulative stresses are as liable to occur in riveted connections as in pin-connections; in fact, more so. It may be said to be impracticable so to construct riveted work that cumulative stresses will not exist, and in this respect pin-connections have the advanatge of riveted connections. Fig. 2 illustrates the matter for a riveted chord, although the advantage is not material. The plate C is common to the whole chord, and all web members are riveted to it, as shown by A and B, so that before the rivets can take their share of the stress in transferring it to the plate and Ls E and D, it (the plate C) will necessarily yield to such an extent that cumulative stresses will exist throughout its whole length to a greater or less degree.

Art. 137. Direct Stress Combined with Bending in Chords.

If direct stress is not applied to the centers of gravity of the ends of a piece subjected to compression, it is clear that bending must take place.

In Figs. 1 and 8 the horizontal components of the oblique

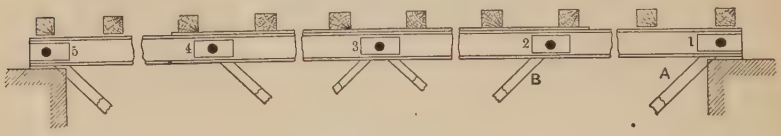


Fig. 8.

forces in the ties A, B, etc., do not act through the centers of gravity of the sections of the chord, hence there must be bending in those chords. If the chords were perfectly straight, and if the centers of the pin-holes were all at the same distance from the centers of gravity of the different cross-sections, as well as in the same straight line, the total direct stress to which the chord is subject at any

section would produce bending at that section, and the lever-arm would be the same for all sections. Camber and deflection from loading, however, so complicate the matter that it is quite impossible to make even a satisfactory approximate computation of the chord bending arising in this manner. All chords in compression, therefore, should be so designed that the axes of the pins may traverse the centers of gravity of their sections, or very nearly so, even though the ties rest directly on the upper chord. It is clear that when this bending exists to a material extent, the proper distribution of direct stress is greater disturbed, though to an indeterminate extent, and is, except in rare cases, faulty construction.

It is evident that if cumulative stresses occur, flexure must necessarily exist, for the simple reason that the direct stress is not uniformly distributed over the cross-section of the chord.

Although this flexure is indeterminate in amount and shows a faulty design, the attempt to utilize it has sometimes been made, and the analysis on which the practice was based will now be given, it being premised that the ties are supposed to rest directly on the chords, as shown in Figs. 8 and 9.

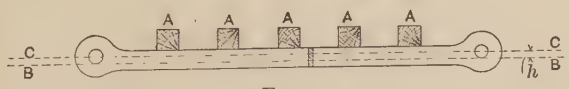


FIG. 9.

Suppose Fig. 10 to be an enlarged cross-section of the chord in Fig. 8, and let *fg* pass through the center of gravity of the cross-section parallel to *ba* and *mn*. Since the increment of the chord stress transmitted through the pin from the ties *AA* is applied to the cross-section of the chord at a distance *h* below the center of gravity, there will be an excess over the mean intensity of stress in the

cross-section at the lower side EE, and a deficiency at the upper side ab.

Let P be the increment of direct compressive stress given to the chord by the ties AA, and let P_1 be the total direct compressive stress in the section, and let S represent the area of the cross-section.

The variation of the intensity of stress from the mean is due to the moment Ph, and since this moment is constant for all points between any two pins, as 1—2 or 2—3,

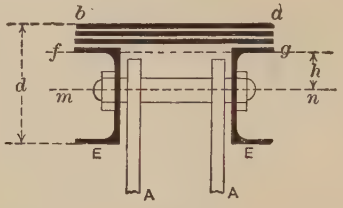


FIG. 10.

Fig. 8, the variation in intensity is also constant between these points.

The moment $Ph = \frac{KI}{d_1}$, in which d_1 is the distance from fg to EE, and K the intensity of stress at EE due to bending, gives, therefore,

$$K = \frac{Phd_1}{I}.$$

The intensity of stress at ab due to bending is

$$\frac{d - d_1}{d_1} K.$$

The total intensity of stress at EE, Fig. 7, is equal to $\frac{P_1}{S} + K$, and that at ba, $\frac{P_1}{S} - K\frac{d - d_1}{d_1}$; the intensity at $fg = \frac{P_1}{S}$, whatever may be the figure of the cross-section.

The intensity at any point of the section may easily be found from K by a simple proportion, and the total intensity by adding that to $\frac{P_1}{S}$.

Deck-bridge with Load Directly on Upper Chord.

If the load rests directly on the upper chord, supposed to be non-continuous at the panel-points, although not so shown in Fig. 8, it will produce tension at the lower side of the chord and compression at the upper by simple flexure, an opposite tendency to that exerted by the moment Ph.

The moments due to the moving load on any panel vary (that is, increase) from the joints to the middle point of the panel, where the moment is a maximum. Denote by K' the greatest intensity of the tensile stress caused by the moment of the moving load, then

$$K' = \frac{d_1 M}{I},$$

in which M expresses the greatest moment of the applied load.

For a uniform load

$$M = \frac{wl^2}{8},$$

and it exists at the center of the panel. Ordinarily, the chord would be required to resist the bending moment expressed by M, but h may be so chosen that for its maximum value $K = K'$, and then no extra metal will be required on account of the flexure produced by the moving load. This value of h is found as follows:

$$\frac{Phd_1}{I} = \frac{d_1 M}{I}; \quad \therefore h = \frac{M}{P}.$$

The value of h, therefore, is independent of the form of cross-sections. If $K<K'$, additional material will be needed in order to prevent an excess of compressive stress at the upper part of the chord and a deficiency at the lower side. When $K>K'$, there is an excess of compressive stress at EE, Fig. 10.

This method of neutralizing the flexure produced by the direct application of the moving load to the chords is unsatisfactory in many ways and should never be used. At all places in the panel except the center the metal is still overstrained by the flexure due to its own stress, and in the vicinity of the panel-points this condition exists to a serious extent.

If the chord is continuous, the objections to the method already mentioned gather increased force. At and near the ends of the panels the fixed and moving load produce flexure in the same direction as the direct-chord stress, and to twice the amount found at the center. It is not necessary, therefore, to consider this case farther.

A considerable saving of material can be effected by placing the ties directly on the upper chord of a deck-bridge, and with a proper design it in no manner conflicts with the best practice. *In all cases the axis of the pin should traverse the center of gravity of the chord section*, or as nearly so as practicable, in order, if possible, to eliminate all flexure due to the direct-chord stress. The chord section should then be so formed that the combined stresses due to flexure in the exterior fibers, and the direct-chord stress shall at no point exceed a proper value per square unit. This value may be taken at 10,000 to 12,000 pounds per square inch for mild-steel upper chords. These values may be taken comparatively high for the reason that a small portion only of the material is subjected to these intensities, and that small portion is well supported against

fatigue by the material about it, which is considerably understrained.

If the notation previously used in this article be still maintained, the maximum external moment M will develop in the most remote fibers at the distance d_1 from the neutral axis the intensity

$$K' = \frac{d_1 M}{I}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If, on the other hand, P_1 is the total direct stress in the chord and S the area of cross-section, while p is the greatest allowable combined stress, then there will result

$$p = \frac{d_1 M}{I} + \frac{P_1}{S}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Eq. (2) shows that in the most efficient design the moment of inertia I of the section must be the greatest possible. At the same time considerations affecting the joint details render it advisable that the center of gravity should lie not far from the mid-depth of the section. These two conditions are fulfilled by placing large portions of the material, and as nearly as possible in equal amounts, at the top and bottom of the chords, as shown in Fig. 11.

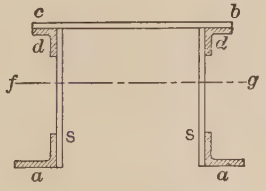


FIG. 11

The cover-plate bc and angles dd are made as light as the circumstances of proper design will permit, but the angles aa are made as heavy as possible. An unequal-legged angle is a good one for aa with the longest leg horizontal,

and it is sometimes necessary to rivet a narrow plate to those horizontal legs in order to properly balance the section. The center of gravity line *fg* will usually lie a little above the center of figure.

As the center of the span is approached from the end the chord section must be increased, but *in no case should that increase be made by thickening the cover-plates or increasing their number*, as such an operation inevitably means cumulative stresses or flexure by direct stress. In rare cases it may be admissible to slightly thicken the cover-plate, if there is but one, but, as a rule, Fig. 10 shows a design to be carefully avoided. All increase of section should be obtained by thickening the side of the web plates, or increasing their number; or, again, by increasing the angles, or, finally, by introducing an interior I beam, as shown in Figs. 6 and 12

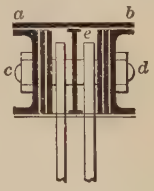


FIG. 12.

If the chord is non-continuous, M is simply the bending for a span equal in length to a panel and due to the track load, own weight, and superimposed moving load, and is easily determined. If the chord is continuous, on the contrary, the analysis for the moving-load bending is not simple. The greatest bending moment for this case, however, may properly and safely be taken at three fourths its value for a non-continuous chord.

The upper-chord section required in the case of combined bending and direct stress is readily found by the aid of eq. (2). The radius of gyration r can be easily and with sufficient accuracy predetermined; so that Sr^2 can

be put for I in that equation. After that substitution is made, there at once results

$$S = \frac{1}{p}\left(\frac{d_1 M}{r^2} + P_1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This is a convenient formula for practical use. It will generally be found with the sections of upper chords ordinarily employed that the radius of gyration r will vary nearly from one third to three eighths of the depth of the chord. If, therefore, as in Fig. 12, d is that depth, there may be taken approximately

$$r = 0.35d.$$

This is sufficiently close for a tentative value, and it will frequently be close enough for a final value.

Art. 138. Combined Bending and Direct Stress in General.

There are a number of practical problems of combined flexure and direct stress of sufficient importance to merit careful examination of their general theory, and among them is the flexure of long columns or posts. In this place the particular cases to be considered are those in which the bending is produced by a uniform load at right angles to the axis of the member, or by eccentricity of longitudinal loading, the direct stress (or external force) being applied in a direction parallel to the same axis. Lower-chord eye-bars and other horizontal or inclined members of either upper or lower chords of bridges belong to this class.

Let M_1 represent the bending moment in the member at that section where the deflection is greatest, produced by loading at right angles to the member's axis or by eccentricity in the application of the longitudinal loading, or by any other means; let w' represent the greatest deflection resulting from the total bending moment and

direct stress; also, let P be the total direct stress acting upon the member whose length is l, while k represents the greatest intensity of the stress due to bending alone and at a distance d_1 of the most remote fiber from the neutral axis of the section at which the deflection w' is found. Finally, let A be the area of cross-section of the member which, together with the moment of inertia I, is supposed to be constant throughout the entire length; and let $q = \frac{P}{A}$, the intensity of uniform stress in the member due to the direct stress or force P.

The resultant maximum bending moment in the member will then be

$$M = M_1 \pm Pw'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If P is tension, it will tend to pull the member straight, thus producing a moment opposite to M_1. In the second member of eq. (1), therefore, the negative sign is to be used for a member in tension and the positive sign for a member in compression.

The greatest resultant intensity of stress, t, in the member will then take the value

$$t = \frac{P}{A} + \frac{Md_1}{I} = \frac{1}{A}\left(P + \frac{Md_1}{r^2}\right). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The quantity r is the radius of gyration, so that $I = Ar^2$.

When the intensity t is prescribed, the required area of section A is

$$A = \frac{1}{t}\left(P + \frac{Md_1}{r^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

These equations are perfectly general and applicable to all cases of combined bending and direct stress. They will be applied in subsequent articles to compression members and eye-bars.

Art. 139. Usual Approximate Method Employed for Combined Bending and Direct Stress in Columns or Posts.

If the ordinary approximate method demonstrated in Art. 145 be employed, eq. (4) of that article is immediately applicable, using the minus sign in the denominator, P being the total direct stress of compression and M_1 the bending moment due to a uniform transverse load or to eccentricity of the line of action of P, if there be any. The greatest intensity of bending stress as represented by that formula would then be

$$K = \frac{M_1 d_1}{I - \frac{Pl^2}{10E}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this equation, d_1 is the distance from the neutral axis of the section to the extreme fiber in which the intensity k exists.

If e be the eccentricity of the line of action of P, and if W be the weight of the compression member whose length is l,

$$M_1 = \frac{Wl}{8} \pm Pe. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

When the moment of P produces bending of the same sign with the transverse load W, the plus sign is to be used in eq. (2), and the minus sign when those moments are opposite. If the line of action of P coincides with the axis of the member, the moment Pe disappears from eq. (2). Again, if the member is vertical, so that there is no transverse bending due to the load W, when the line of action of P has the eccentricity e,

$$M_1 = Pe. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This latter case exists very frequently in the columns of the buildings.

Eq. (1) is thus seen to represent the greatest intensity of bending stress with M_1 taken from either eq. (2) or eq. (3) for the cases of transverse loading, no transverse loading, eccentric longitudinal loading, or any combination of those cases.

The resultant intensity of stress, i.e., the greatest intensity of compressive stress in the entire compression member, will be

$$t = \frac{P}{A} + \frac{M_1 d_1}{I - \frac{Pl^2}{10E}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

As A is the area of cross-section, $I = Ar^2$, r being the radius of gyration of the cross-section of the compression member. If $q = \frac{P}{A}$, eq. (4) will take the form

$$t = \frac{P}{A} + \frac{M_1 d_1}{Ar^2 - \frac{Pl^2}{10E}} = q + \frac{M_1 d_1}{Ar^2 - \frac{Pl^2}{10E}}. \quad . \quad . \quad . \quad (5)$$

In the use of this equation the intensity q must obviously never exceed the working value given by the column formula employed. Indeed, if there is suitable eccentricity q may be much less than that working long-column value.

In practical operation the principal use of eq. (5) may be the determination of the area of cross-section A with some prescribed value of t.

It is usually feasible to assign general outside dimensions of the proposed column section and that will enable a close approximate value of r to be assigned. If, at the

same time, an approximate value of q may also be taken, the resolution of the first and third members of eq. (5) will at once give

$$A = \frac{P}{10E}\frac{l^2}{r^2} + \frac{M_1}{t-q}\cdot\frac{d_1}{r^2}. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If, on the other hand, such an assignment of q may not be made, it will be necessary to solve the first and second members of eq. (5), as a quadratic equation, for A. Bringing both terms of the second member of eq. (5) over a common denominator and solving the resulting equation of the second degree in the usual manner, the following general value of A will be found:

$$A = \frac{1}{2}\left(\frac{P}{10E}\frac{l^2}{r^2} + \frac{P}{t} + \frac{M_1 d_1}{tr^2}\right) \pm \sqrt{\frac{1}{4}\left(\frac{P}{10E}\frac{l^2}{r^2} + \frac{P}{t} + \frac{M_1 d_1}{tr^2}\right)^2 - \frac{P}{10Et}\frac{l^2}{r^2}}. \quad . \quad (7)$$

Frequently there may be written $d_1 = \frac{h}{2}$ and $r = 0.4h$. Hence

$$\frac{d_1}{r^2} = \frac{3}{h} \text{ (nearly).}$$

If, again, $d_1 = \frac{h}{2}$ and $r = 0.35h$,

$$\frac{d_1}{r^2} = \frac{4}{h} \text{ (nearly).}$$

The preceding values of the radius of gyration r represented in terms of the depth h of the compression member are closely approximate for practical design work.

Eqs. (6) and (7) will give the desired area of section of the compression member carrying both direct stress and bending produced by transverse loading under the

assumptions of the method ordinarily employed. Those formulæ are sufficiently accurate for their purposes, but it may be desirable to use the more exact formulæ given in the next article.

Art. 140. Exact Method for Combined Compression and Bending.

The exact procedure for combined compression and bending is identical with that demonstrated in Art. 146, the formulæ determined there simply being adapted to a compressive longitudinal force instead of a force of tension. It is to be observed, as in the case of the tension member, that the compression member may be horizontal or inclined, so as to be subjected to bending either from its own weight or from some other form of loading in addition to its own weight. The member may also be subjected to uniform bending throughout its length by the eccentric application of the longitudinal force P concurrently with the preceding cross-bending, or, as in the case of a vertical column carrying eccentric loading, by that force P alone.

It is essential to recognize in this connection that while the columns may occasionally be in the pin-end condition, usually their ends are essentially in a condition of at least partial fixedness, although the degree of fixedness is indeterminate. It will conduce to simplicity of treatment if the transverse bending, either from distributed loading or by the eccentricity of application of the column load, be treated as if the ends of columns are hinged. It may readily be shown that the center deflection of a beam of given length and cross-section with ends simply supported and with the loading uniformly distributed is five times as great as when the ends of the same beam are fixed. In the following analysis, therefore, the

bending from both the sources named may be considered as produced in a column with hinged ends by a total uniformly distributed load W, sufficient in amount to cause one fifth of the actual bending moment acting on the column with ends fixed. In this manner the fixed or constrained end condition of the actual column is provided for, while the simplicity of the hinged-end computations is retained. The bending moment produced by P, acting with the lever-arm of the greatest deflection, will concur with the bending moment produced by the own weight of the member or other vertical uniform loading instead of being opposed to it, as in the case of the tension member of Art. 146. The work performed, therefore, by P and the uniform loading W will be equal to the resilience or elastic work performed in the member in changing the deflection from w_1 to w', it being remembered, in this case, that w' may be less than w_1. Under these conditions, then, eq. (10) of Art. 146, expressing the work done on the beam in changing the deflection from the w_1 to w', will become the following, the second member representing the resilience or the work done by the elastic stresses throughout its volume:

$$\frac{8}{3}\frac{P}{2}\left(\frac{w'^2 - w_1^2}{l}\right) + \frac{8}{25}W(w' - w_1) = \frac{W^2 l^3}{240EI}\left(\frac{w'^2}{w_1^2} - 1\right). \quad (1)$$

Dividing both members of this equation by $(w' - w_1)$, then solving for w', the following value of the latter will immediately result:

$$w' = \frac{w_1}{\frac{6}{25}\frac{Wl}{P}\frac{1}{w_1} - 1}, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which

$$w_1 = \frac{5Wl^3}{384EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Having found the deflection w', the general equation for the resultant maximum bending moment, eq. (1) of Art. 138, will take the following form, in which the coefficient c is introduced to provide for the end conditions. If the ends are hinged, corresponding to the end condition of a beam simply supported, $c=1$, but if the ends are fixed, c may be taken as 0.5.

$$M=c\left(\frac{Wl}{8}\pm P(e\pm w')\right), \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In this equation care must be exercised in using the double signs, observing that both plus signs are to be taken together as are both minus signs; also, that the eccentricity e in a vertical column is taken in a direction opposite to the deflection w', in which case e is to be considered positive and the lever-arm of P is $(e+w')$. In the upper chord of bridges e may be given such value that

$$M=\frac{Wl}{8}-P(e-w')=0 \text{ (nearly)}. \quad . \quad . \quad . \quad (5)$$

For reasons already explained it is usually not advisable to resort to this procedure.

In the case of vertical columns like those in buildings, ordinarily the term $\frac{Wl}{8}$ disappears, leaving the bending moment in the column.

$$M=P(e+w'). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In the great majority of cases w' is so small in comparison with e as to make it negligible, so that

$$M=Pe. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

These various values of the bending moment M cover all that usually occur in practical operations.

If, in accordance with the preceding notation, t is the maximum resultant intensity of stress in the member, there will result

$$t = \frac{P}{A} + \frac{Md_1}{I} = \frac{1}{A}\left(P + \frac{Md_1}{r^2}\right). \quad . \quad . \quad . \quad (8)$$

Evidently the uniform intensity of compressive stress $\frac{P}{A}$ must not exceed the intensity of working stress given by a suitable long-column formula. When the greatest working intensity t is prescribed, the desired area of cross-section of the compression member will be

$$A = \frac{1}{t}\left(P + \frac{Md_1}{r^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The closely approximate values of $\frac{d_1}{r^2}$ given immediately following eq. (7) of Art. 139, may be used in a precisely similar manner in the above eq. (9), so as to simplify the practical use of that equation.

Art. 141. Riveted Joints and Pressure on Rivets.

In riveted bridge work the pitch of rivets (i.e., the distance from center to center) should not be less than about three diameters; if possible it should be from four to eight diameters of the rivet, provided that value does not exceed about fourteen to sixteen times the plate thickness. The diameter of the rivet is determined by the amount of stress which the joint is to carry, so that the intensity of pressure against the surface of the rivet in contact with the plate shall not exceed a given value. If the rivet and hole were in perfect contact, this intensity

could easily be found, having given the amount of stress which the rivet is to carry, but such is never the case. The only resort left, therefore, is to assume that the rivet does fit perfectly, and fix a low enough value for the intensity of pressure against its surface to make the joint safe.

In Fig. 13 suppose EF to be a part of a plate in which is drilled or punched the rivet-hole $ADBK$, and suppose the stress to be exerted on the plate in the direction of the arrow at K, then the surface of contact between the plate and rivet will be projected in ADB. Contact will not take place throughout the whole semi-circumference

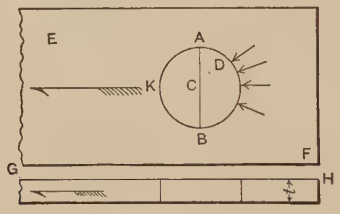


FIG. 13.

when the plate is not subjected to stress, unless the rivet fits the hole with absolute accuracy.

Since all material is elastic to some degree, there will be a surface of contact when the plate is subject to stress, even when the rivet does not accurately fit thehole, and this surface will evidently increase with the stress in the plate.

If it be supposed that the rivet exactly fits the hole, the pressure on the surface of contact, ADB, will be of uniform intensity, and the case will be similar to that of fluid pressure on a cylinder. Let p denote this intensity whose direction is normal to ADB at every point (friction is omitted from consideration), then the total pressure in the direction of the arrow at K exerted by the plate on the rivet for each unit of length of the latter is equal to pAB.

Let t be the thickness of the plate, as shown, and let d be the diameter AB, then the total pressure against the rivet in the direction of the arrow is

$$P = ptd.$$

The quantity p is the greatest mean value of the intensity of compressive stress which it is desirable to put upon the material under the given circumstances. It is usually taken as 15,000 to 20,000 pounds per square inch for steel. The actual maximum value of the intensity immediately in front of the center, C, is much greater than either 15,000 pounds or 20,000 pounds. If T' is the amount of stress which the joint is required to carry, the number of rivets, so far as the previous consideration is concerned, is equal to

$$\frac{T'}{ptd} = n.$$

The riveted joint itself, as shown in Fig. 14, may now be examined. The distance c should be at least $1\frac{1}{2}$ to 2

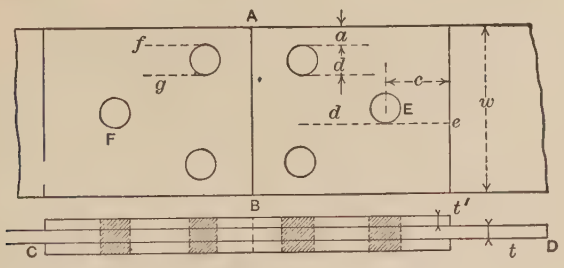


FIG. 14.

diameters of the rivet. By the arrangement of the rivets shown, when the pitch is from four to eight diameters, the strength of the plate of the width w may be decreased only by about the amount of metal taken out in one rivet-

hole, although experiments to settle this point definitely are wanting.

After having determined the pitch and distance of c, as above, there are five methods of rupture of the joint only which need serious attention. These five are: (1) tearing of the plate through the rivet-hole E; (2) tearing of the cover-plates through the rivet-holes at the middle of the joint, two in the figure, (3) shearing of the rivets, (4) and (5) rupture by compression at the surface of the contact between the rivets and the plates. The safe shearing stress to which rivets are subjected in bridge structures is usually taken at 9,000 to 10,000 pounds per square inch. This gives a safety factor of from 5 to 6.

If S is the intensity of the maximum safe shearing stress on rivets (10,000 pounds for steel), p the intensity of the maximum compressive stress (15,000 to 20,000 for steel), and T the maximum working tensile stress; also, n' the number of rivets on the line through the middle of the joint (two in the figure), while t and t' represent the thickness of the plate and covers as shown, then equal liability to rupture in the five ways mentioned is expressed as follows:

$$Tt(w-d) = 2Tt'(w-n'd) = \frac{\pi nd^2}{4} \cdot 2 \cdot S = ntdp = 2nt'dp = T'.$$

It almost always happens that these quantities are not each equal to T', but none of these should be less. If only one cover-plate is used, the $2Tt'$ should be replaced by Tt', $2S$ by S, and $2nt'$ by nt'. The form of this equation of condition may be somewhat changed by piling of the plates, etc., but it will remain essentially the same and serves to illustrate the principle which must govern in all cases.

It is seen, therefore, that in obtaining the amount of pressure which should be put upon a rivet, p must be

multiplied by its diameter, and not by the semi-circumference ADB, Fig. 13.

Art. 142. Riveted Connections between Web Members and Chords.

When web members, as A and B, Fig. 15, are riveted to the chord, the center line (i.e., the line joining the centers of gravity of the sections of the members) should pass through the center of gravity of a system of points situated at the centers of the rivet-holes; otherwise the intensity of stress in any section of the member will not be uniform, and it (the web member) will be subjected to flexure. It is supposed that each rivet carries the same amount of

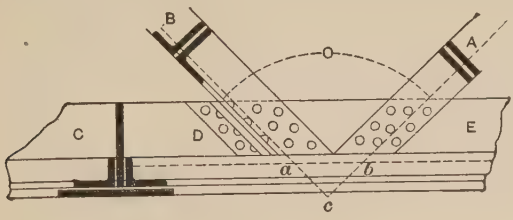


FIG. 15.

stress. This is probably seldom true, but it is the best assumption that can be made. Fig. 15 represents a proper distribution of rivets in reference to the center lines Ac and Bc.

It will be observed that the strut, composed of two unequal-legged angles, has its connection with the chord through both legs by means of angle lugs. This should always be done in similar cases, for in no other way can an angle-brace develop its full strength. The practice of riveting single legs, only, of angle-braces to chords is highly objectionable, for the reason that the actual resistance of the brace is far below the nominal.

When three or more pieces are riveted together at the same joint, all the center lines of stress should intersect at one point if flexure is to be avoided.

If T is the total tension in the member A of Fig. 15, and C the compression in B, then there will be developed at a the bending moment

$$T \times ac \sin \theta,$$

and at b the bending moment

$$C \times bc \sin \theta,$$

it being supposed that ac, bc, and ab are the center lines of stress of the two members and chord.

It is true that the metal is well supported in the vicinity of the joint, but unless provision is made for the flexure, as shown in Art. 138, which is seldom or never the case, some of the metal will be overstrained. Hence this flexure should always be made a minimum and reduced to zero if possible.

In pin-connections this bending at the joints is entirely obviated when all center lines of stress intersect at the center of the pin.

Art. 143. Eye-bars, or Links.

An "eye-bar," or "link," is a tension member of a pin-connection bridge, fitted at each end with an eye for the insertion of a pin. Two views of an eye-bar are shown in Fig. 16; A is the body of the bar, D the neck, and C the eye. The head of an eye-bar is the enlarged portion in which the pin-hole is made. The eye-bar is one of the most important members of a pin-connection bridge, and

the determination of the relative dimensions of the head has been the subject of much experimenting. A mathematical investigation, however, with the same object in view is a matter of considerable complexity, although an approximate solution of the problem may be obtained,

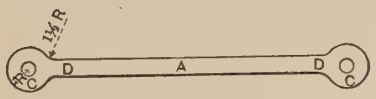


FIG. 16.

and its agreement with the results of experiment is quite close.

Before taking a general view of the stresses which may arise in an eye-bar head, it must be premised that a difference of $\frac{1}{32}$ inch to $\frac{1}{64}$ inch between the diameter of the pin and that of the pin-hole is good practice. Before the eye-bar is strained, therefore, there is a line of contact only between the pin and eye-bar head, but on account of the elasticity of the material, this line changes to a surface when the bar is under stress and increases with the degree of stress to which the bar is subjected. The line and surface of contact is in the vicinity of K, Fig. 17, i.e.,

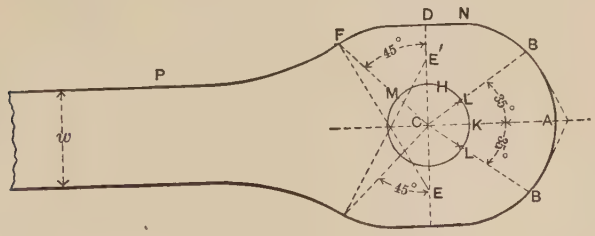


FIG. 17.

on that side of the pin toward the nearest end of the bar. The consequence of this is that, when the bar is strained, the portion about KA, Fig. 17, is subjected to direct compression and extension; that about BL, DH, and FM to

direct tension and bending, while in the vicinity of CN there is a point of contraflexure, and the stress in the direction of the circumference changes from compression to tension as H is approached from K.

If w represents the width of an eye-bar, as shown, then its thickness, t, is generally included between the limits $\frac{1}{3}w$ and $\frac{1}{8}w$. These limits of the relative values of the quantities are seldom exceeded.

Fig. 18 represents a method of laying down an eye-bar head determined by experiments by a member of the British Institution of Civil Engineers. It was much used

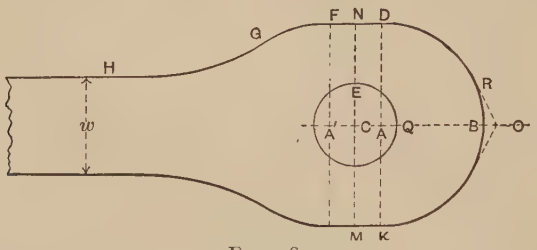


FIG. 18.

in early American bridge practice. Let r represent the radius of the pin-hole and w the width of the bar.

Then take $EN = 0.66w$. The curve $DRBK$ is a semi-circle with a radius equal to $r + 0.66w$, with a center, A, so taken on the center line of the bar that $QB = 0.87w$. GF is a portion of the same curve, with A' as the center ($A'C = AC$); GH is any curve with a long radius joining GF gradually with the body of the bar. HG should be very gradual in order that there may be a large amount of metal in the vicinity of CG, for there the metal is subjected to flexure as well as direct tension. FD is a straight line parallel to the center line of the bar.

Fig. 17 shows another method founded on the results of a mathematical investigation.* Take r and w as before.

* Trans. Am. Soc. C. E., Vol. VI, p. 127.

Then $BC = AC = r + 0.87w$, $DH = \frac{2}{3}w = 0.66w$, $ED = EF = 2r + w$. DF is described with ED until $DCF = 45°$. BAB is described with BC until $BCA = 35°$. BN is drawn from L as a center located in such a position as to cause that arc to be at the same time tangent to DN and AB. DN is a straight line drawn parallel to the axis of the bar. PF is any easy curve which will appear the best. The dotted lines in both Fig. 17 and Fig. 18 show the slope that should be given in order to clear a die.

The outline of the head is now usually formed of a portion of the circumference of a circle whose center is the center of the pin-hole. In such a case no dimension of the head should be less than the corresponding one determined by either of the methods just given. Every large bridge shop now (1905) has its standard methods of forming eye-bar heads and its standard dimensions for bars of various sizes. Those dimensions depend both upon the width or depth of the bar and the diameter of the pin. Fig. 16 illustrates the general outline adopted by the American Bridge Co.

Fig. 19 shows the head thickened in such a manner that the mean maximum intensity of pressure between pin and



FIG. 19.

pin-hole shall not exceed a given amount, p. Let T represent the maximum intensity of tension in the body of the bar; then, as has been shown in discussing the pressure against the bodies of rivets,

$$wtT = 2rpt'; \quad \therefore\ t' = \frac{wtT}{2rp}.$$

Eye-bar heads are now seldom or never thickened.

Art. 144. Eye-bar Subjected to Bending by its Own Weight or Other Vertical Loading.

Let Fig. 20 represent a lower-chord eye-bar of a pin-connected bridge with the length l and carrying the total tension P. The depth of the bar is h and the thickness b, so that the area of the normal section is bh. The bar acts as a beam carrying its own weight as a uniform load over the span l. That load deflects the bar as a beam while the direct stress of tension, P, decreases that deflec-

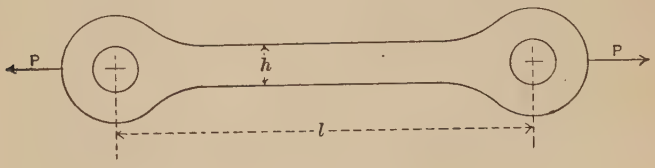


FIG. 20.

tion by tending to pull the bar straight. The problem is to determine the greatest stress in the bar and incidentally its center deflection.

There are several methods of procedure. The first and simplest method is approximate in its results, although sufficiently close for some purposes. It consists in treating the bending and direct stresses as existing independently, so that the results are obtained by simply adding the bending to the direct intensities. This method will be treated first.

The more exact method consists in recognizing the bending moment as the resultant of those due to the transverse load acting on the bar as a simply supported beam, and to the direct stress P acting with the greatest deflection as its lever-arm.

Approximate Method.

Although reference will be made to Fig. 20, the formulæ as written will be equally applicable to compression members in which P would be the total force of compression.

If the total weight of the bar or compression member is W, and if I is the moment of inertia of its cross-section about the neutral axis, while k is the greatest intensity of bending stress at the distance d_1 from the same axis, the theory of flexure gives

$$M_1 = \frac{W \cdot l}{8} = \frac{kI}{d_1}; \quad \therefore k = \frac{Wld_1}{8I} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the area of cross-section is represented by A, while the radius of gyration is r, $I = Ar^2$. Again, the quantity $I \div d_1$ is called the "section modulus," and tabulated values of it for rolled sections may be found in hand-books. Let m be that modulus, then eq. (1) may take the form

$$k = \frac{Wld_1}{8Ar^2} = \frac{Wl}{8m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The intensity of direct tension is

$$q = \frac{P}{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Obviously k will be tension on the lower side of the bar or other member and compression on the upper side. The greatest intensity of stress in the piece will be the sum of q and k. Eqs. (2) and (3) will, therefore, give the value of that greatest tensity, t, of stress as follows:

$$t = q + k = \frac{1}{A}\left(P + \frac{Wld_1}{8r^2}\right) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

When the greatest value of t is prescribed, the required area of section, A, can be at once writted from eq. (4),

$$A = \frac{1}{t}\left(P + \frac{Wld_1}{8r^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In case of an eye-bar with the cross-section bh, $d_1 = \frac{h}{2}$ and $\frac{d_1}{r^2} = \frac{6}{h}$. Hence

$$t = \frac{1}{bh}\left(P + \frac{3}{4}\frac{Wl}{h}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and

$$bh = \frac{1}{t}\left(P + \frac{3}{4}\frac{Wl}{h}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If the bar carries any other uniform load than its own, it is only necessary to make W represent the total uniform load, including the weight of the bar itself.

Finally the direct force P may act with the eccentricity e. In this case the moment Pe produces uniform bending throughout the length of the bar, and it is only needful to write $\left(\frac{Wl}{8} \pm P \cdot e\right)$ for $\frac{Wl}{8}$ in the preceding formulæ, the double sign showing that Pe may act either with or against the moment of the uniform load.

The formulæ of this article are not sufficiently exact for the usual cases of engineering practice.

Art. 145. The Approximate Method Ordinarily Employed.

The method commonly employed in practical work for the treatment of compound bending and direct stress is a much closer approximation than the preceding method,

although not exact. Its chief feature is the introduction of the bending moment produced by the direct or longitudinal force multiplied by the actual maximum deflection. In the same manner the moment due to the eccentricity of the line of action of that force is introduced wherever necessary.

The following expression may readily be found * for the deflection w' at the center of a beam, due to pure bending and in terms of the greatest intensity of bending stress k, a being a constant depending, among other things, upon the distribution of loading:

$$w' = a\frac{kl^2}{Ed_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The same analysis by which eq. (1) is established shows, for a beam simply supported at each end and loaded uniformly, that $a = \frac{5}{48}$, and for the same beam loaded by a single weight only at the center of the span, $a = \frac{4}{48}$. The cases which occur in practice conform nearly to that of a load uniformly distributed over the length l. Hence for such a beam there is ordinarily taken

$$w' = \frac{kl^2}{10Ed_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The moment produced by the direct force or stress P acting with the lever-arm w' will have the opposite sign to that of M_1 (the moment due to transverse loading or to eccentricity) if the member is in tension, but if the member is in compression those two moments will have

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* Eq. (6*a*), Art. 21, of "The Elasticity and Resistance of Materials," by Wm. H. Burr.

the same sign. The resultant equation of moments may, therefore, be written

$$M = \frac{kI}{d_1} = M_1 \pm Pw'. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

As stated, the plus sign is to be used for a compression member and the negative sign for a tension member.

If the value of w', given by eq. (2), be substituted in eq. (3), the following value of k will result:

$$k = \frac{M_1 d_1}{I \mp \frac{Pl^2}{10E}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In eq. (4) the plus sign is to be used for tension members and the minus sign for compression members. This equation is general and adapted to all forms of cross-section under the conditions virtually assumed. Although not explicitly stated, it is essentially assumed that the ends of the member remain absolutely fixed in distance apart. This is frequently not the case, especially in the lower-chord eye-bar of a pin-connected bridge subjected to direct tension and to bending due to its own weight, the bar usually being horizontal.

If the ends of the beam or member, uniformly loaded, are fixed, $a = \frac{1}{10}$, when k is the greatest intensity of bending stress at the mid-point of the member, or $\frac{1}{32}$ if k is the intensity of the bending stress at the fixed ends. One of those values (usually $\frac{1}{32}$) is to be substituted therefore for $\frac{1}{10}$ in the formulæ which follow when the fixed-end condition exists.

The resultant maximum intensity of stress t in the member will obviously be

$$t = k + q, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which equation q is the uniform intensity $P \div A$.

Eq. (4) will be immediately applicable to any particular case by substituting in it the values of I and M_1 for that special case.

If the case of the lower-chord eye-bar mentioned in a preceding paragraph be considered, the total weight of the bar being W, while b and h represent its thickness and depth respectively, $I=\frac{bh^3}{12}$ and $M_1=\frac{Wl}{8}$. These values substituted in eq. (5) will give the desired value of the resultant intensity, as follows:

$$t=\frac{P}{bh}+\frac{W\frac{l}{h}}{\frac{4}{3}bh+\frac{8Pl^2}{5Eh^2}}. \quad \ldots\ldots \quad (6)$$

Eq. (6) gives the value of the maximum intensity of the tension in the extreme lower fibers of the eye-bar when subjected to the total direct tension P and to the bending due to its own weight.

The greatest intensity of bending stress in the bar is evidently the second term of the second member of eq. (6), and it has the following value if the weight of the bar per unit of length is $\frac{W}{l}=g$, or if the weight of a cubic unit of the metal is i:

$$k=\frac{\frac{5}{8}\frac{gE}{b}}{\frac{5}{6}E\frac{h^2}{l^2}+\frac{P}{bh}}=\frac{\frac{5}{8}ih\cdot E}{\frac{5}{6}E\frac{h^2}{l^2}+q}. \quad \ldots\ldots \quad (7)$$

If in the third member of eq. (7), $i=0.286$ pound and $E=29{,}000{,}000$ pounds,

$$k=\frac{5{,}183{,}750h}{24{,}167{,}000\frac{h^2}{l^2}+q}.$$

It is frequently important to observe what depth of bar with a constant area of cross-section, subjected to a prescribed working stress, will give the maximum bending stress due to its own weight when the length is fixed. That depth can readily be determined by taking the first derivative of k, as is given by eq. (7), with h as the variable. By performing that operation and placing $\frac{dk}{dh}=0$, there will at once result

$$h=\frac{l}{\sqrt{\frac{5}{6}E}}\sqrt{q}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The value of h resulting from an application of eq. (8) gives the depth of bar which, with a given value of l, will under the conditions of the case yield the greatest intensity of bending stress k; it indicates, therefore, a limit of depth to be avoided as far as practicable.

Steel is the usual structural material for eye-bars for which E may be taken at 29,000,000. For this value of E, h will become, by eq. (8),

$$h=\frac{l}{4{,}900}\sqrt{q}.$$

In this equation $q=\frac{P}{A}$ is the intensity of uniform stress in the bar, or the "working stress."

By placing the value of h, as given by eq. (8), in the value of k (eq. 7), there will result the maximum possible bending stress in a bar of given length l and given area of cross-section A:

$$k=\tfrac{1}{3}i\sqrt{E}\frac{l}{\sqrt{q}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If $E = 29{,}000{,}000$ and $i = 0.286$ pound per cubic inch for steel, eq. (9) will take the value, for the corresponding values of h in the equation preceding eq. (9),

$$k = \frac{510l}{\sqrt{q}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The following table shows at a glance the greatest possible fiber stresses in eye-bars of different lengths and depths when the working tensile stresses in pounds per square inch are those given in the extreme left-hand column of the table:

Working Tensile Stresses in Pounds per Square Inch.	Length of Eye-bars in Feet.											
	15		20		25		30		35		40	
	Depth, Inches.	Fiber Stress, Pounds per Square Inch.	Depth, Inches.	Fiber Stress, Pounds per Square Inch.	Depth, Inches.	Fiber Stress, Pounds per Square Inch.	Depth, Inches.	Fiber Stress, Pounds per Square Inch.	Depth, Inches.	Fiber Stress, Pounds per Square Inch.	Depth, Inches.	Fiber Stress, Pounds per Square Inch.
8,000	3.3	1030	4.4	1370	5.5	1710	6.6	2050	7.7	2400	8.8	2740
10,000	3.7	920	4.9	1220	6.1	1530	7.3	1840	8.6	2140	9.8	2450
12,000	4.0	840	5.4	1120	6.7	1400	8.0	1680	9.4	1960	10.7	2240
14,000	4.3	780	5.8	1030	7.2	1290	8.7	1550	10.1	1810	11.6	2070
16,000	4.6	730	6.2	970	7.7	1210	9.3	1450	10.8	1690	12.4	1940

In using the preceding formulæ it is to be remembered that the ordinary unit of length, as well as the unit of cross-section, is the linear inch, and that the weight i of a cubic unit will then be the weight of a cubic inch. This investigation will yield results sufficiently accurate for all the usual cases of engineering practice, although it does not provide for the straightening effect of the pull P, except as producing a bending moment opposite to that of the uniformly distributed load W.

Allowance for any other distributed loading than the

weight of the bar itself, and for any eccentricity of the line of action of P that may exist, is made precisely as explained in the two paragraphs following eq. (7) of Art. 144.

Art. 146. Exact Method of Treating Combined Bending and Direct Stress.

In this method of finding the results of direct stress combined with bending it is necessary to determine an expression for the center deflection of the bar, or compression member, considered as simply supported at each end. As the line of action of the direct stress P is supposed to coincide with the original center line of axis of the bar, if g is the weight per linear unit or the latter, the bending moment M_1 in the second member of eq. (1), Art. 139, becomes

$$M_1 = \frac{g}{2} \times (l - x).$$

As this case is one in which P is tension, the general eq. (1) of Art. 139 will take the following form from a general equation* of the theory of flexure:

$$\frac{d^2w}{dx^2} = \frac{1}{EI}\left(\frac{g}{2} \times (l - x) - Pw'\right) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this equation $g = \frac{W}{l}$ is the weight per linear inch, or other unit, of the bar or member producing a bending moment opposite to that induced by the direct stress P acting with the lever-arm w'. The integration indicated in eq. (1) may be completed, but as it is not a simple

* See Eq. 7 of Art. 9 of "The Elasticity and Resistance of Materials," by Wm. H. Burr.

integration it will not be made here. As the greatest bending stress is found at the center of span the center of deflection only is needed and a different procedure may be followed.

Let w_1 represent the center deflection of the member considered, a beam simply supported at each end and carrying its own weight only, or any other total weight W uniformly distributed. It is necessary to use the expression for the work performed, or resilience of the beam in being deflected at the center by the amount w_1. Eq. (8) of Art. 29 * gives that resilience as

$$\text{Resilience} = \frac{W^2 \cdot l^3}{240EI}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In producing the center deflection w_1 the center of gravity of the weight W will descend through the distance w_0 found by placing Ww_0 equal to the resilience given eq. (2). Hence

$$w_0 = \frac{Wl^3}{240EI}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Also, since by eq. (26) of Art. 22,* $w_1 = \frac{5Wl^3}{384EI}$,

$$\frac{w_0}{w_1} = \frac{8}{25}; \quad \therefore w_0 = \frac{8}{25}w_1. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence the resilience becomes

$$\text{Resilience} = W \cdot \frac{8}{25}w_1. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

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* This reference is to "The Elasticity and Resistance of Materials," by Wm. H. Burr.

If the value of W in terms of w_1 be taken from eq. (26) of Art. 22* and substituted in eq. (5),

$$Resilience = \frac{3,072EI}{125l^3} w_1^2. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence the resilience of a bent beam varies as the square of the center deflection.

If the actual center deflection of the bar or member considered be w', the resilience of the beam when deflected to that extent will be

$$Resilience = \left(\frac{w'}{w_1}\right)^2 \frac{W^2 l^3}{240EI}. \quad . \quad . \quad . \quad . \quad (7)$$

The curvature of the bar or member being slight, the lengths (equal to each other) of the neutral surface with the deflections w' and w_1 will be, if l' and l_1 are the corresponding lengths of span or horizontal projections of the neutral surface,

$$l'\left(1 + \frac{8}{3}\frac{w'^2}{l'^2}\right) = l_1\left(1 + \frac{8}{3}\frac{w_1^2}{l_1^2}\right). \quad . \quad . \quad . \quad . \quad (8)$$

Hence

$$l' - l_1 = \frac{8}{3}\left(\frac{w_1^2}{l_1} - \frac{w'^2}{l'}\right). \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The difference $l' - l_1$ represents the movement toward or from each other of the two ends of the bar or member under the action of the direct stress or force P.

In the case of the eye-bar, the pull of the force P removes a part of the deflection w_1, and in so doing performs work in aiding to lift the weight W of the bar, the remanider of the work of lifting W being performed by the

*This reference is to "The Elasticity and Resistance of Materials," by Wm. H. Burr.

elastic efforts of the bar to straighten itself from the deflection w_1 to w', the latter portion of the work being represented by the quantity $\frac{W^2 l^3}{240EI}\left(1-\frac{w'^2}{w_1^2}\right)$. Hence the following equation of work may be written

$$\frac{P}{2}\cdot\frac{8}{3}\left(\frac{w_1^2-w'^2}{l}\right)+\frac{W^2\cdot l^3}{240EI}\left(1-\frac{w'^2}{w_1^2}\right)=W\tfrac{8}{25}(w_1-w'). \quad (10)$$

The conditions under which the work represented by eq. (10) is performed are such that either (w_1-w') or (w_1+w') may be written in the second member. The resulting numerical value of w' will be the same in both cases but affected by different signs. As the equation is written the numerical value of w' will be negative.

In eq. (10) there is taken $l'=l_1=l$, the length of panel, which may be done with essential accuracy.

Dividing both sides of eq. (10) by (w_1-w') and solving for w',

$$w'=\frac{w_1}{1+\frac{6}{25}\frac{Wl}{P}\frac{1}{w_1}}. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The deflection $w_1=\frac{5Wl^3}{384EI}$ appearing in eq. (11) is a known quantity.

After w' is determined, the resultant bending moment at the center of the bar will be

$$M'=\frac{Wl}{8}-Pw'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

If the area of cross-section of the bar is A, the maximum intensity of stress t in it will be, by eq. (2) of Art. 138,

$$t=\frac{1}{A}\left(P+M'\frac{d_1}{r^2}\right). \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Or, if the maximum value of t is specified,

$$A = \frac{1}{t}\left(P + \frac{M'd_1}{r^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

If the section is rectangular, so that $A = bh$ and $d = \frac{h}{2}$,

$$t = \frac{1}{bh}\left(P + \frac{6M'}{h}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and

$$bh = \frac{1}{t}\left(P + \frac{6M'}{h}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

When the depth of the bar is small in comparison with the length l, it may happen that the resultant or final deflection w' will be such as to make the bending moment M' equal to zero. Or

$$M' = \frac{Wl}{8} - Pw' = 0; \quad \therefore w' = \frac{Wl}{8P}. \quad . \quad . \quad . \quad . \quad (17)$$

When w' found by eq. (17) is less than w' given by eq. (11), eq. (17) is to be employed. The result shows that the bar will be subject to no bending, but that it will hang like a flexible cable. The conditions thus developed are those which indicate when a horizontal or inclined bar stressed in tension ceases to act partially as a beam and becomes purely and wholly a tie.

These formulæ are perfectly general for all cases of bars or members in tension, even for small sections as wire. Their application to individual cases will show that excessive intensities will not exist where simple tension members are held under stress in a nearly horizontal position.

Art. 147. Size of Pins.

The exact analytical determination of the pin diameter in any particular case is, like many other matters, involving the elasticity of materials impracticable, although the problem in its simplest form was subject to a mathematical investigation by Charles Bender, C.E., in *Van Nostrand's Magazine* for October, 1873. One or two reasonable assumptions, which, in a great majority of cases, must be nearly accurate, give the problem a simple character. The first of these assumptions is that the pressure applied to any pin has its center at the center of the surface of contact. Fig. 21 represents the half of a pin-connected joint, *LL* being the center line, and by this assumption the center of pressure between each of the eye-bars *A*, *B*, *C*, *D*, etc., and post bearing *P*, and the pin is located half-way between the faces of those members normal to the axis of the pin.

If, however, a pin is held by a compression member, such as an upper chord or post, the center of pressure in that member may be taken as the center of such a surface as will reduce the bearing intensity to its maximum limit.

It is to be premised that the general considerations touching the distribution of pressure between rivets and plates given in Art. 141 hold equally true for pins. The greatest allowable bearing intensity between pins and eye-bars of steel generally ranges from 15,000 to 20,000 pounds per square inch of the surface found by multiplying the diameter of pin by thickness of bar. The latter product is always considered the bearing surface.

If two bars only, such as *A* and *D*, act on each end of a pin it is clear that the center line of the latter will be convex toward *D*. The result will be a movement of the centers of pressure of those bars toward each other,

so that the lever-arm of A will be less than half the thickness of that member plus half that of B. The second assumption given above seems thus reasonable, and may be extended to the case of a pair of eye-bars, only, at the end of a pin. When, on the other hand, a number of

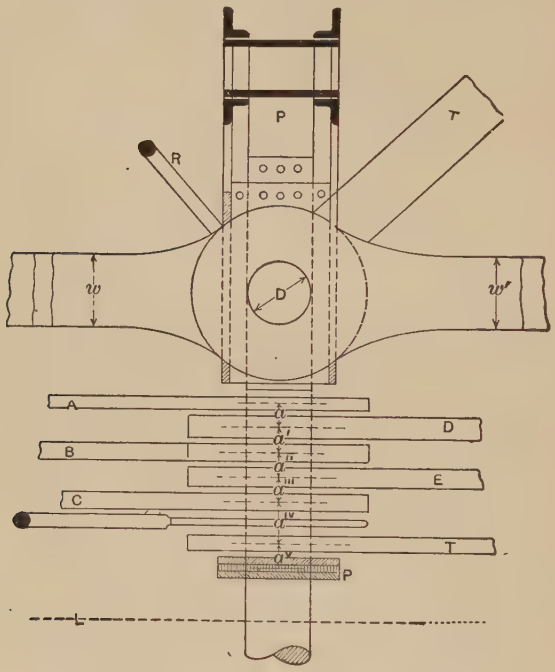


FIG. 21.

eye-bars of various sizes take hold of a pin, particularly if the bending moments have different directions at different sections of the pin, the axis of the latter may be essentially straight and the centers of pressure should be taken according to the first assumption. This is, in reality, the best practice in all cases, for if the center of pressure departs from the axis of the bar, the latter will be subjected to a bending moment equal to the tension in the bar mul-

tiplied by the distance of the center of pressure from its axis. Hence the necessity of so fixing the diameter of pin that it shall be as stiff as possible.

In Fig. 21 let a be the distance between the centers of eye-bars A and D; a' that between D and B; a'' that between B and E, etc. These distances a, a', a'', etc., should always be taken as the thickness of the head plus one eighth of an inch, the latter amount representing about the proper clearance in the best work.

Then let T_a, T_d, T_b, etc., represent the total tensions in the bars A, D, B, etc. The bending moments about the centers of those bars will then be

About center of D. $.aT_a$;
" " " B. $.(a+a')T_a - T_d a'$;
" " " E. $.(a+a'+a'')T_a + T_b a'' - T_d(a'+a'')$;
etc., etc.

The rod R is a counter and does not usually act when the pin receives its greatest bending.

The preceding moments are all similarly found and are about vertical axes until the center of the post bearing, P, is reached. The tension T of the main tension brace T, produces a moment about an axis normal to its own. Let it be supposed that the resultant moment of all the chord members A, B, C, D, and E about the center of P is right-handed looking vertically down, as shown by M' in Fig. 22.

Let M' represent that moment by any convenient scale. The moment of T by a_v, or Ta_v, will be right-handed looking upward; and let M_T represent that moment by the same scale as before. The latter line is drawn normal to the axis of the member T. The line M will now represent by the same scale the moment to which the pin is subjected at the center of P, and its direction is that of the axis of the moment.

The greatest pin bending in the lower chord will usually take place with the greatest chord stresses, but the upper chord pins will receive their greatest moments by the greatest web stresses.

When a number of bars are coupled to the pin in such a joint as that shown in Fig. 21, it is usually necessary to test a number of sections in order to find the greatest moment, unless the bars are nearly of the same size and placed alternately as shown, when the greatest moment will be found at the center of the pin.

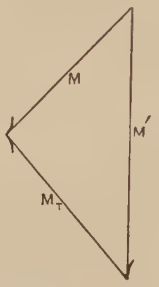


FIG. 22.

It is frequently advisable, however, to employ different sized bars in order to reduce the bending moments, a small bar being placed at the end of the pin.

The same reduction of bending moments is brought about even more effectually by the arrangement of lower-chord bars shown in Fig. 23.

In that figure it will be observed that the lower-chord eye-bars are so grouped on any one pin that the stresses in them for each half of the pin form couples which have opposite signs and thus, to a great extent, neutralize each other.

By varying the sizes or thicknesses of the bars and by resorting to the method of grouping shown in Fig. 23 (which represents a portion of an actual lower chord) the bending

moments in lower-chord pins may easily be reduced to any desired extent in any case. This method of grouping, however, prevents painting the contiguous sides of two bars in contact throughout their length and it is rarely used.

It is evident that the resultant moment shown in Fig. 22 could be obtained by resolving the stress T into its vertical and horizontal components* and combining

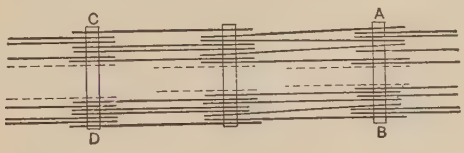


FIG. 23.

their moments with those of the lower-chord stresses, making the components of M vertical and horizontal instead of vertical and inclined, and that is the method generally employed.

The bending of pins is much increased by thickened eye-bar heads, since the thickening increases the lever-arm of the tensile stress in the eye-bar.

The preceding operations illustrate the general method of finding the bending moment to which a pin is subjected in all cases; the component moments are determined from the stresses in the individual truss members, and the resultant is then found by the moment triangle or polygon. The pin diameter is then readily found.

If M is the external bending moment, I the moment of inertia of the normal section of the pin about its diameter D, and K the intensity of stress in the fibers most

* For a detailed illustration of pin design see the authors' "Influence Lines," Chapter VI.

remote from D, it is known from the theory of flexure, since $I = \frac{\pi D^4}{64}$, that

$$M = \frac{K\pi D^3}{32}; \quad \therefore D = 2.2\sqrt[3]{\frac{M}{K}}. \quad . \quad . \quad . \quad (1)$$

If K is known, eq. (1) gives D at once after M is found by the general method exemplified by Fig. 22, or in any other manner.

For steel pins in the trusses of railway bridges, K may be taken at 20,000 pounds. This value in eq. (1) gives

$$D = 0.0821\sqrt[3]{M}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

There are numerous tables showing the bending moments of pins of all usual diameters with given values of K, so that in practice the computations expressed in eqs. (1) and (2) are seldom necessary. The value of M is determined for any particular case, after which, by the simple inspection of a table, the proper diameter may be chosen.

It is seen by eq. (1) that the diameter of a pin varies directly as the cube root of M and inversely as the cube root of K.

It may sometimes happen that M_T, in Fig. 22, is so small that it may be neglected, in which case $M = M'$.

No pin should possess a diameter less than eight tenths the width of the widest bar coupled to it.

When bending and bearing are properly provided for a safe shearing resistance will be amply secured. If the apparent moment in the pin is sufficient to cause failure by flexure, it does not, by any means, follow that failure will actually take place, for the distortion of the pin beyond the elastic limit will relieve the outside eye-bars of a larger portion (in some cases perhaps all) of the stress in them.

This result will produce a redistribution of stress in the eye-bars, by which some will be understrained and the others correspondingly overstrained. Thus, although the pin may not wholly fail, the safety of the joint may be sacrificed by the overstrained metal in the eye-bars.

Art. 148. Camber.

Camber is the curve given to the chords of a bridge, causing the center to be higher than the ends, or rather it is the amount of rise of the center above the ends. It is given to a truss so that the chords may not fall below a horizontal line when the load is applied. Fig. 24 represents a truss with exaggerated camber. The actual amount varies from $\frac{1}{1000}$ to $\frac{1}{1800}$ of the span.

Camber may be given to a truss either by lengthening the upper chord or shortening the lower one; the latter

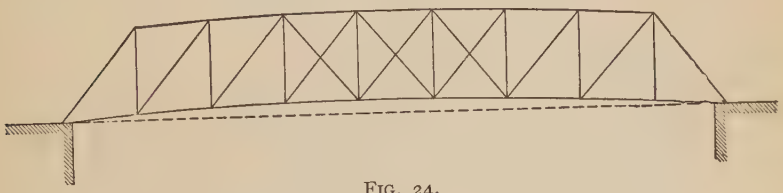


FIG. 24.

method may be preferable because the upper chord is sometimes not horizontal, and different panel lengths would have to be shortened by different amounts.

On account of the unavoidable play at the joints of all work, the shortening of the lower chord, or lengthening of the upper, must be increased by about $\frac{1}{32}$ of an inch per panel in order to secure the desired camber.

The lower-chord shortening is made uniformly throughout its length; that is, each panel length is shortened by a constant quantity. The true chords will, therefore,

become arcs of circles of large radii, and vertical posts will become radial.

By means of the equation of the circle, $y^2 = 2Rx - x^2$, R being the radius, the amount of shortening or lengthening of chord to produce a given camper may be determined, if the play at the joints be omitted. In the equation above y represents the half-span and x the camber desired, hence the radius

$$R = \frac{y^2 + x^2}{2x}.$$

This is the radius of the lower chord when cambered. Generally it will be near enough to put $R = \frac{y^2}{2x}$.

The angular length of the lower chord will be

$$\alpha = 2 \sin^{-1} \frac{y}{R} = 2 \sin^{-1} \frac{2xy}{y^2 + x^2},$$

and the length in feet

$$l = R\alpha.$$

The length of the upper chord will then be

$$l' = (R + d)\alpha.$$

The difference in length of chords will be

$$D = l' - l = d\alpha.$$

This is the amount by which the lower chord is to be shortened, or the upper lengthened, in order to produce the required camber, if no play or strains exist.

Since x is very small compared with y,

$$\alpha = 2 \sin^{-1} \frac{2xy}{y^2+x^2} = \frac{4xy}{y^2+x^2} = \frac{4x}{y} \text{ (nearly).}$$

If the span is l_1,

$$\alpha = \frac{8x}{l_1}; \quad \text{and} \quad D = \frac{8dx}{l_1}.$$

If r is such a ratio that $d = rl_1$,

$$D = 8rx.$$

On account of the play at the joints, x should be taken a little larger than the camber desired.

Frequently r is about one eighth, and for such a value

$$D = x;$$

or, neglecting the play at the joints, the difference in lengths of the chords should equal the camber.

If the chords are to be horizontal under the greatest loads, while T and C represent the supposed uniform intensities of tension and compression in the lower and upper chords respectively, E and E' representing the coefficients of elasicity,

$$\tfrac{1}{2}D = \frac{T}{E}l = \frac{C}{E'}l'.$$

This formula can only be approximate, for the chords are never exactly uniformly stressed.

Since d, the depth of truss, does not vary, these formulæ apply only to trusses of uniform depth.

A through-truss has been supposed, but the same formulæ exactly apply to a deck-bridge.

It is to be borne in mind that one half the horizontal distance between the centers of end pins is to be taken for y in determining R. If this distance is assumed in designing the truss, then the panel length is to be found by dividing l or l' by the number of panels.

If the panel length is first assumed, and the camber produced by shortening or lengthening it, this horizontal distance is essentially equal to the assumed chord length diminished or increased by $D = d\alpha$.

In order to hold the camber in a truss, the diagonals must be shortened, as shown in Fig. 25. The diagonal

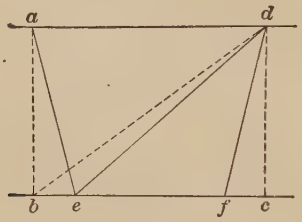


FIG. 25.

which was bd before cambering becomes ed afterward. ad and bc are supposed to be panels in the upper and lower chords respectively before putting in the camber; afterward bc becomes ef, while ad remains the same; the lower chord is supposed to be shortened. Let x be the amount of shortening of each panel of the lower chord $= 2be = 2fc$, d the depth of the truss, and p the original panel length equal to ad. Then

$$ed = \sqrt{dc^2 + ec^2} = \sqrt{d^2 + \left(p - \frac{x}{2}\right)^2}.$$

If the camber is produced by lengthening the upper chord, then ef is the original panel length, and ad the new one, and

$$ed = \sqrt{dc^2 + ec^2} = \sqrt{d^2 + \left(p + \frac{x}{2}\right)^2}.$$

In a triangular truss the diagonal gc, Fig. 26, is changed to

$$gf = \sqrt{d^2 + \tfrac{1}{4}(p - x)^2}.$$

If the upper chord is lengthened, cg is the diagonal desired, and ff' the original panel length p. Hence

$$cg = \sqrt{d^2 + \tfrac{1}{4}(p + x)^2}.$$

Each diagonal is to be shortened to the length ed.

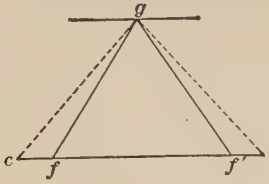


FIG. 26.

In a drawbridge, each arm, in giving the camber, can be considered one span, but the whole amount of shortening in the lower chord of one arm must also be taken out of the upper chord at the center. If this is not done, the ends will sink below their original positions.

Art. 149. Economic Depth of Trusses with Parallel Chords.

The so-called economic depth of truss for a given span is that depth which involves the least material or weight of metal in the bridge. This depth depends upon the intensity of moving load for each truss, the length of panel, the greatest allowable stresses, etc. Various mathematical investigations have been made with a view to the

determination of this depth of truss in terms of the length of span, but on account of the exceedingly intricate character of the problem, any feasible analysis must be based upon assumptions which simplify the analytical operations, but render the results only approximately true. These investigations, however, and the experience of American engineers, show that a depth varying from one fifth to one seventh the length of span will give the least weight of truss; the former for very heavy loads, as in two-truss double-track bridges, and the latter for light loads.

When the span becomes long, i.e., 400 to 500 feet, the depth of truss increases to an unusual height, and the cost of erection is correspondingly large. The depth is then frequently taken not larger than one eighth the span, or even less.

Again, local conditions, such as the necessarily uniform depth (for the sake of appearance) of adjacent spans of varying length, sufficient depth of short spans for overhead bracing (necessary for lateral stability), etc., in the majority of cases exclude the use of the economic depth, even if it were exactly known.

It is to be borne in mind also that the lightest truss is not necessarily the cheapest. That bridge is the most economical which can be made ready for traffic for the least money.

Facility in working up details, and the least possible amount of time in the shop, are important elements in every design.

In fact the lightest weight does not make the most economical bridge, for the reason that the shop cost per pound is greater than with a somewhat increased weight of metal. When it is borne in mind that a considerable variation may be made from the depth of least weight without affecting that weight to any considerable extent

(as actual computations show to be the case), it is easy to understand that the truly economic depth is materially less than that which gives precisely the least weight of material.

Long panels are an economic feature of any bridge possessing a system of floor-beams and stringers, as well as conducive to other points of merit. The resulting concentration of metal not only leads to less weight and rate of cost in the shop, but enhances, also, the stiffness and stability of the individual members.

For economy in weight, long panels require a greater depth than shorter panels.

This much may be said in regard to continuous trusses: On account of the existence of the points of contraflexure, they require considerably less depth than trusses that are not continuous, used on the same points of support. The depth of the latter, therefore, is a limit which should never be reached by the depth of the former.

Art. 150. Safety Factors and Working Stresses.

Although the subjects of safety factors and working stresses properly belong to the domain of the resistance of materials, they may here be touched upon in a general manner.

The fixed weight of a long span-bridge is much greater per foot than that of a short span. Again, it has been seen in a preceding article that the moving load per foot for a long span may be less than that for a short span. For both these reasons the variations of stress in passing from a loaded to an unloaded condition are much greater in the material of a short span than that of a long one. Consequently the material will be much more fatigued in a short span than in a long one.

Although the subject of fatigue of metals is yet in

an unsettled state, it is clearly established that these conditions of stress in short spans demand a larger safety factor or smaller working stress than those in long spans.

Again, in any bridge or truss whatever carrying a moving load, some parts are subject to a much greater variation of stress in the process of first being subject to and then relieved of loads than others.

Counterbraces may not be, and probably are not, strained at all by the fixed load; but they take a proper working stress under the action of the moving load.

The condition of loading for greatest stress in any main web member, except those at the ends, is a partial covering of the span, but the fixed load is distributed over the whole span. Hence the variation of stress in the main web members will be greatest at the middle of the span and least at the end. At the center, however, the variation is much less than in the counterbraces.

The fatigue of the material, therefore, requires that the greatest safety factors or least working stresses be found in the counterbraces, and that the working stresses in the main web members at the center be greater than those in the counterbraces, but less than those in the main web members at the ends of the truss.

The disposition of the moving load for the greatest chord stresses is, in all cases, essentially the same as that of the fixed load. Hence the variation of stress will be essentially the same throughout the chords, and the safety factor or working stress may be uniform throughout each chord, the safety factor being the same as that in the end web members sustaining the same kind of stress.

If a structure is to carry a fixed load only, the safety factor may be three for wrought iron and steel and but little greater for good qualities of cast iron and timber. As a rule, however, cast iron and timber require a much larger safety factor than wrought iron and steel. Local

circumstances affect, to a great extent, working stress. If the risk (respecting life and property) attending failure is small, the safety factor may be small also. But if the risk is great, the safety factor must be correspondingly great.

In the truss members of long-span bridges steel of the safety factor may vary from three and a half or four to five; but in short spans of the same material they should vary from about five to six.

Good cast-iron members should be found with safety factors varying from six to ten, while those for timber may vary from eight to twelve.

It is not to be supposed from these large safety factors that the determination of the stresses or the character of the various materials is so excessively uncertain. It is certainly true that there is some indetermination in these respects, but only a little in comparison with that connected with the mode of application of the moving load.

With a perfect condition of track, a rapidly moving train is supposed by many to approximate very closely to a suddenly applied load, although it is quite certain that it does not. For this reason some engineers have doubled the moving loads in making their calculations and then fixed the values of the safety factors as if all loads were gradually applied.

But no track is in perfect condition, and all rough places, or lack of continuity, such as rail joints more or less open, produce shocks which cause greater stress than any suddenly applied loads. The amounts of these last stresses are indeterminate, for the extent of their causes can scarcely be determined.

Again, Mr. J. W. Cloud, C.E., at the Philadelphia meeting of the American Institute of Mining Engineers, February, 1881, pointed out the existence of certain hitherto un-

recognized stresses, such as those caused by the vertical component of the thrust of the connecting-rod of a locomotive, which alternates in direction twice in each revolution of the driving-wheels, thus producing a pulsating effect, as well as those which arise from the lack of balance of the driving-wheels in a vertical direction.

All these causes produce stresses which it is impossible to measure, and the safety factor must cover all uncertainties.

It is possible that a more highly perfected track and the production of more nearly uniform material in connection with an extended experience may justify the reduction of safety factors.

Art. 151. The Resistance of Solid Metallic Rollers.

An approximate expression for the resistance of a roller may easily be written, and although the approximation may be considered a loose one, it furnishes an excellent basis for an accurate empirical formula.

The following investigation contains the improvements by Prof. J. B. Johnson and Prof. H. T. Eddy on the method originally given by the author.

The roller will be assumed to be composed of indefinitely thin vertical slices parallel to its axis. It will also be assumed that the layers or slices act independently of each other.

Let E' be the coefficient of elasticity of the metal over the roller;

E be the coefficient of elasticity of the metal of the roller;

R be the radius of the roller and R' the thickness of the metal above it;

w = intensity of pressure at A;

p = " " " " any other point;

P = total weight which the roller sustains per unit of length;

x be measured horizontally from A as the origin;

$d = AC$;

$e = DC$.

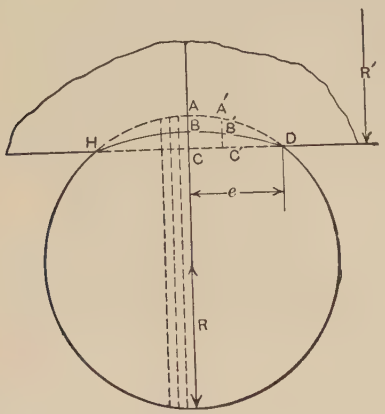


FIG. 27.

From Fig. 27,

$$AB = \frac{wR}{E}; \quad A'B' = \frac{pR}{E}.$$

$$BC = \frac{wR'}{E'}; \quad C'B' = \frac{pR'}{E'}.$$

$$\therefore d = AC = AB + BC = w\left(\frac{R}{E} + \frac{R'}{E'}\right), \quad . \quad . \quad (1)$$

and

$$A'C' = A'B' + B'C' = p\left(\frac{R}{E} + \frac{R'}{E'}\right). \quad . \quad . \quad . \quad . \quad (2)$$

Dividing eq. (2) by eq. (1),

$$p = A'C'\frac{w}{d}.$$

But

$$P=\int_{-e}^{+e} pdx=\frac{w}{d}\int_{-e}^{+e} A'C'dx.$$

If the curve DAH be assumed to be a parabola, as may be done without essential error, there will result

$$\int_{-e}^{+e} A'C'dx=\frac{4}{3}ed.$$

Hence

$$P=\frac{4}{3}we. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

But

$$e=\sqrt{2Rd-d^2}=\sqrt{2Rd} \text{ (nearly).}$$

By inserting the value of d from eq. (1) in the value of e, just determined, then placing the result in eq. (7),

$$P=\frac{4}{3}\sqrt{2w^3R\left(\frac{R}{E}+\frac{R'}{E'}\right)}. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If $R=R'$,

$$P=\frac{4}{3}R\sqrt{2w^3\frac{E+E'}{EE'}}. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The preceding expressions are for one unit of length. If the length of the roller is l, its total resistance is

$$P'=Pl=\frac{4}{3}l\sqrt{2w^3R\left(\frac{R}{E}+\frac{R'}{E'}\right)}. \quad . \quad . \quad . \quad . \quad (6)$$

Or, if $R=R'$,

$$P'=\frac{4}{3}Rl\sqrt{2w^3\frac{E+E'}{EE'}}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In ordinary bridge practice eq. (7) is sufficiently near for all cases.

A simple expression for conical rollers may be obtained by using eqs. (4) or (5).

As shown in Fig. 28, let z be the distance parallel to the axis of any section from the apex of the cone; then consider a portion of the conical roller whose length is dz.

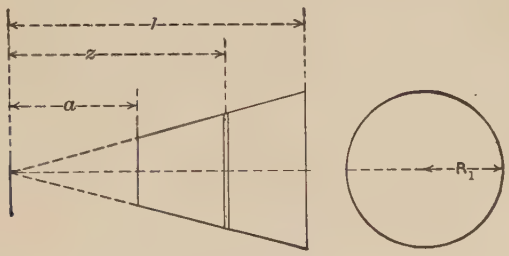


FIG. 28.

Let R_1 be the radius of the base. The radius of the section under consideration will then be

$$R = \frac{z}{l} R_1,$$

and the weight it will sustain, if $R_1 = R'$,

$$dP' = \frac{4}{3} \frac{R_1}{l} \sqrt{2w^3 \frac{E + E'}{EE'}} \cdot z dz.$$

Hence

$$P' = \int_a^l dP' = \frac{4}{3} \frac{l^2 - a^2}{2l} R_1 \sqrt{2w^3 \frac{E + E'}{EE'}}. \quad . \quad . \quad (8)$$

Eqs. (6), (7), and (8) give ultimate resistances if w is the ultimate intensity of resistance for the roller.

It is to be observed that the main assumptions on

which the investigation is based lead to an error on the side of safety.

If for steel $w = 15{,}000$ pounds per square inch, and $E = E' = 30{,}000{,}000$ pounds, eq. (5) gives

$$P = \frac{8}{3} R \sqrt{\frac{w^3}{E}} = 900R.$$

Theodore Cooper's specifications require, § 53, that the pressure per linear inch of steel rollers shall not exceed $1200\sqrt{d}$, d being the diameter of the roller in inches.

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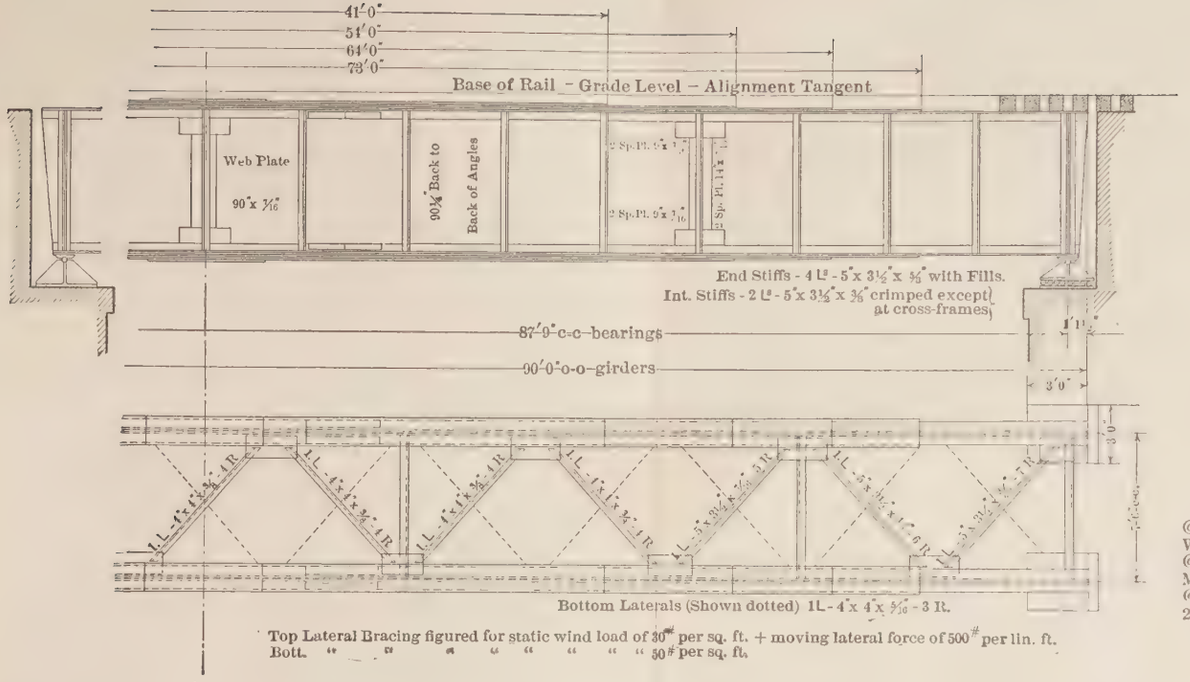
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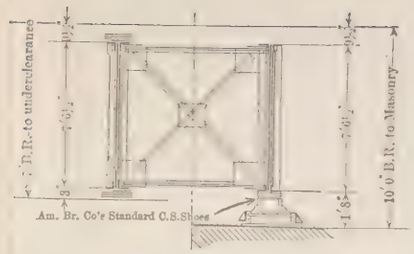
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INTER. FRAME.
Top and bottom—1 angle 4″ × 4″ × 5/16″.
Diags.—1 angle 4″ × 4″ × 5/16″—3 R.

END FRAME.
Top and bottom—1 angle 4″ × 4″ × ¾″.
Diags.—1 angle 4″ × 4″ × ¾″—4 R.

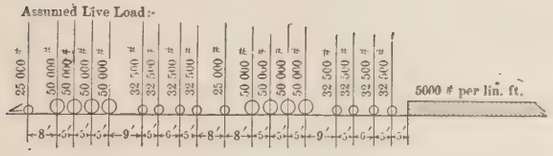
GIRDERS: 87′ 9″ center to center of bearings.

MAX. SHEARS.
D = 34700 lbs.
L = 168000 "
I = 130200 "
332900 lbs.
@ 10000 lbs. = 33.3 sq. ins.
Web plate, 90″ × 7/16″ = 39.4 sq. ins.
@ 400 lbs. = 832 sq. ins.
Masonry bearing, 36″ × 36″ = 1296 sq. ins.
@ 2400 lbs. = 139 lin. ins., 4″ rollers.
2½″ rivet spacing at end of flange.

MAX. MOMENTS.
D = 9[illegible]000 in.-lbs.
L = 38265000 "
I = 29580000 "
76970000 in.-lbs.
÷ 90″.5 = 850000 lbs. @ 17000 lbs. = 50.0 sq. ins.
⅛ web = 4.9 sq. ins.
2 angles 6″ × 6″ × ¾″ 16.9 sq. ins. gross or 13.9 sq. ins. net.
1 cov. pl. 16″ × ⅜″ = 9.0 " " " " 7.9 " "
1 " " 16″ × [illegible] = [illegible] " " " " [illegible] " "
1 " " 16″ × ⅜″ = 9.0 " " " " 7.9 " "
1 " " 16″ × 9/16″ = 9.0 " " " " 7.9 " "
50.4 sq. ins.

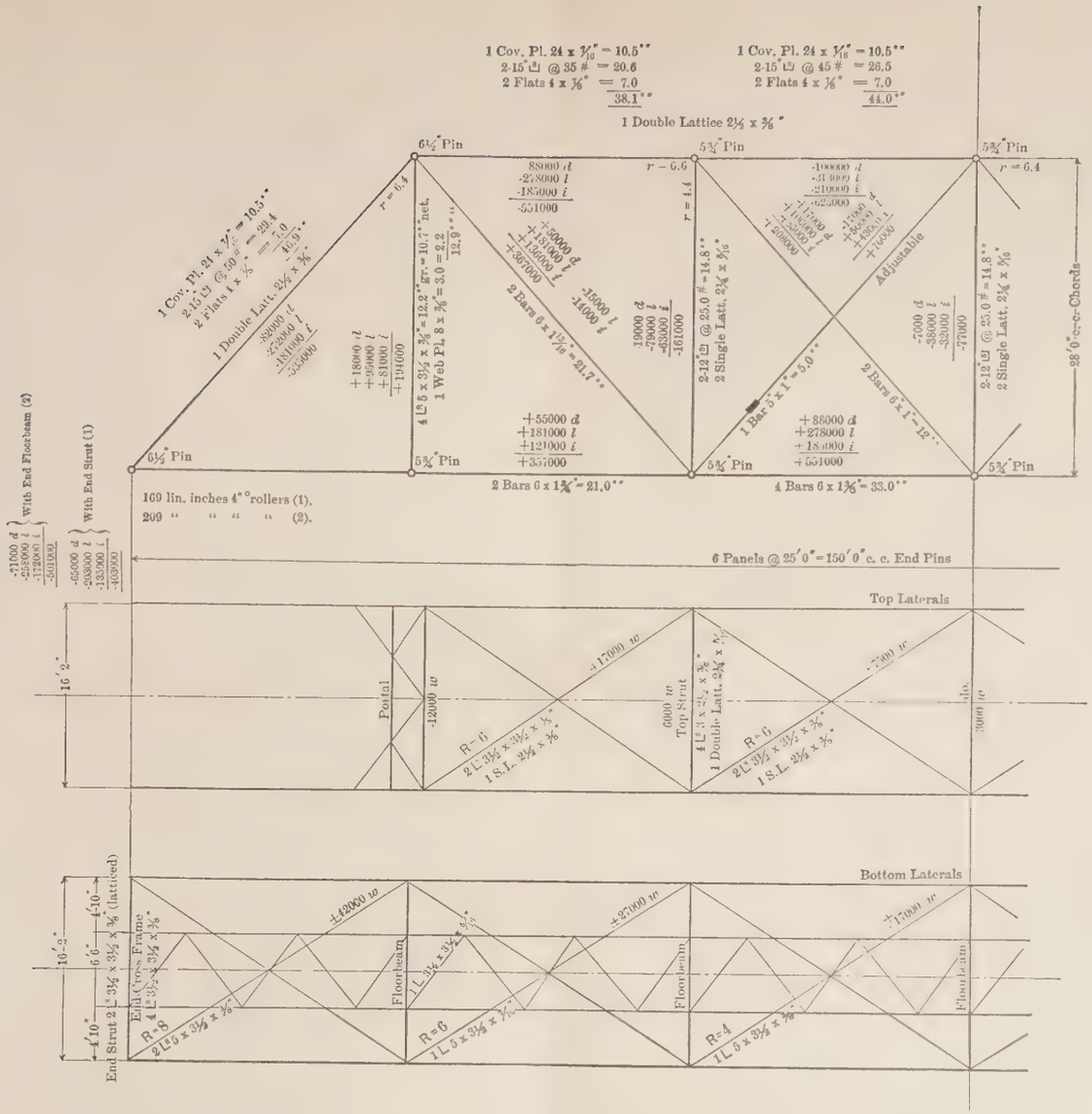
ASSUMED DEAD LOAD:
Track, 500 lbs.
Steel, 1080 "
1580 lbs. per lin. ft.

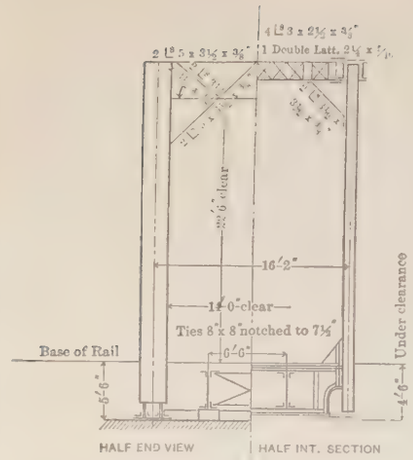
Assumed Live Load:-



SPECIFICATIONS: Am. Br. Co., Rwy., 1900.
Material, medium open-hearth steel.
Rivets, ⅞″ diameter.
Ties, notched to 9½″.

STRESS SHEET FOR A SINGLE-TRACK, DECK PLATE-GIRDER SPAN, 90′ 0″ OUT TO OUT. COOPER'S E 50 LOADING. STANDARD DESIGN.
AMERICAN BRIDGE CO.





STRINGERS: 25′ 0″ center to centre floor-beams.
Weight of track = 400 lbs. per lin. ft. = 5000 lbs. per stringer.
Weight of one Stringer = 4500 "
9500 lbs.

Max. Shear: 5000 lbs. *d*; 71000 " *l*; 66000 " *i*
142000 lbs. × 11.8/14.4 = 116000.

Max. Moment: 356000 in.-lbs. *d*; 4575000 " *l*; 4223000 " *i*
9154000 in.-lbs.

Flange stress = 9154000 ÷ 37.8 = 242200 lbs.
" area = 242200 ÷ 17000 = 14.2 sq. ins.
2 angles 6″ × 6″ × 9/16″ = 12.8 sq. ins. gross = 11.8 sq. ins. net
⅜ web = 2.6 " " "
14.4 sq. ins.
Web plate, 41 × ½ ″ = 20.5 sq. ins. No intermediate stiffeners.

12 rivets required through flange in depth of web
28 field rivets required in end connection.
End connections, 5 × 3½ × 7/16″ angles.

INTERMEDIATE FLOOR-BEAMS: 16′ 2″ center to center trusses.
Max. Shear: 11000 lbs. *d*; 95000 " *l*; 81000 " *i*
187000 lbs. × 9.5/12.5 = 140000.

Max. Moment: 638000 in.-lbs. *d*; 5510000 " *l*; 4690000 " *i*
10838000 in.-lbs.

Flange stress = 10838000 ÷ 50.2 = 215000 lbs.
" area = 215000 ÷ 17000 = 12.6 sq. ins.
Web plate, 52″ × ½″ = 26 sq. ins. × ⅛ = 3.3 sq. ins. net.
2 angles 6″ × 4″ × ⅝″ = 10.6 sq. ins. gr. = 9.5 " " "
12.8 sq. ins. net.

17 rivets required through flange between post and stringer. 36 field rivets required in end connection.
End connections, 5″ × 3½″ × 7/16″ angles

END FLOOR-BEAMS: 12′ 4″ b. b. End angles.
Max. Shear: 6000 lbs. *d*; 77000 " *l*; 71000 " *i*
154000 lbs. × 6.4/8.8 = 112000.

Max. Moment: 210000 in.-lbs. *d*; 2690000 " *l*; 2480000 " *i*
5380000 in.-lbs.

Flange stress = 5380000 ÷ 56.8 = 94600 lbs.
" area = 94600 ÷ 17000 = 5.6 sq. ins.
Web plate, 58½″ × ⅜″ = 22.0 sq. ins. × ½″ = 2.8 sq. ins. net.
2 angles 6″ × 4″ × ⅝″ = 7.2 sq. ins. gross = 6.4 " " "
9.2 sq. ins. net.

End floor-beam to be used only when specified.

ASSUMED DEAD LOAD:
Track = 400 lbs. per lin. ft. = 5000 lbs. per panel per truss
Floor = 520 " " " " = 6500 " " " " "
Trusses = 1060 " " " " = 13200 " " " " "
24700 lbs. per panel per truss

Exposed wind surface of bridge = 120 sq. ft. per panel per top chord.
200 " " " " " bottom "

Top lateral load	= 50 lbs. per sq. ft.
Bottom lateral static load	= 30 " " " "
Bottom lateral moving load	= 500 " " " "

Assumed Live Load-E 50



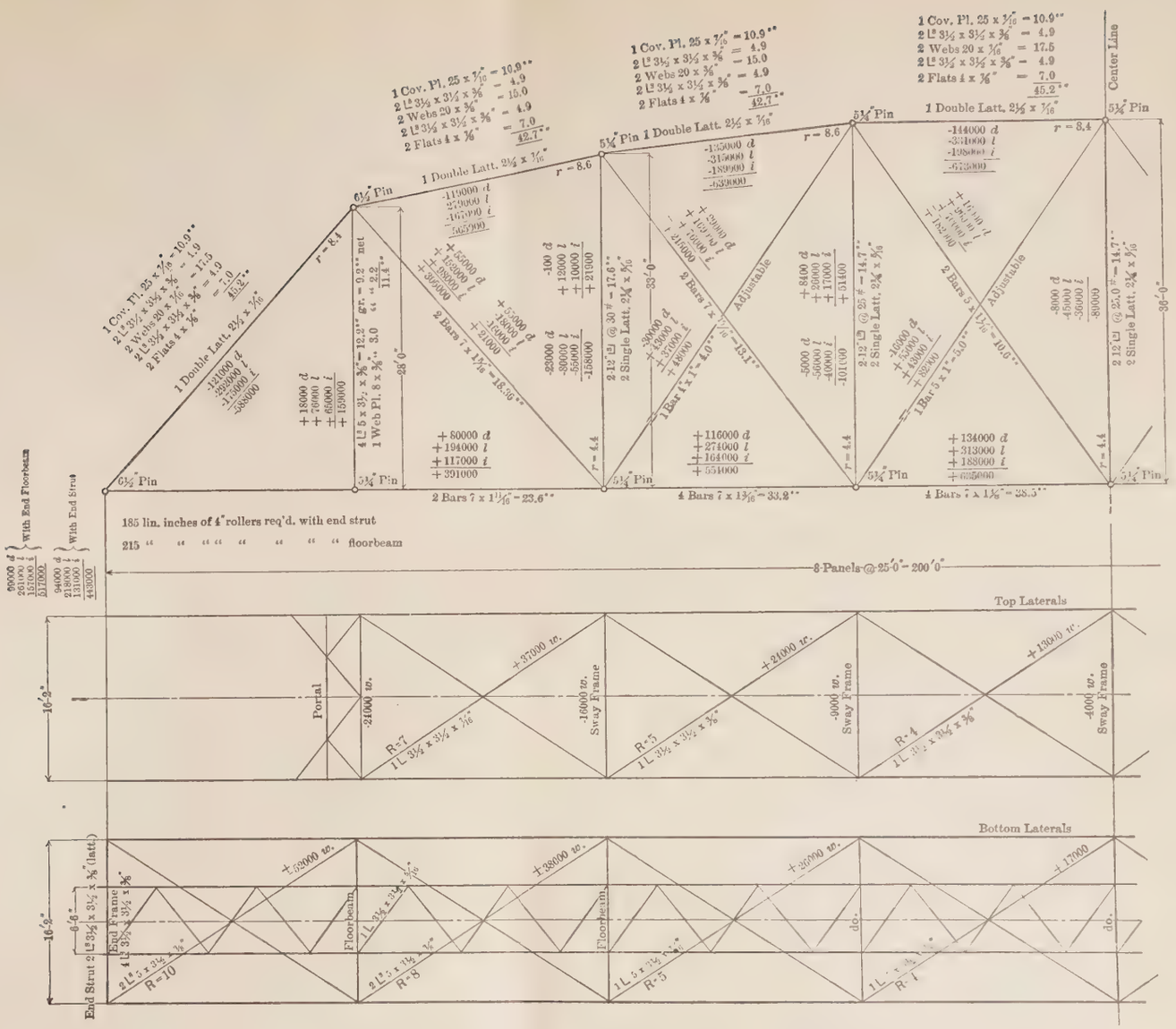
SPECIFICATIONS: American Bridge Co., 1900.
Material, medium open-hearth steel.
Rivets in flanges of 12″ channels and in angles under 3½″ to be ¾″ diameter.
All others to be ⅞″ diameter.

+	denotes tension;
−	" compression;
d	" dead-load stress;
l	" live-load stress;
i	" impact stress;
w	" wind stress;
r	" least radius of gyration;
R	" number of ¾″ field rivets;
R_1	" " " ⅞″ " "
e	" eccentricity.

AMERICAN BRIDGE COMPANY.

STRESS SHEET FOR A STANDARD SINGLE-TRACK, THROUGH, PIN-CONNECTED SPAN, 150′ 0″, CENTER TO CENTER END PINS.

COOPER'S E 50 LOADING.


Center Line
Under Clearance
Base of Rail
HALF END VIEW
HALF INT. SECTION
Top Laterals
Bottom Laterals
Portal
Sway Frame
Floorbeam
End Strut
8 Panels @ 25'0" = 200'0"
185 lin. inches of 4" rollers req'd. with end strut
215 " " " " " " " " floorbeam
6½" Pin
5¼" Pin
Adjustable
1 Double Latt. 2½ x 7/16"
2 Bars 7 x 1 15/16" = 23.6"
4 Bars 7 x 1 15/16" = 33.2"
4 Bars 7 x 1⅛" = 28.5"
2 Bars 7 x 1 7/16" = 13.1"
1 Bar 7 x 1" = 7.0"
1 Bar 5 x 1" = 5.0"
2 Bars 3 x 1" = 6.0"
1 Web Pl. 8 x 3/8" = 3.0 " = 22.2" net
1 Cov. Pl. 25 x 7/16" = 10.9"
2 L 3½ x 3½ x 3/8" = 4.9
2 Web 20 x 3/8" = 15.0
2 L 3½ x 3½ x 3/8" = 4.9
2 Flats 4 x 7/8" = 7.0
42.7"
45.2"
2 L 5 x 3½ x 7/8"
2 L 3½ x 3½ x 7/16"
2 L 3½ x 3½ x 3/8"
2 L 3½ x 3½ x 5/16"
2 L 3½ x 3½ x 3/8" (latt.)
2 L 3½ x 3½ x 5/16"
2 L 3½ x 3½ x 3/8"

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